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Original Research Article

Prediction of the Reliability of a System with Multiple Redundant Elements

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Abstract

The present paper focuses on the problem of active redundancy in the general case where several participating elements are kept on standby. This study assumes that the failure distribution of all redundant elements, as well as the switch and the failure detector, all follow the exponential law characterized by a constant failure rate. The analysis of the different success modes of the system considered, first in the specific case of a system with a single element on standby, then with two, and finally with three elements on standby, this analysis has allowed us to establish a recurrence relation that provides the analytical expression of the reliability of an active redundancy system in the general case where several elements are kept on standby.

Keywords: Reliability; Safety; Standby Redundancy; Active Redundancy; Passive Redundancy.

Nomenclature

E_1	Main primary operating element
$E_2 \ldots E_n$	Standby elements
E_d	Failure detection
E_C	Switching mechanism in the redundant system
$R(t)$	Reliability function
$f(t)$	Failure density function
λ	Failure rate of main and standby element
λ_d	Failure rate of the detector
R_c	Constant reliability of Commutator
$\cap$	Symbol of events intersection
$\bar{C}$	Logical complement of event C

1. Introduction

During the design phase, the prediction of the reliability of a complex system to accomplish its mission up to a well-determined time is very important [1-5], especially when dealing with non-repairable systems that operate until the first failure [6-13].

Sometimes, despite the improvement of operating conditions and even despite the improvement of the technology of the constituent elements [8,9,14,15], the system considered may not accomplish its task within the planned time from the design phase. In these conditions, redundancy is the only technical way to increase reliability and extend its average lifetime [5,8,9], [16-23].

Technical systems with redundancy are, therefore, systems for which the failure of any element of their structure does not lead to the failure of the system considered, and consequently, the system with redundancy can accomplish its mission in a relatively longer time than when the system operates without redundancy.

Depending on the operating mode of the main device (the one imposing a mission from the outset) with the auxiliary device (the one on standby), redundancy can be either:

- Passive (hot), related to a main device and an auxiliary device operating [24-25]
- Active (cold), related to a main device and an auxiliary device operating successively, one after the other [26-32]

The literature on this subject is extensive, but it is mainly based on purely theoretical assumptions; it assumes that the switch and the detector are ideal and that the number of standby units is limited [6-7], [18-20].

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The present article deals with the case of active redundancy in the general case where, instead of a single one, several devices are kept on standby (Figure 1). This type of redundancy requires a means of failure detection which, in real-time, monitors the availability status of the operating element, and which, in the event of a failure of the latter, informs the switch to switch the mission to the next element.

In what follows, the study of this system will be limited first to the case where the system considered has a single element in standby, then two elements in standby, and finally, three elements in standby are considered.

In this study, all elements are assumed to be characterized by a constant failure rate λ, the switch characterized by a constant switching probability R_c throughout the system's operating period, and the failure detector by a constant failure rate λ_d. [33-39]

The analysis and the aborted illustration of the different success modes of an active redundant system, first in the easy case when only one standby element, second case with two standby elements and finally with three standby elements, this analysis has led us to determine analytically the general expression of the reliability function of the considered system with any number of standby elements.

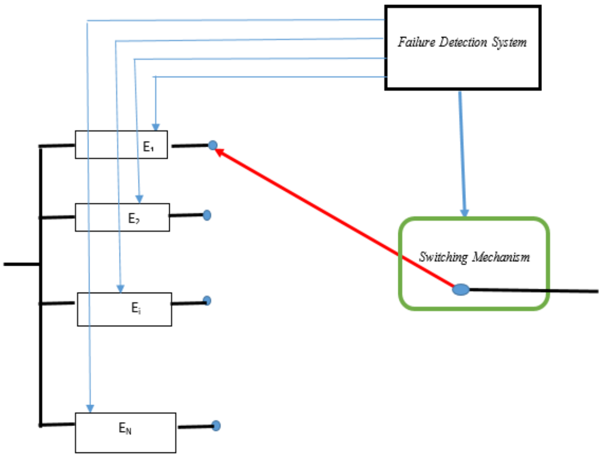

Figure 1. The generalized structure of a system with multiple redundant elements

2. A General View of Distribution Models

The proposed models in many textbooks and journal papers are numerous, such as the famous Weibull model, which represents an extension and generalization of the exponential distribution described in this paper.

