

# Reliability Modeling and Performance Evaluation of Sachet Water Production System Using Gumbel- Hougard Family Copula

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## Abstract

For both big and small flows, reverse osmosis is particularly effective at treating brackish, surface, and groundwater. Pharmaceutical, boiler feed water, food and beverage, metal finishing, and semiconductor production are a few examples of businesses that employ RO water. This research establishes a method for testing the performance reliability of RO systems. The RO can turn unrestricted amounts of impure water into portable drinkable water without releasing carbon dioxide or other contaminants into the atmosphere. Because of these advantages, RO has been increasingly incorporated to meet pure water demand. In the present research, we consider a reverse osmosis system that is made up of six components. Raw water tank, with two units of tanks, one of them needs to be operational at a time. The second subsystem is the sand filter. The sand filter is needed for the system operation. The third subsystem is the activated carbon filter. Two out of three consecutive units of the activated carbon filter are necessary for the operation. Subsequently, the precision filter has one unit. And the unit is essential for the operation. The RO membrane is the next subsystem. In this paper, one out of the three is essential for the system to be in operation. Finally, the last subsystem is the water-producing tank. One out of one of the water-producing tanks is necessary for operation. Availability, mean time to failure (MTTF), cost analysis, and reliability are discussed in the paper.

**Keywords:** Reverse osmosis; reliability; water; availability; filter.

## Notation

$K_n$  Failure rate for the subsystems. Where  $n=1,2,3,4,5,6$ .

$\lambda_m$  Repair rate for the subsystems with incomplete failure Where  $m=1,2,3,4,5,6$ .

$\psi(k)$  Repair rate for the subsystems with complete failure. Where  $k= x, y, z, l, m, \text{ and } n$ .

$P_0$  Denote initial state where the system is working perfectly.

$P_1$  Denote state with an incomplete failure in subsystem 1 due to failure of first unit and copula repair is busy in repairing the failed unit.

$P_2$  Denote state with a complete failure in subsystem 2 due to failure of first unit and repair machine is busy in repairing the failed unit

$P_3$  Denote state with a complete failure in subsystem 2 due to failure of the only unit in the subsystem.

$P_4$  Denote state with an incomplete failure in subsystem 3 due to failure of first unit and repair machine is busy in repairing the failed unit.

$P_5$  Denote state with complete failure in subsystem 3.

$P_6$  Denote state with an incomplete failure in subsystem 4

$P_7$  Denote incomplete state of system due to failure of a second unit from subsystem 5

$P_8$  Denote complete state of system due to failure of a the third unit from subsystem 5

$P_9$  Denote complete failure state in subsystem 6

$P_{10}$  Denote state with an incomplete failure in subsystem 1 and 3. Repair machine is busy in repairing the failed units.

$P_{11}$  Denote state with an incomplete failure in subsystem 1,3 and 5. Repair machine is busy in repairing the failed units.

$P_{12}$  Denote complete failure state due to failure of any unit from either of the subsystem 1, 3 or 5

## 1. Introduction

Because of rising demand and limited water supplies, direct reuse of recovered wastewater is becoming increasingly crucial. It has been performed in many nations where water reuse is required to meet the demand for various reasons. Abubakar and Singh [1] investigated the performance evaluation of an industrial system using a copula linguistic method. Dhillon and Mishra [2] conducted a comparison of two hollow fiber module configurations (inside/out and outside/in). Dhillon and Rrayayoti, analyze the reliability of non-maintained parallel systems subject to hardware failure and human error [3]. Barosz et al. [4] studied the efficiency analysis of manufacturing lines with industrial robots and Human Operators. Teresa and Sponza [5] also investigated the effects of permeate flow and pressures on pollutant parameter removals. E. O. Ezugbe and S. Rathilal [6] Experiments were conducted using pure water and NaCl solutions ranging from 15 g/L to 300 g/L, as well as two distinct fiber materials and structures. Gahlot et al. [7] used copula linguistics to analyze the performance of a repairable system in series design under various forms of failure and repair strategies. Garud R. M [8], and Gupta et al. [21] investigated different reliability metrics of STP generators using the RAMD method at the component level. For all generating subsystems, mathematical models based on the Markovian birth-death process have been constructed. For example, Gulati et al. [9] reported the performance study of a complex system in the series configuration under multiple failure and repair disciplines using copula. Malik et al. [10] presented the reliability study of a repairable network system consisting of three computer laboratories linked to a server in a 2-out-of-3: G arrangement. Lado and Singh [11] investigated the cost evaluation of a complicated repairable system composed of two subsystems in series utilizing the Gumbel–Hougaard family copula. Using Gumbel-Hougaard family copula distribution. Singh et al. used copula linguistics to evaluate the efficiency of a repairable device in a series configuration under various types of failure and repair policies [12]. Singh et al. [13] investigated the cost analysis of an engineering system involving two subsystems in a series configuration with controllers and human failure. Lindhe, A. et al. [14] studied a computation of an approximate dynamic fault tree for modeling water supply hazards System Safety and Reliability Engineering. Maihulla and Yusuf [15] studied the Reliability, availability, maintainability, and dependability analysis of the photovoltaic system. The study regarding the reliability modeling and performance evaluation of the solar photovoltaic system using the Gumbel–Hougaard family copula was carried out by Maihulla et al. [16]. Maihulla and Yusuf [17] also demonstrated the Markov modeling for a photovoltaic system using the method adopted in the present research. The computer model is based on diffusion and convection transport mechanisms, as well as the concentration

