Reliability Improvement in Distribution Systems Via Game Theory

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Abstract
This paper presents a new competitive approach to provide reliability for distribution system customers. The model is based on the Cournot game and utilizes the Nash equilibrium concept to find the output of the problem. Reliability in the proposed framework is an ancillary service and the customers who participated in the program must pay for reliability provision. The proposed model also considers regulatory concerns of reliability insuring the average reliability of the system is not incurred. Based on the proposed model, customers will compete for their reliability enhancement considering all the constraints related to the network, regulator and each customer. The expected outage time for each customer is considered the reliability index in this paper. The model is investigated in a sample case study and the results show how a customer would behave if they participated in the reliability improvement program of distribution systems. Our results also show that there would exist a high motivation for both parties (utility and customers) to implement the proposed model for the reliability enhancement of the distribution system.

Keywords: Distribution system; Cournot Game, Load point Reliability; Reliability Enhancement.

1. Introduction
Commonly methods of distribution system reliability enhancement are based on improving the average reliability of the system and they do not use load point reliability indices. Average indices can potentially bias investment decisions towards areas of the system with adequate reliability [1].

There are four methods that distribution utilities usually use for reliability enhancement over the system. In the first category, the utility target a reliability level and invest in the distribution system to reach the targeted goal [2], [3]. In this method, it might reliability cost is estimated and included in the overall cost function of distribution company for distribution network improvement, or reliability is directly considered in the system improvement problem’s constraint to meet a targeted level. In the second category, the sum of customer costs and distribution utility investment costs are minimized to determine the optimal reliability value of the system. This approach is named value-based planning (VBP) and different objective functions and constraints can be applied to the problem in this method [4]–[6]. In this approach, reliability is the main objective of the distribution system improvement, and the objective function optimized is usually social cost or social welfare. In the third category, distribution utilities improve distribution system reliability based on regulatory enforcement. Performance-based rates (PBRs) are the most popular regulatory scheme that rewards utilities for good reliability and penalizes them for poor reliability [7]–[11]. In this approach, depending on the PBR model, the distribution company seeks a reliability level over the system for maximizing the rewards by regulation or minimizing the penalties that may occur. The papers in the above three categories, most consider the average system reliability, and the final decision for reliability enhancement is usually based on economical or hierarchical analysis.

Among average reliability indices, SAIDI, SAIFI, CAIDI, ASAI, and EENS are usually used by distribution companies for reliability assessment of distribution systems, and especially SAIDI is famous for the reliability level of the distribution system in reliability improvement problems.

Also, the approaches in the three categories cannot correctly address customer reliability requirements and so they are not useful to provide differentiated reliability over the system. In the fourth category, load point reliability indices are focused. A reliability insurance scheme (RIS) is the simplest way of allowing customers to choose a reliability level and participate in reliability programs [12], [13]. Widespread implementation of RIS
to customers is problematic due to free-riding behavior and gaming behavior [14]. In RIS, limited levels of reliability are provided over the system but customer choices are restricted to the reliability contract designed by utilities which are not satisfactory for all customers. Therefore, the main problem is how a distribution company can provide reliability at the distribution system level while considering customers’ reliability preferences.

The first step is incorporating load point reliability indices to a reliability provision program of distribution utilities. Obviously, considering customer needs for reliability means the utilities should use an approach to be able to differentiate reliability levels between different customers at different load points. The second step is utilizing an approach which handles customer contribution in the reliability provision program.

It is shown that customer choice for reliability leads to a competition among customers for the desired reliability levels [15]. So, the authors in [16] proposed a market based scheme for reliability provision in the distribution systems according to strategic bidding of reliability by customers. The market framework is based on the demand function equilibrium in game theory.

In [17], a new market-based approach for the reliability enhancement of distribution system customers is proposed which is based on the popular Cournot game model. The paper presents a theoretical view of the gaming problem for reliability enhancement.

As this model has the advantage of applicability in large systems, we used the Cournot game model in this paper to provide valuable insight into the distribution utilities about how customers behave and if they can influence the reliability of the system through their choices. Further, this paper clearly shows how the regulatory limitation on the system reliability level affects the proposed competitive model for reliability which is not addressed so far. The model has the capability to manage and control the reliability level of customers at load points and it explains the free-riding behavior of the customers suitably. It should be noted that the framework proposes the reliability as an ancillary service and hence, reliability enhancement of the system in the normal condition of the distribution system operation is not the subject of this paper.