- *Exponential distribution:* given by the failure distribution function:

$$F(t) = 1 - \exp(-\lambda t) \quad (1)$$

When λ represents the constant hazard rate.

Weibull distribution: given by the failure distribution function:

$$F(t) = 1 - exp\left[-\left(\frac{t}{a}\right)^b\right] \quad (2)$$

characterized by 2 parameters:
(a: scale parameter, b: shape parameter)

Complementary Weibull distribution: given by the failure distribution function:

$$F(t) = exp\left[-\left(\frac{t}{a}\right)^{-b}\right] \quad (3)$$

Also, like the Weibull distribution, characterized by 2 parameters (a: scale parameter, b: shape parameter)

Power distribution (Firkowicz distribution): given by the failure distribution function:

$$F(t) = \left(\frac{t}{a}\right)^b \quad (4)$$

This type of distribution describes the general case of the bathtub curve of failure rate when b<1

Mackay-Hame distribution: given by the failure distribution function:

$$\mathrm{F(t)} = 1 - \exp\left[1 - \exp\left(\frac{t}{a}\right)^{b}\right] \quad (5)$$

Gaussian distribution (Normal distribution): given by the failure density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left[-\frac{Z^2}{2}\right] \quad (6)$$

when

$$z = \frac{t-\mu}{\sigma} \quad (7)$$

Log-Normal Distribution: given by the failure density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left[-\frac{z^2}{2}\right] \quad (8)$$

With

$$z = \frac{\ln(t)-\mu}{\sigma} \quad (9)$$

The limitation of the exponential model is that the failure rate of components or systems is independent, and the failure rate is considered as a constant during the lifetime.

This approach represents an approximation of reality and gives us the possibility to evaluate and calculate the characteristics of reliability of more complex systems and situations.

The reason is that it has been documented quite fully that in most cases, components failure rate is time-dependent, it results, of course, in time-dependent failure rate of the components and units. For example, the electronic components show a strictly decreasing failure rate. In contrast, mechanical components, as well as assembles, show a strictly increasing failure rate.

In these cases, when the constituent components have failure rates that deviate from the exponential model and instead follow, for example, the Weibull distribution, obtaining reliability characteristics for systems presents especially analytical challenges. That is why we assumed in the following analysis to follow the exponential model distribution.

3. Case of an active redundancy system with a single standby element

The illustration in Figure 2 allows a clear visualization of the two possible success modes for an active redundancy system with a single standby element.

- ✓ Success Mode 1:
  - The main element E_1 operates successfully for the entire mission duration until time t, or
- ✓ Success Mode 2:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C then transfers the mission to the standby element E_2 and
  - Element E_2 continues and completes the remaining mission, from time t_1 until time t.

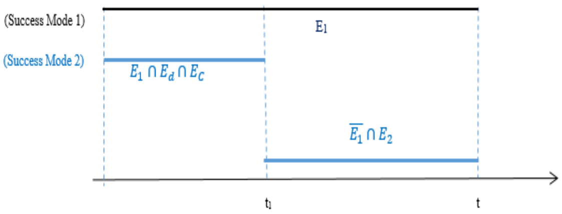

Figure 2. Illustration of success modes for an active redundancy system with a single standby element