polarization idea, and is used to estimate the performance of a RO membrane when varied feed water concentrations and feed flow rates are used. Maihulla and Yusuf [18] studied the filtration system of a reverse osmosis machine using the Copula approach. Vacuum membrane distillation (VMD) is a method of desalinating saltwater. Maihulla et al. [19] looked at the series-parallel system and evaluated it as a model with five input factors (feed temperature, feed total dissolved solids (TDS), trans-membrane pressure (TMP), feed flow rate, and time) and two output parameters (permeate TDS and flow rate) for predicting the performance of a seawater reverse osmosis system (SWRO). Muhammad et al. [20] analyzed the reliability and performance Analysis of two units that Active Parallel System Attended by two repairable machines. Some phenols chosen as phytochemical indicators were quantified using ultrahigh-performance liquid chromatography (UHPLC). To treat the pollutants (Total Organic Carbon (TOC), Dissolved Organic Carbon (DOC), Total Phosphorus (TP), Total Nitrogen (TN), and total polyphenols) present in the olive mill wastewater, a sequential Direct Contact Membrane Distillation (DCMD) and a Reverse Osmosis (RO) hybrid membrane system were used. Nelson [22] presented An Introduction to Copulas. Niwas and Garg [23] studied the Selection of Optimal Software Reliability Growth Models Using an Integrated Entropy-TOPSIS Approach. Tailor [24] created a parallel system's availability and reliability under imperfect repair and replacement: analysis and cost optimization. P. Biniiaz et al [25] explored how an ecologically acceptable, cost-effective, and energy-efficient membrane distillation (MD) method may reduce pollution produced by industrial and residential wastes. The reliability analysis of two state repairable parallel redundant systems under human failure by Gupta and Sharma [26]. R. Tundis et al. [27] studied The chemical profile, as well as the antioxidant and anti-obesity effects, of concentrated fractions derived from micro-filtered OMW processed by direct contact membrane distillation (DCMD), was investigated. Rajesh et al. [28] investigated the reliability and availability analysis of a three-unit gas turbine power-producing system with seasonal effect and FcFs maintenance pattern. Ram et al. [29] performed a performance ability analysis of a system with a 1-out-of-2: G scheme with flawless reworking. S. Sadri [30] created feed water pressure, membrane specifications, and feed water characteristics. Srivastava et al. [31]. Y. Li and K. Tian [38] conducted a study of the principles and categorization of membrane distillation, with an emphasis on the variables influencing it and ways to improve its efficacy. Singh and Rawal [32] investigated on Cost Analysis of an Engineering System involving subsystems in series Configuration. Singh and Ayagi [34] provided a copula-based analysis of dependability metrics for a system composed of two subsystems in series. Gulati et al. [35] investigated the performance and cost evaluation of a repairable complex system with two subsystems connected in series. Tamraz [36] analyzed

the Availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization. Kumar et al. [37] presented their research on the dependability of non-repairable phased-mission systems with shared bus performance. Lee et al. [38] study the artificial neural network model for optimizing the operation of a seawater reverse osmosis desalination plant. Siong et al. provided a performance evaluation of a fuzzy logic-based congestion minimization method for sensor networks [39]. Lee et al. [40] created a desalination plant that was then used to simulate feed water temperature. The two designs' pure water permeability and global heat transfer coefficient were compared. Yang et al. [41] investigated the reliability and availability of backup systems with working vacations and failed component retrials. Yang et al. presented their research on the reliability and availability analysis of standby systems with working vacations and retry of failed components [42]. Yang and Abbaspour [43] studied an In Beijing, an investigation of wastewater reuse scenarios was conducted. Yusuf [44] presented the reliability analysis of a communication network with redundant relay stations under partial and total failure. Reliability for the performance of two active units using the copula approach was carried out by Yusuf et al. [45]. Maihulla and Yusuf [46] presented the reliability modeling and performance analysis of a reverse osmosis machine in water purification using Gumbel–Hougaard family copula.

The findings indicate that the potential for wastewater reuse is significant at competitive pricing. They also determined that the agricultural irrigation and urban leisure sectors are the primary consumers of treated wastewater. It is described how hydrodynamics affects global heat and mass transport coefficients. The freshwater shortage has been identified as one of the major issues that humans must solve in the twenty-first century. RO is a technique that separates and removes dissolved solids, organics, pyrogens, submicron colloidal debris, color, nitrate, and bacteria from water using semi-permeable spiral wound membranes. Under pressure, feed water is supplied via a semipermeable membrane, where water penetrates the membrane's minute holes and is delivered as filtered water known as permeate water. These models are quite useful in analyzing generator dependability, maintainability, and availability. Numerous scholars have already been recognized for their contributions in the subject of reliability theory by analyzing the performance of complicated repairable systems under various forms of failure and repair distribution. The previous literature presented their work on the reliability and performance study of certain serial systems, claiming improved system performance. Little is known about assessing the reliability and performance of a serial-parallel system of reverse osmosis (R.O). We provide a unique model in this paper. This model is made up of six serial subsystems. The transition diagram is used to generate a system of partial differential equations, which are then solved using the Copula approach to

produce system strength reliability characteristics such as reliability, availability, mean time to failure (MTTF), and profit function. The purpose of this research is to develop dependability models in order to evaluate the system's strength. This article's results will be beneficial to plant managers, industries, and manufacturing systems that desire to use RO machines.

Because of a serious issue with system and subsystem failure in the water purification industries, we are interested in reverse osmosis machine systems. Aside from its significance in people's lives all around the world, water filtration technology is moving slowly forward. Industries are putting a lot of effort into keeping up with the increasingly complicated machine systems. However, in the existing literature, the evaluation of the performance of reverse osmosis system for sachet water production scares. The needs, therefore, arise especially using the Gumbel-Hougaard Family copula approach. The paper is divided into several parts. The introduction is defined in Section 1, with an emphasis on the significant literature review conducted for the study of the proposed model. The state description and notation that will be used to analyze the suggested model are covered in section two. The dependability models and controlling equations for the system are covered in section three. The result analysis is covered in Section 4 of the text. This includes creating tables with the Maple program and the corresponding graphs. A conclusion in Section Five brings the essay to a close.