In fact, reliability has characteristics of a private good while overall reliability of the system or security issues is considered a public good [18]. Also in [19] the authors address customer-oriented reliability planning and try to provide a higher level of reliability for some customers while maintaining average reliability improvement of the system. Although this paper can provide differentiated reliability over the system, customer-oriented reliability is provided through the reliability improvement framework by a distribution company. In [20], also authors introduce a new framework for customer reliability enhancement in the distribution system based on a new market mechanism. The paper assumes the reliability provision cost can be stated based on reliability level. Then a bid function similar to the power market bidding approach is made and the reliability level and price are cleared by maximizing social welfare. These two latest papers also cannot address the free-riding problem properly.

2. MODEL EXPRESSION

2.1 Model assumptions

The reliability improvement model in this paper is based on game theory. Game theory has found vast applications in power systems analysis because of its capability in solving decision-making problems with multiple agents and objectives. The most relevant areas that game theoretical methods have been applied in power system analysis include power markets, power system planning, power system dispatch, power system control, microgrids, demand response, power system security, and power system evolution [21]. However, in reliability problems, game theory mostly has been used in reliability maintenance planning [22], and also there is a rare application of game theory in risk assessment [23], [24]. We assumed that there are n customers at n load points.

In fact, the model is implemented at a medium-voltage distribution system. This is not necessarily the only case to be analyzed. The results and characteristics of the model can generally discuss the competitive behavior of customers for reliability at each level of the distribution system. In addition, if there is more than one customer at a load point, the customer cost function must be aggregated and then embedded into the model. In such a case, the customers at a load point can be considered as cooperative players which is out of the scope of this paper. The distribution utility’s strategy for reliability enhancement is usually based on investing in switch and tie line installation. The output of the model would be determined by switch locations and load point reliability levels.

Expected outage time at load points is considered as the reliability index to be improved. The customers at load points are the game players who interact with each other to reach their optimum reliability level. It is supposed that utility is not a rival of customers as players but it influences the game by pre-determined restoration strategies, distribution system constraints, and model parameters. Here, it is assumed that customers only know about their own cost functions and consider other customer actions fixed in their optimizations– a Cournot game assumption. Nash equilibrium is calculated to determine the output of player competition.

2.2 Model formulation

Customer cost function (payoff function) for reliability is determined as

\[ C_c = \alpha D_i + \beta C_a(D_{i-1}) - C_a(D_i) \]

This pay off function comprises three items. The first term, \( C_a(D_i) \), determines the expected customer
damage function (CDF) of customer \( i \) during interruption time of \( D_i \). CDF is considered as a quadratic function as shown in following [14].

\[
CDF(D_i) = \frac{1}{2}a D_i^2 + b D_i + c_i \quad ($) \quad (2)
\]

Second term in (1) defines a percentage decrease in customer outage cost, proportional to \( \alpha_{ij} \), which is paid to the power distribution utility, when the interruption time of load-point \( i \) changes from \( D_{ij} \) to \( D_i \). In fact, the restoration activity of distribution utility causes the reliability level of customers changes. The level of increase or decrease might be different for individual load points. It means that some of the load points might benefit more than others by a specific utility’s investment for reliability enhancement. This term considers this different effect of restoration activity on load point reliability and allocates cost to each customer corresponding with the change in their costs. On the other side, this term also emphasizes load importance for reliability enhancement. In fact, more sensitive customers are expected to need a higher level of reliability and so pay more than others accordingly. This term cause that investment for reliability is in accordance with the reliability value of the distribution systems.

\( D_i \) in (1) can be stated as a function of failure rates of sections as

\[
D_i = \sum_{j=1}^{n} \lambda_j \quad (3)
\]

Where \( \lambda_j \) indicates interruption time at load point \( i \) due to a failure in section \( j \). The third term of (1) states the cost of providing reliability by distribution utility for each customer; a portion of this cost at load point \( i \), proportional to \( \beta_i \), is paid by customer \( i \) to the distribution utility. In fact, this term guarantee that each customer contributes to the reliability provision cost of the system according to its usage of utility facilities for provided reliability.