Therefore, the reliability of this redundant system with one main element and one standby element can be expressed as a sum of the probabilities of these two success modes [8-10]:

$$R_2(t) = Prob(T_1 > t) + Prob[(T_1 \le t_1) \cap (T_d > t_1) \cap (T_c > t_1) \cap (T_2 > t - t_1)] \quad (10)$$

when

$$Prob(T_i > t) = R_i(t) \quad (11)$$

And

$$Prob(T_i \le t) = f_i(t) = -\frac{dR_i(t)}{dt} \quad (12)$$

From the previous relations (12) and (13), R_2 (t) can be expressed as:

$$R_2(t) = R_1(t) + \int_0^t f(t_1) R_d(t_1) R_c(t_1) R(t - t_1) dt_1 \ (13) \quad (13)$$

When $R_i(t)$ represents the reliability of a single element, and the failures follow an exponential distribution characterized by a constant failure rate λ_i, then:

$$R_i(t) = e^{-\lambda_i t} \quad (14)$$

and the failure density function

$$f_i(t) = -\frac{dR_i(t)}{dt} = \lambda_i e^{-\lambda_i t} \quad (15)$$

Then

$$R_2(t) = e^{-\lambda t} + \int_0^t e^{-\lambda_d t_1} R_C \, e^{-\lambda t_1} e^{-\lambda(t - t_1)} dt_1 \quad (16)$$

After integration and simplification expression (16), the reliability of this redundant system with one main element and one standby element can be expressed as:

$$R_2(t) = e^{-\lambda t}\left[1 + R_C \frac{\lambda}{\lambda_d}\left(1 - e^{-\lambda_d t}\right)\right] \quad (17)$$

2. Case of an active redundancy system with two standby elements

The illustration in Figure 3 allows clear visualization of the three possible success modes for an active redundancy system with one main element E_1, and two standby elements E_2 and E_3.

- ✓ Success Mode 1:
  - The main element E_1 operates successfully for the entire mission duration until time t, or
- ✓ Success Mode 2:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C then transfers the mission to the standby element E_2 and
  - Element E_2 continues and successfully completes the remaining mission, from time t_1 until time t without needing the second standby element E_3, or
- ✓ Success Mode 3:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C then transfers the mission to the standby element E_2 and
  - The second standby element E_2 fails at a time t_2 (with $t_1 < t_2 < t$) and
  - The failure detector E_d detects this failure at time t_2 and
  - The switch E_C then transfers the mission to the second standby element E_3 and
  - The second standby element E_3 continues and completes the remaining mission from time t_2 until time t.

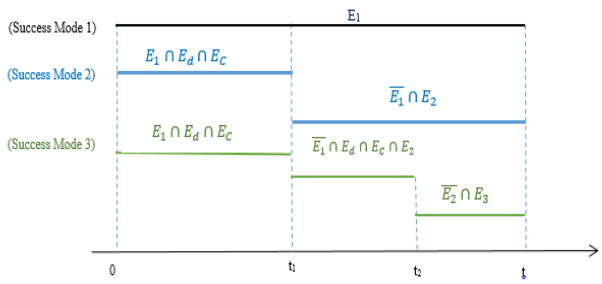

Figure 3. Illustration of success modes for an active redundancy system with two standby elements

Therefore, the reliability of this redundant system with one main element and two standby elements can be expressed as a sum of the probabilities of these three success modes:

$$R_3(t) = Prob(T_1 > t) + Prob[(T_1 \leq t_1) \cap (T_d > t_1) \cap (T_c > t_1) \cap (T_2 > t - t_1)] + Prob[(T_1 \leq t_1) \cap (T_d > t_1+t_2) \cap (T_c > t_1+t_2) \cap (T_2 \leq t_2) \cap (T_3 > t - (t_1+t_2))] \quad (18)$$

Let's now observe that the first two terms of the relation given by (18) represent exactly the reliability function of a system with a single standby element as given by relation (17) consequently.

$$R_3(t) = R_2(t) + P_3(t) \quad (19)$$

with

$$P_3(t) = Prob[(T_1 \leq t_1) \cap (T_d > t_1+t_2) \cap (T_c > t_1+t_2) \cap (T_2 \leq t_2) \cap (T_3 > t - (t_1+t_2))] \quad (20)$$