## 2. State Description and Notation

### 2.1 Subsystem 1. (Raw water tank)

Raw water tanks are used to temporarily hold raw water before it is treated. This versatile tank may be completely and conveniently folded for travel. There is no need for rods or poles because the tank is pop-up. Failure of the second unit in a pair of parallel subsystems can result in system failure in its entirety.

### 2.2 Subsystem 2 (sand filter)

**Sand Filter:** This filter widely employed in water filtration. Sand filters remove suspended debris using a totally different method [47]. The water travels through a bed of filter medium, typically 0.75 mm sand 750 mm deep, as opposed to small orifices through which particles cannot pass. It is one of a series of interconnected subsystems, and if it fails, the entire system may also fail.

### 2.3 Subsystem 3 (activated carbon filter) Activated

**carbon filter:** Siong et al. [39] employed this filter to clean water without leaving any dangerous chemicals behind. A prototype is being created employing a UV radiation system and activated carbon for the purification of water. Analysis of surface area and porosity. To compare the surface morphology of GAC-A and GAC-B, a magnified image of each was obtained using scanning

electron microscopy (SEM). The activated carbon filter has three parallel subsystems; failure of any two of them can result in the failure of the other two.

**2.4 Subsystem 4. (Precision filter)**

**Precision filter:** The cylinder's exterior is typically composed of stainless steel, while the interior uses tubular filter elements made of PP melt-blown, wire-burned, folded, titanium, activated carbon, and other materials as needed to meet the requirements for effluent quality. For the precision filter, one out of every series of subsystems, the failure of anyone can result in the failure of the entire system.

**2.5 Subsystem 5. (RO membrane)**

**Reverse Osmosis Membrane:** The high velocity of the wastewater flowing along the filter maintains the flow turbulent, which helps manage the thickness of the solids on the filter and lowers filter clogging [24]. This is how RO membranes are typically used as cross-flow filters. One of the three parallel RO membrane subsystems was taken into consideration; if any one of the other two fails, the entire system will also fail.

**2.6 Subsystem 6. (Water producing tank)**

**Water producing Tank:** Now pure, the water is poured into a tank and maintained under pressure there until the faucet is opened. Two bladders pressurize the water in the tank so that it can enter and exit as needed. The water barely fills the tank to around two-thirds of the water intake pressure even though the tank is always under pressure. At the bottom of the tank is a bladder containing compressed air, and at the top is a butyl water bladder, which is a thick material like the liner of a steel food can. When you turn on the faucet, the intake valve opens to let more water in while also maintaining a steady amount of pressure that forces the water out in a constant stream. The Water Producing Tank has one out of a number of subsystems, and if anyone fails, the system as a whole may also fail.

failure occurs when any of the subsystems fail. When the system fails completely, the copula is used to fix it. There are fourteen states in the system, seven of which are operating and seven of which are full failure states (see Figure 2). Table 1 gives a brief explanation of the states.

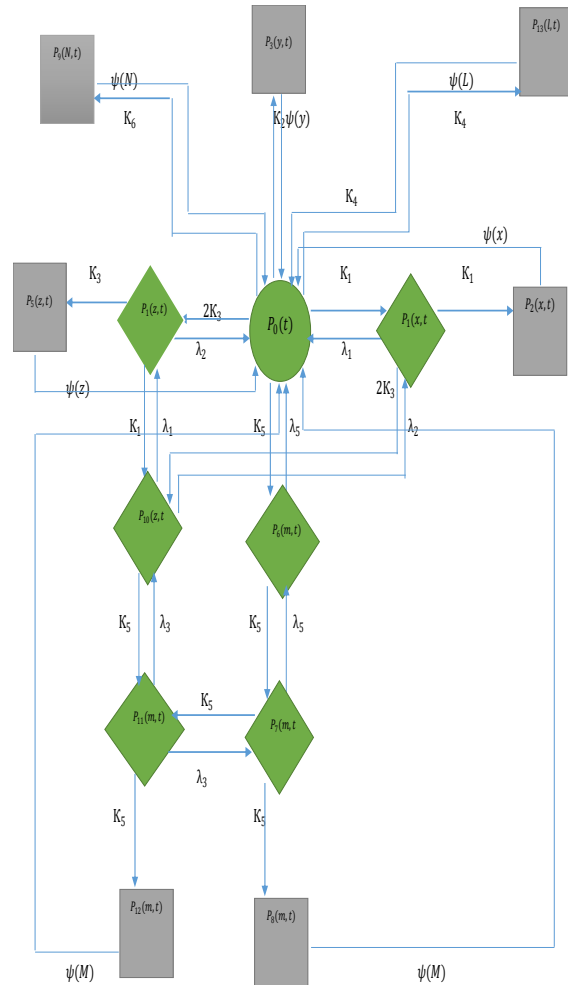


Figure 1. State transition diagram

**3. SYSTEM RELIABILITY MODEL AND DESCRIPTION OF THE MODEL**

The model depicted in Figure 1 consists of six subsystems A, B, C, D, E and F configured as series-parallel. Subsystem A (Raw water tank) has two identical units working as 1-out-of-2, subsystem B (Sand filter) has one unit working as 1-out-of-1, subsystem C (Carbonated filter) has three units working as 2-out-of-3. Subsystem D (Precision filter) arranged in parallel working as 1-out-of-1. Subsystem E (Reverse osmosis membrane) has three units working as 2-out-of-3. Finally, subsystem F (Water production tank) has three units working as 2-out-of-3. There are two types of system failures: partial and total failure. Partial failure happens when a unit in a subsystem fails but the system continues to function, whereas total