\( \alpha_{ij} \) and \( \beta_{ij} \) are the model parameters which are defined by distribution utility. These two parameters can significantly change the output of the game. In fact, help distribution utilities to have the control and authority on customer behaviors in competition among customers. \( C_{io} \) in the third term indicates reliability provision cost at load point \( i \) and is a function of outage times at load points. This cost is dependent to the reliability profile of customers after the implementation of the game. It can be shown that this cost is a linear function of customer outage costs [16]. So it can be stated as follow:

\[
C_{io} = \sum_{j=1}^{n} k_{ij} D_j \quad (4)
\]

Furthermore, there are some constraints must be satisfied. The constraint (5) and (6) are limitation on the location of switches and section lengths. They can also be expressed as a function of load point outage times.

\[
\sum_{j=1}^{n} \lambda_j = \lambda_j \quad (5)
\]

where \( \lambda_j \) is the failure rate of section \( j \). Note that expected outage time of load points (reliability equations) can be stated as a linear function of section’s failure rate according to (3). Then, we also can express failure section rates as a function of load point outage times. In such a case, constraints (5) and (6) can be stated based on customer outage times or problem decision variables. So we have:

\[
\sum_{j=1}^{n} \lambda_j D_j = \lambda_j \quad (7)
\]

\[
\lambda_j \leq \sum_{k=1}^{m} m_k D_k \leq \lambda_j \quad (8)
\]

This equation implies that SAIDI must be preserved on a certain level. In fact, considering load point indices for reliability enhancement should not cause the average reliability indices reduction under an acceptable level, as reliability has characteristics of public goods [18].

The model presented in (1)- (9) includes \( n \) optimization problem minimizing the objective (pay off) functions determined in (1). The decision variable of customer \( i \)’s optimization is \( D_i \) but each objective function is dependent to decision variable of other players’ problems which are minimized in the separated optimizations. It concludes \( n \) correlated problems which should be solved simultaneously (Cournot assumption). Therefore, the problem is more complicated than common optimization problem with \( n \) variables. In our problem, the solution of \( n \) optimization problems is the Nash equilibrium point in which there is no motivation for each customer to change its action unilaterally. To find such a point, we follow the KKT-optimally-condition approach for all \( n \) optimization problems and solve all of them simultaneously to achieve the output of the model.

3. Numerical example

We have tested the model for a 5-bus system to show the model features. Consider the general distribution system in Figure 1 with 5 customers (\( n=5 \)), we assume that there is only one tie line connected to load point 5 which is able to restore the load power at load points 3, 4, and 5 (due to capacity constraints).

![Figure 1. A distribution feeder with five load points](image-url)
It is supposed that the customers are industrial (Ind) or commercial (Com). The customer data is presented in Table 1.

Table 1. Customer Data

<table>
<thead>
<tr>
<th>Load point</th>
<th>Power (kW)</th>
<th>Customer type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>Com</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>Com + Ind</td>
</tr>
<tr>
<td>3</td>
<td>270</td>
<td>Com</td>
</tr>
<tr>
<td>4</td>
<td>320</td>
<td>Com</td>
</tr>
<tr>
<td>5</td>
<td>550</td>
<td>Ind</td>
</tr>
</tbody>
</table>

Furthermore, the customer damage function (CDF) of each type is shown in Table 2 [3].

To show the model characteristics, we change the model parameters (α and β) and find the output of Cournot based reliability planning (CBRP). The results of CBRP are compared with social cost minimization approach (SCMA). Parameters α and β in the simulations corresponds to vectors of α₁ and βᵢ. Initially, both α and β are set on 0.3. Then, αᵢ and βᵢ (simultaneously) are changed from 0.2 to 0.6 and resulting customer and social costs are reported in Figure 2. In this figure, the model also tested for different level of limitation on SAIDI according to (9). SAIDI, as mentioned before, is a regulatory concern to prevent descending reliability of system from a standard level. However, considering this constraint can limit flexibility of the system to respond to the desired reliability level of customers at load points.

Furthermore, the result for a special profile for α and β is reported in Table 3. In this profile, the output of the CBRP is reported for unconstrained (NC) and constrained SAIDI (the limited level highlighted in the Table) when α and β are chosen as S₁ = [0.6, 0.6, 0.42, 0.3, 0.3]. In fact, these results represent one of the outputs in the Figure 2.

In a distribution system, upstream customers have the advantage for receiving a higher level of reliability due to their location (free riding can happen) in comparison to downstream customers [13]. The output of SCMA confirms this statement. As Table 3 shows, a higher level of reliability is allocated to upstream users. Customers 1 and 2 receive higher level of reliability in comparison to others (the order is D₁>D₂>D₃>D₄>D₅), While a big load power is located at the end of the feeder. But, by setting the higher level of α and β for upstream users, this property can be controlled. In fact, the utility can manage the output of the game by proper setting of the model parameters. For example, if α and β are set on S₁, as it is reported in Table 3, reliability levels allocated to customers can be changed. Customers at load points 4 and 5 (downstream customers) will receive higher levels of reliability compared with customers on upstream customers (the order is D₁<D₄<D₅<D₃<D₂). This property can also be observed in customer costs. As Fig. 2 shows, by increase in α and β for upstream customers (middle of the figures), the customer costs for downstream part of the feeder is decreased (Figure 2-4, Figure 2-5) in unconstrained CBRP model (solid line). Further, this achievement is provided by only 1.5% and 7.3% increasing in the social cost of the system and SAIDI, respectively.