Taking into account the notations given by relations (11) and (12), we can proceed as follows:

$$P_3(t) = \int_0^t \int_0^{t-t_1} f(t_1) R_d(t_1 + t_2) R_C(t_1 + t_2) f(t_2) R(t - (t_1+t_2)) dt_2 dt_1 \quad (21)$$

By replacing the expressions of $R_i(t)$ and $f_i(t)$ given by relations (14) and (15) in expression (21), we obtain:

$$P_3(t) = \int_0^t \lambda e^{-\lambda t_1} \int_0^{t-t_1} e^{-\lambda_d (t_1+t_2)} R_C e^{-\lambda t_2} e^{-\lambda (t-t_1-t_2)} dt_2 dt_1 \quad (22)$$

Finally

$$P_3(t) = \left(\frac{\lambda}{\lambda_d}\right)^2 R_C e^{-\lambda t} \left[1 - e^{-\lambda_d t} - t \lambda_d e^{-\lambda_d t}\right] \quad (23)$$

The finally expression of the system reliability function of an active redundancy system with one main element and two standby elements can be obtained from the expressions of $R_2(t)$ given by relation (17) and $P_3(t)$ given by relation (23).

4. Case of an Active Redundancy System with Three Standby Elements

The illustration in Figure 4 allows clear visualization of the three possible success modes for an active redundancy system with one main element E_1, and three standby elements E_2, E_3 and E_4.

- ✓ Success Mode 1:
  - The main element E_1 operates successfully for the entire mission duration until time t, or
- ✓ Success Mode 2:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C transfers the mission to the standby element E_2 and
  - Element E_2 continues and successfully completes the remaining mission, from time t_1 until time t without needing the second standby elements E_3 and E_4, or
- ✓ Success Mode 3:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C then transfers the mission to the standby element E_2 and
  - The second standby element E_2 fails at a time t_2 (with $t_1 < t_2 < t$) and
  - The failure detector E_d detects this failure at time t_2 and
  - The switch E_C then transfers the mission to the second standby element E_3, and
  - The thirst standby element E_3 continues and completes the remaining mission, from time t_2 until time t or
- ✓ Success Mode 4:
  - The main element E_1 fails at a time t_1 (with $t_1 < t$) and
  - The failure detector E_d detects this failure at time t_1 and
  - The switch E_C then transfers the mission to the standby element E_2 and
  - The second standby element E_2 fails at a time t_2 (with $t_1 < t_2 < t$) and
  - The failure detector E_d detects this failure at time t_2 and
  - The switch E_C then transfers the mission to the second standby element E_3 and
  - The thirst standby element E_3 fails at a time t_3 (with $t_2 < t_3 < t$) and
  - The failure detector E_d detects this failure at time t_3 and
  - The switch E_C then transfers the mission to the thirst standby element E_4 and
  - The fourth standby element E_4 continues and completes the remaining mission from time t_3 until time t.

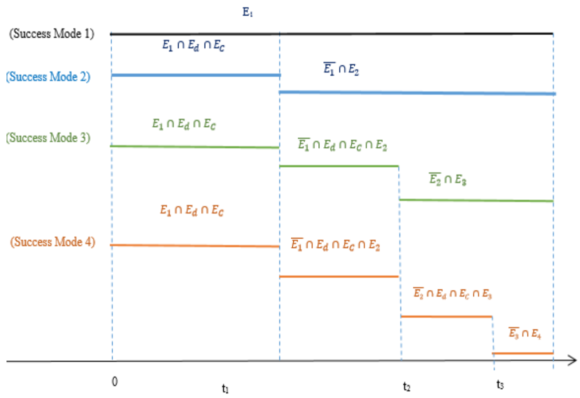

Figure 4. Illustration of success modes for an active redundancy system with three standby elements