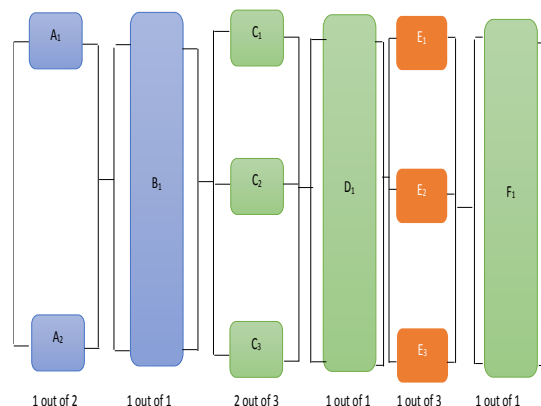


Figure 2. Reliability block diagram of the system

### 3.1 Formulation and Solution of Mathematical Model

The Gumbel-Hougaard copula is an asymmetric Archimedean copula that has more positive tail reliance than negative tail dependence. By the probability of considerations and continuity of arguments, the following set of difference-differential equations is associated with the above mathematical model. Using the method adopted by Nelson (2006), Gulati et al. [35], Singh and Ayagi [34], Gahlot et al. [7], Lado et al. [13], Lado and Singh [12], Singh and Poonia [33], and Maihulla and Yusuf [46]. The system of differential-difference equations obtained from Figure 2 is presented below:

$$\left[ \frac{\partial}{\partial t} + K_1 + K_2 + 2K_3 + K_4 + K_5 + K_6 \right] P_0(t) = \int_0^\infty \lambda_1 P_1(x, t) dx + \int_0^\infty \lambda_2 P_4(z, t) dz + \int_0^\infty \lambda_3 P_6(m, t) dm + \int_0^\infty \psi(x) dx + \int_0^\infty \psi(x) P_2(x, t) dx + \int_0^\infty \psi(l) P_{13}(l, t) dl + \int_0^\infty \psi(y) P_3(y, t) dy + \int_0^\infty \psi(N) P_9(N, t) dN + \int_0^\infty \psi(z) P_5(z, t) dz + \int_0^\infty 2\psi(m) P_8(m, t) dm \tag{1}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + K_1 + 2K_3 \right] P_1(x, t) = 0 \tag{2}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \psi(x) \right] P_2(x, t) = 0 \tag{3}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y) \right] P_3(y, t) = 0 \tag{4}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_1 + K_1 + K_3 \right] P_4(z, t) = 0 \tag{5}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \psi(z) \right] P_5(z, t) = 0 \tag{6}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \lambda_3 + 2K_5 \right] P_6(m, t) = 0 \tag{7}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \lambda_3 + K_5 \right] P_7(m, t) = 0 \tag{8}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \psi(m) \right] P_8(m, t) = 0 \tag{9}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial N} + \psi(N) \right] P_9(N, t) = 0 \tag{10}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_1 + \lambda_2 + K_5 \right] P_{10}(z, t) = 0 \tag{11}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial m} + 2\lambda_3 + K_5 \right] P_{11}(m, t) = 0 \tag{12}$$

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \psi(m) \right] P_{12}(m, t) = 0 \tag{13}$$

The above equations (1) to (13) represent the transition states for each of the 13 units of the transition diagram. Using their corresponding boundary conditions below, the set of differential equations can be solved using the Gumbel-Hougaard Family copula by taking the Laplace transformation.

**Boundary Conditions**

- $P_1(0, t) = K_1 P_0(t)$  (14)
- $P_2(0, t) = K_1^2 P_0(t)$  (15)
- $P_3(0, t) = K_2 P_0(t)$  (16)
- $P_4(0, t) = 2K_3 P_0(t)$  (17)
- $P_5(0, t) = K_4 P_0(t)$  (18)
- $P_6(0, t) = K_5 P_0(t)$  (19)
- $P_7(0, t) = K_5^2 P_0(t)$  (20)
- $P_8(0, t) = K_5^3 P_0(t)$  (21)
- $P_9(0, t) = K_6 P_0(t)$  (22)
- $P_{10}(0, t) = 4K_1 K_3 P_0(t)$  (23)
- $P_{11}(0, t) = (K_5 K_1 K_3 + K_5^2) P_0(t)$  (24)

$$P_{12}(0, t) = (K_5^2 K_1 K_3 + K_5^2) P_0(t) \tag{25}$$

$$\text{Initial condition } P_0(t) = 1 \tag{26}$$

$$\text{Other } P_0(t) = 0$$

Taking Laplace transformation of equation (1)–(25) and using the equation with the help of (26), one can obtain

$$\left[ s + K_1 + K_2 + 2K_3 + K_4 + K_5 + K_6 \right] \bar{P}_0(s) = \int_0^\infty \lambda_1 \bar{P}_1(x, s) dx + \int_0^\infty \lambda_2 \bar{P}_4(z, s) dz + \int_0^\infty \lambda_3 \bar{P}_6(m, s) dm + \int_0^\infty \psi(x) dx + \int_0^\infty \psi(x) \bar{P}_2(x, s) dx + \int_0^\infty \psi(l) \bar{P}_{13}(l, s) dl + \int_0^\infty \psi(y) \bar{P}_3(y, s) dy + \int_0^\infty \psi(N) \bar{P}_9(N, s) dN + \int_0^\infty \psi(z) \bar{P}_5(z, s) dz + \int_0^\infty 2\psi(m) \bar{P}_8(m, s) dm \tag{27}$$

$$\left[ s + \frac{\partial}{\partial x} + \lambda_1 + K_1 + 2K_3 \right] \bar{P}_1(x, s) = 0 \tag{28}$$

$$\left[ s + \frac{\partial}{\partial x} + \psi(x) \right] \bar{P}_2(x, s) = 0 \tag{29}$$

$$\left[ s + \frac{\partial}{\partial y} + \psi(y) \right] \bar{P}_3(y, s) = 0 \tag{30}$$

$$\left[ s + \frac{\partial}{\partial z} + \lambda_1 + K_1 + K_3 \right] \bar{P}_4(z, s) = 0 \tag{31}$$

$$\left[ s + \frac{\partial}{\partial z} + \psi(z) \right] \bar{P}_5(z, s) = 0 \tag{32}$$

$$\left[ s + \frac{\partial}{\partial m} + \lambda_3 + 2K_5 \right] \bar{P}_6(m, s) = 0 \tag{33}$$

$$\left[ s + \frac{\partial}{\partial m} + \lambda_3 + K_5 \right] \bar{P}_7(m, s) = 0 \tag{34}$$

$$\left[ s + \frac{\partial}{\partial m} + \psi(m) \right] \bar{P}_8(m, s) = 0 \tag{35}$$

$$\left[ s + \frac{\partial}{\partial N} + \psi(N) \right] \bar{P}_9(N, s) = 0 \tag{36}$$

$$\left[ s + \frac{\partial}{\partial z} + \lambda_1 + \lambda_2 + K_5 \right] \bar{P}_{10}(z, s) = 0 \tag{37}$$

$$\left[ s + \frac{\partial}{\partial m} + 2\lambda_3 + K_5 \right] \bar{P}_{11}(m, s) = 0 \tag{38}$$

$$\left[ s + \frac{\partial}{\partial m} + \psi(m) \right] \bar{P}_{12}(m, s) = 0 \tag{39}$$

**Boundary Conditions for the Laplace transform**

- $\bar{P}_1(0, s) = K_1 \bar{P}_0(s)$  (40)
- $\bar{P}_2(0, s) = K_1^2 \bar{P}_0(s)$  (41)
- $\bar{P}_3(0, s) = K_2 \bar{P}_0(s)$  (42)
- $\bar{P}_4(0, s) = 2K_3 \bar{P}_0(s)$  (43)
- $\bar{P}_5(0, s) = K_4 \bar{P}_0(s)$  (44)
- $\bar{P}_6(0, s) = K_5 \bar{P}_0(s)$  (45)
- $\bar{P}_7(0, s) = K_5^2 \bar{P}_0(s)$  (46)
- $\bar{P}_8(0, s) = K_5^3 \bar{P}_0(s)$  (47)
- $\bar{P}_9(0, s) = K_6 \bar{P}_0(s)$  (48)
- $\bar{P}_{10}(0, s) = 4K_1 K_3 P_0(t)$  (49)
- $\bar{P}_{11}(0, s) = (K_5 K_1 K_3 + K_5^2) \bar{P}_0(s)$  (50)
- $\bar{P}_{12}(0, s) = (K_5^2 K_1 K_3 + K_5^2) \bar{P}_0(s)$  (51)

Solving equation (27) to (39) with the help of boundary condition (40) to (51) and applying the below shifting properties of Laplace.

$$\bar{P}_1(s) = \bar{P}_0(0, s) \left\{ \frac{1 - s\phi(s + \lambda_1 + K_1 + 2K_3)}{s + \lambda_1 + K_1 + 2K_3} \right\} \tag{52}$$

$$\bar{P}_2(s) = \bar{P}_1(0, s) \left\{ \frac{1 - s\psi(s)}{s} \right\} \tag{53}$$

$$\bar{P}_3(s) = \bar{P}_3(0, s) \left\{ \frac{1 - s\psi(s)}{s} \right\} \tag{54}$$

$$\bar{P}_4(s) = \bar{P}_4(0, s) \left\{ \frac{1 - s\phi(s + K_1 + K_3)}{s + K_1 + K_3} \right\} \tag{55}$$

$$\bar{P}_5(s) = \bar{P}_5(0, s) \left\{ \frac{1 - s\psi(s)}{s} \right\} \tag{56}$$

$$\bar{P}_6(s) = \bar{P}_6(0, s) \left\{ \frac{1 - s\phi(s + 2K_5)}{s + 2K_5} \right\} \tag{57}$$

$$\bar{P}_7(S) = \bar{P}_7(0, s) \left\{ \frac{1 - \bar{s}_\phi(s + K_5)}{s + K_5} \right\} \quad (58)$$

$$\bar{P}_8(S) = \bar{P}_8(0, s) \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \quad (59)$$

$$\bar{P}_9(S) = \bar{P}_9(0, s) \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \quad (60)$$

$$\bar{P}_{10}(S) = \bar{P}_{10}(0, s) \left\{ \frac{1 - \bar{s}_\phi(s + K_5)}{s + K_5} \right\} \quad (61)$$

$$\bar{P}_{11}(S) = \bar{P}_{11}(0, s) \left\{ \frac{1 - \bar{s}_{2\phi}(s + K_5)}{s + K_5} \right\} \quad (62)$$

$$\bar{P}_{12}(S) = \bar{P}_{12}(0, s) \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \quad (63)$$