Therefore, the reliability of this redundant system with one main element and three standby elements can be expressed as the sum of the probabilities of these four success modes:

$$R_4(t) = Prob(T_1 > t) + Prob[(T_1 \leq t_1) \cap (T_d > t_1) \cap (T_c > t_1) \cap (T_2 > t - t_1)] + Prob[(T_1 \leq t_1) \cap (T_d > t_1+t_2) \cap (T_c > t_1+t_2) \cap (T_2 \leq t_2) \cap (T_3 > t - (t_1+t_2))] + Prob[(T_1 \leq t_1) \cap (T_d > t_1+t_2+t_3) \cap (T_2 \leq t_2) \cap (T_3 \leq t_3) \cap (T_4 > t - (t_1+t_2+t_3))] \quad (24)$$

Let's now observe again that the first three terms of the relation given by (24) represent exactly the reliability function of a system with two standby elements expressed by $R_3(t)$ as given by relation (17), (19) and (23) consequently:

$$R_4(t) = R_3(t) + P_4(t) \quad (25)$$

with

$$P_4(t) = Prob[(T_1 \leq t_1) \cap (T_d > t_1+t_2+t_3) \cap (T_2 \leq t_2) \cap (T_3 \leq t_3) \cap (T_4 > t - (t_1+t_2+t_3))] \quad (26)$$

Taking again into account the notations given by relations (11) and (12), we can proceed as follows:

$$P_4(t) = \int_0^t \int_0^{t-t_1} \int_0^{t-t_1-t_2} f(t_1)\, R_d(t_1 + t_2 + t_3) R_C(t_1 + t_2 + t_3) f(t_2) f(t_3) R(t - (t_1+t_2+t_3)) dt_3 dt_2 dt_1 \quad (27)$$

By replacing again the expressions of $R_i(t)$ and $f_i(t)$ given by relations (14) and (15) in expression (27), we obtain:

$$P_4(t) == \int_0^t \int_0^{t-t_1} \lambda e^{-\lambda t_1} \lambda e^{-\lambda t_2} \lambda e^{-\lambda t_3} R_c e^{-\lambda_d (t_1+t_2+t_3)}. \, . e^{-\lambda_d (t-t_1-t_2)} dt_3 dt_2 dt_1 \quad (28)$$

After integration and simplification expression (19), we obtain:

$$P_4(t) = \left(\frac{\lambda}{\lambda_d}\right)^3 R_C . e^{-\lambda t} . \left[1 - e^{-\lambda_d . t} (t . \lambda_d) e^{-\lambda_d . t} \frac{1}{2} (t . \lambda_d)^2 e^{-\lambda_d . t}\right] \quad (29)$$

The final expression of the system reliability function of an active redundancy system with one main element and three standby elements can be obtained from the expressions of $R_3(t)$ given by relation (19) and $P_4(t)$ given by relation (29). Therefore, we can conclude that, in general, the reliability function of a system with n redundant elements can be expressed by the following recurrence relation:

$$R_n(t) = R_{n-1}(t) + P_n(t) \quad (30)$$

with

$$P_n(t) \left(\frac{\lambda}{\lambda_d}\right)^{n-1} R_C . e^{-\lambda t} \sum_{k=0}^{n-2} \left[1 \frac{(t . \lambda_d)^k}{k!} e^{-\lambda_d . t}\right] \quad (31)$$

5. Conclusion and future scope

The analysis of the different success modes of an active redundant system has led us to determine analytically the expression of the reliability function, first in the case of a system with one and two standby elements and then for a system with any number of standby elements. Furthermore, the generalization of the result obtained by recurrence constitutes an extension of the various existing works, which, in fact, only represent a particular case of the results obtained in the present paper.

The same technique adopted for this analysis can also applied for other more complicated failure rate functions, such as the Weibull distribution. Rather too complex to derive analytically the prediction system reliability function, therefore, statistical modeling commonly called Monte Carlo simulation can be employed.

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Conflicts of Interest

The authors declare no conflict of interest.

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