Substituting (40) to (51) into (52) to (63) we have

$$\bar{P}_1(S) = K_1 \left\{ \frac{1 - \bar{s}_\phi(s + \lambda_1 + K_1 + 2K_3)}{s + \lambda_1 + K_1 + 2K_3} \right\} \bar{P}_0(S) \quad (64)$$

$$\bar{P}_2(S) = K_1^2 \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (65)$$

$$\bar{P}_3(S) = K_2 \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (66)$$

$$\bar{P}_4(S) = 2K_3 \left\{ \frac{1 - \bar{s}_\phi(s + K_1 + K_3)}{s + K_1 + K_3} \right\} \bar{P}_0(S) \quad (67)$$

$$\bar{P}_5(S) = K_4 \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (68)$$

$$\bar{P}_6(S) = K_5 \left\{ \frac{1 - \bar{s}_\phi(s + 2K_5)}{s + 2K_5} \right\} \bar{P}_0(S) \quad (69)$$

$$\bar{P}_7(S) = K_5^2 \left\{ \frac{1 - \bar{s}_\phi(s + K_5)}{s + K_5} \right\} \bar{P}_0(S) \quad (70)$$

$$\bar{P}_8(S) = K_5^3 \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (71)$$

$$\bar{P}_9(S) = K_6 \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (72)$$

$$\bar{P}_{10}(S) = 4K_1K_3 \left\{ \frac{1 - \bar{s}_\phi(s + K_5)}{s + K_5} \right\} \bar{P}_0(S) \quad (73)$$

$$\bar{P}_{11}(S) = (K_5K_1K_3 + K_5^2) \left\{ \frac{1 - \bar{s}_{2\phi}(s + K_5)}{s + K_5} \right\} \bar{P}_0(S) \quad (74)$$

$$\bar{P}_{12}(S) = (K_5^2K_1K_3 + K_5^2) \left\{ \frac{1 - \bar{s}_\psi(S)}{s} \right\} \bar{P}_0(S) \quad (75)$$

$$D(s) = [s + K_1 + K_2 + 2K_3 + K_4 + K_5 + K_6] - [K_1\bar{s}_\phi(s + \lambda_1 + K_1 + 2K_3) + 2K_3\bar{s}_\phi(s + K_1 + \bar{s}_\phi(s + K_1 + K_3) + K_5\bar{s}_\phi(s + 2K_5) + K_1^2\bar{s}_\psi(S) + K_4\bar{s}_\psi(S) + K_2\bar{s}_\psi(S) + K_6\bar{s}_\psi(S) + K_4\bar{s}_\psi(S) + K_5^3\bar{s}_\psi(S))] \quad (76)$$

$$\bar{P}_0(S) = \frac{1}{D(S)} \Rightarrow D(S) = \frac{1}{\bar{P}_0(S)} \quad (77)$$

$$\bar{P}_{up}(S) = \bar{P}_0(S) + \bar{P}_1(S) + \bar{P}_4(S) + \bar{P}_6(S) \quad (78)$$

$$\bar{P}_{down}(S) = 1 - \bar{P}_{up}(S) \quad (79)$$

## 4. ANALYTICAL STUDY OF MODEL FOR PARTICULAR CASES

### 4.1 Formulation and Analysis of System's Availability

Taking

$$S_{\alpha_0}(s) = \bar{S}_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^\theta}(s) = \frac{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}$$

$$, \bar{P}_\phi(s) = \frac{\phi}{s + \phi} \text{ but } \phi = 1 \text{ and} \quad (80)$$

Setting all repairs to 1. i.e.  $\phi(x) = \phi(y) = \phi(z) = \phi(m) = \phi(N) = 1$

And Taking the values of different parameters as  $\lambda_1 = 0.001, \lambda_2 = 0.002, \lambda_3 = 0.003, \lambda_4 = 0.004, \lambda_5 = 0.005, \lambda_6 = 0.006$

$$\bar{S}_\mu(S) = \frac{2.7183}{s + 2.7183}, \frac{1 - \bar{s}_\phi(S)}{s} = \frac{1}{s + \phi}$$

Taking the values of different parameters as  $\lambda_1 = 0.001, \lambda_2 = 0.002, \lambda_3 = 0.003, \lambda_4 = 0.004, \lambda_5 = 0.005, \lambda_6 = 0.006$

In (76) then taking the inverse Laplace transform, we can obtain, the expression for availability as:

$$D(s) = s + 0.024 - \left[ \frac{0.001}{s + 1.007} + \frac{0.006}{s + 1.004} + \frac{0.005}{s + 1.01} + 0.0016026 \left( \frac{2.7183}{s + 2.7183} \right) \right] \quad (81)$$

$$\bar{P}_{up}(S) = \left[ 1 + \frac{0.001}{s + 1.007} + \frac{0.006}{s + 1.004} + \frac{0.005}{s + 1.01} \right] \quad (82)$$

$$\text{Availability} = \{ 0.0004590357359e^{-2.71935133t} + 0.000224371224e^{-1.007944728t} + 0.00000382703479e^{-1.00766883t} + 0.0000557425631e^{-1.0091996995t} + 0.09993702262e^{-0.004734699t} \} \quad (83)$$

Table 1. System Availability with time

Time (in days)	Availability
0	1.00000
10	0.90113
20	0.81254
30	0.73267
40	0.66064
50	0.59570
60	0.53714
70	0.48433
80	0.43672
90	0.39379
100	0.355079

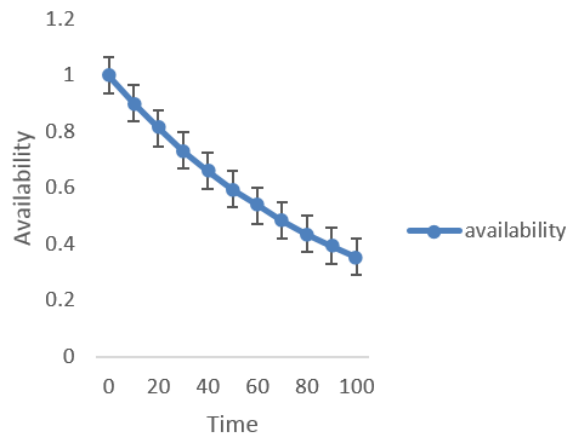


Figure 3. Variation of availability with time

### 4.2 Reliability Analysis

Taking all repair rate:  $(x) = \phi(y) = \phi(z) = \phi(m) = \phi(N) = 0$ , In equation (76) and for same values of failure rate as  $K_1 = 0.001, K_2 = 0.002, K_3 = 0.003$  and  $K_4 = 0.004, K_5 = 0.005, K_6 = 0.006$

And then taking inverse Laplace transform, one may have the expression for reliability for the system. Expression for reliability of the system is given as;

$$D(S) = S + 0.024 \tag{84}$$

$$\bar{P}_{up}(S) = \left[ 1 + \frac{0.001}{S+0.0016} + \frac{0.004}{S+0.0051} + \frac{0.007}{S+0.018} \right] \tag{85}$$

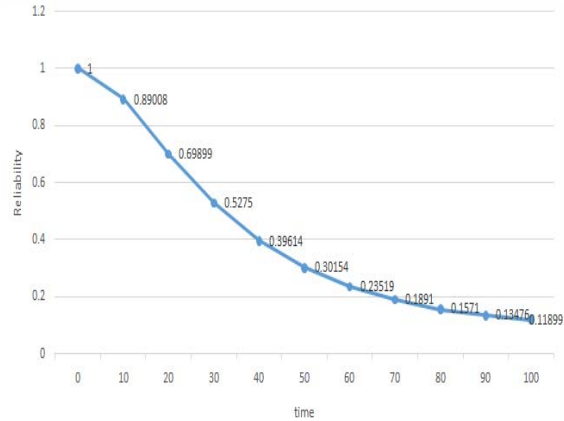
$$\text{Reliability} = \{ -0.5975038840e^{-0.103000000t} + 1.284000000e^{-0.400000000t} + 0.0050000000e^{-0.0120000000t} + 0.01063829787e^{-0.0070000000t} + 0.0618557600e^{-0.004000000t} \} \tag{86}$$

For different values of time t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100.

Unit of time, we may get different of  $\bar{P}_{up}(t)$  with the help of (86) as shown in Table 1 and corresponding Figure.

**Table 2.** Variation of reliability with respect to time

Time (in days)	Reliability
0	1.00000
10	0.89008
20	0.69899
30	0.52750
40	0.39614
50	0.30154
60	0.23519
70	0.18910
80	0.15710
90	0.13476
100	0.11899



**Figure 4.** Variation of reliability with time

### 4.3 Formulation and Analysis Mean Time to Failure

Setting repairs to zero in equation (84), the expression for MTTF is defined as follows:

Fixing  $\alpha_1 = 0.0001$ ,  $\alpha_2 = 0.0002$ ,  $\alpha_3 = 0.0003$ ,  $\alpha_4 = 0.0004$ ,  $\alpha_5 = 0.0005$ ,  $\alpha_6 = 0.0006$ , varying failure rate in equation (84), MTTF is computed with respect to failure rate as presented in Table 3 below.

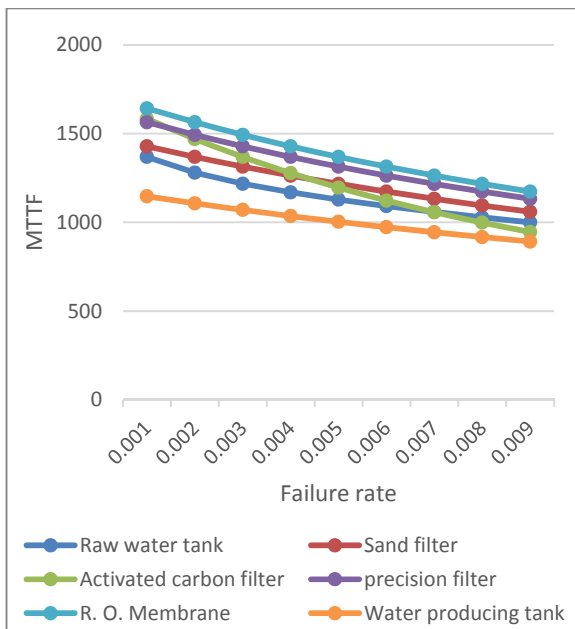
**Table 3.** Variation of MTTF with failure rates

Failure rate	MTTF					
	(a)	(b)	(c)	(d)	(e)	(f)
0.001	1369.04	1428.57	1583.33	1564.63	1642.85	1146.98
0.002	1280.00	1369.05	1469.70	1493.50	1564.63	1107.43
0.003	1217.95	1314.29	1369.05	1428.57	1493.51	1070.51
0.004	1169.31	1263.74	1277.78	1369.05	1428.57	1035.98
0.005	1128.24	1216.93	1195.89	1314.29	1369.05	1003.61
0.006	1091.95	1173.47	1122.71	1263.74	1314.29	973.19
0.007	1058.97	1133.00	1057.29	1216.93	1263.74	944.57
0.008	1028.49	1095.24	998.65	1173.47	1216.93	917.58
0.009	1000.00	1059.91	945.91	1133.00	1173.47	892.09



**Table 4.** Expected profit as a function of time

Time	$E_p(t)$					
0	0	0	0	0	0	0
10	3.49	4.49	5.49	6.49	7.49	8.49
20	6.05	8.05	10.05	12.05	14.05	16.05
30	7.77	10.77	13.77	16.77	19.77	22.77
40	8.73	12.73	16.73	20.73	24.73	28.73
50	9.01	14.01	19.01	24.01	29.01	34.01
60	8.67	14.67	20.67	26.66	32.67	38.67
70	7.77	14.77	21.77	28.77	35.77	42.77
80	6.37	14.37	22.37	30.37	38.37	46.37
90	4.52	13.52	22.52	31.52	40.52	49.52
100	2.26	12.26	22.26	32.26	42.26	52.26



**Figure 5.** Variation of MTTF with failure rates

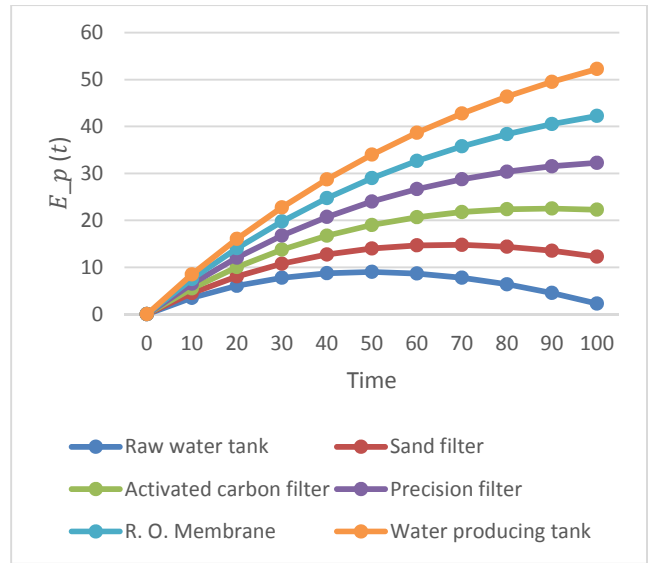
#### 4.4 Cost Analysis

The expression for the expected profit incurred in  $[0, t)$

$$E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t \tag{87}$$

Taking fixed values of parameters of equation (87), the subsequent equation (88) follows;

$$E_p(t) = \left. \begin{matrix} -0.018340e^{-2.87072t} + 0.008479e^{-1.20122t} \\ +0.000094e^{-1.15618t} - 598.072776e^{-0.00016t} \\ +0.000183e^{-1.13000t} + 0.000237e^{-1.12000t} \\ +5985.0821 \end{matrix} \right\} -k_2(t) \tag{88}$$



**Figure 6.** Box plot of Expected profit against  $K_2 \in \{0.01, 0.02, 0.03, 0.04, 0.05\}$

### 5. Interpretation of The Result and Conclusion

Table 1 and Figure 3 provide information on how availability and time change when failure rates are fixed at different values. When failure rates are fixed at lower values = 0.001, = 0.002 = 0.003, and the availability of the system decreases with the passage of time and ultimately becomes steady to the value zero after a sufficiently long interval of time. Hence one can safely depict the future behavior of complex systems at any time for any given set of parametric values, as is evident by the graphical consideration of the model. Figure (4) shows that the incorporation of copula improves the reliability of the system significantly. One can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model. Figure 4 of the analysis focused on the reliability of the system when the repair is not supplied. With the comparison of the values of availability and reliability in Table 1 and Table 2, it is evidently proved that when the repair<sup>(87)</sup> provided the performance of the system is quite better than replacement.

Tables 3 and figure 5 yield the mean-time-to-failure (MTTF) of the system with respect to variation in the failure rates, and respectively when other parameters have been kept constant. Shown by color graphs (green, Pink, ash, yellow, dark green, and purple) respectively. The system is also analyzed by employing Gumbel-Hougaard family copula. The study shows that the incorporation of a copula improves the reliability of the system significantly.

Cost analysis of the system is done in the analytic part of the paper. Figure 6 shows the variation of



cost with variation in the values of parameters. Table 3 displayed the result of MTTF with respect to the failure rates  $K_k$  from this table, MTTF is decreased with increasing in  $K_k$ .

For the analysis of profit the following are used:

- (a) Fixing  $K_2 = 0.01$  varying time  $t$  from 0 to 100
- (b) Fixing  $K_2 = 0.02$  varying time  $t$  from 0 to 100
- (c) Fixing  $K_2 = 0.03$  varying time  $t$  from 0 to 100
- (d) Fixing  $K_2 = 0.04$  varying time  $t$  from 0 to 100
- (e) Fixing  $K_2 = 0.05$  varying time  $t$  from 0 to 100

Table 4 displayed the result of expected profit  $E_p(t)$  with respect to  $K_2$ . From the table, it is evident that expected profit increases as  $K_2$  decreases.

## 6. Conclusions

With the non-availability of data for the RO system, the present paper introduced a reliability modeling approach in order to study the overall strength, efficiency, and performance of the reverse osmosis (RO) system. The strength considered in this paper can be seen in terms of reliability, availability, MTTF, and profit function. In this paper, we have introduced a new model of reverse osmosis (RO) system consisting of six subsystems namely, raw water tank, sand filter, activated carbon filter, precision filter, RO membrane, and water producing tank. According to the study's findings, dependability modeling may be used to assess the strength, efficiency, and performance enhancement of a reverse osmosis (RO) system. Where the reverse osmosis (RO) system's strength, efficiency, and performance improvement are determined. This work may be expanded to include a system with many subsystems and multiple repair machines to minimize congestion in the repair facility and solve utilizing supplemental variable approaches. The current work will be beneficial to water manufacturing and industrial uses that are dangerous to humans, among other things.

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