In addition, the good point is that the utility has a high motivation to implement the proposed framework. In fact, in all scenarios for α and β, the utility cost would decrease compared with SCMA (due to cost allocation process in the proposed CBRP). For example, in scenario S₁ utility cost decreases to 45% according to Table 3.

In addition, the average of CBRP’s outputs for all scenarios in Fig.2 is presented in the last column of Table 3. Accordingly, the average of utility cost for all scenarios is equal to 2493$ (in comparison with 4735$ of SCMA) which confirms that CBRP is beneficial for utility without considering how the model parameters are set.

Table 3. The Output of CBRP and SCMA

<table>
<thead>
<tr>
<th>Output</th>
<th>SCMA</th>
<th>CBRP (Strategy:S₁)</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>NC</td>
<td>Constrain on SAIDI</td>
</tr>
<tr>
<td>D₁ (h)</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>D₂ (h)</td>
<td>0.83</td>
<td>2.02</td>
<td>2.02</td>
</tr>
<tr>
<td>D₃ (h)</td>
<td>1.55</td>
<td>1.82</td>
<td>1.5</td>
</tr>
<tr>
<td>D₄ (h)</td>
<td>1.51</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>D₅ (h)</td>
<td>1.65</td>
<td>1.18</td>
<td>1.46</td>
</tr>
<tr>
<td>SAIDI (h)</td>
<td>1.23</td>
<td>1.32</td>
<td>1.3</td>
</tr>
<tr>
<td>Social Cost ($)</td>
<td>24520</td>
<td>24900</td>
<td>24820</td>
</tr>
<tr>
<td>Utility Cost ($)</td>
<td>4735</td>
<td>2616</td>
<td>2492</td>
</tr>
</tbody>
</table>

Furthermore, different goals can be targeted by the model. As Fig. 2-6 shows, by proper setting of α and β in a range, the social cost in CBRP is equal to that of SCMA. But, there is a big difference. Customer costs for some of them are different by the two approaches. The cost of customers 3, 4, and 5 (downstream customers) is rather closed in both models but customers 1 and 2 have higher cost in CBRP in comparison to SCMA. In such a...
situation, utility cost decreases although the social cost is similar in both models.

This is an important feature because reliability provision costs are aggregated together and so it is not clear how much cost is needed to provide reliability for each customer. Therefore, free riding is possible. But here customers share in their reliability provision costs and so, better management for reliability is possible. Even in Scenario S1, as Table 3 and Fig. 2-1 show, C1 in CBRP is greater than that of in SCMA, although the reliability of customer 1 is identical in both methods. In fact, in all scenarios as S1, as mentioned above, customers are not neutralized to their reliability provision cost.

Furthermore, Figure 2 indicates how the limitation on SAIDI affects the output of the CBRP model. Increasing the average reliability of the system (by decreasing the maximum level of SAIDI) would cause the output of CBRP differs in the case that there is no constraint on SAIDI.

The constraint especially affect the results for downstream customers. According to Fig.2 (4&5 sections) customer costs at load points 4 and 5 are be increased by decreasing the SAIDI level. Furthermore, the cost of upstream customer at load point 2 (Fig.2-2) are decreased and the load point 3 at the middle of the feeder dose not affected considerably. Although in all scenarios for SAIDI limitation, the output of the CBRP model is be closing to the output of conventional reliability improvement of SCMA according to Fig 2-6. This is an important feature that emphasizes the competitive framework might be limited by regulatory purposes and it needs to allow the average reliability level of the system reduces to a reasonable level to insure that desired load point reliability levels could be provided. Although, according to Table 3 and Fig. 2, no considerable increase is observed in average indices (i.e. social cost and SAIDI) due to the competitive manner of the proposed framework.

It is also possible to reach SAIDI of SCMA by CBRP without constraining SAIDI of the system. By setting $\alpha$ and $\beta$ on $S_2= [0.6, 0.32, 0.3, 0.3, 0.3]$, the SAIDI would be identical in both methods but importantly, customer interruption times differ.

The last important point is that the flexibility of the model to respond to the customer preferences for reliability depends on the technical flexibility of the system. In fact, it is needed that utilities provide the minimum requirements for reliability enhancement to be able to implement a competitive framework. For instance, if we limit the tie line capacity for restoring the load powers on the feeder, the ability of the model to distinguish between load point reliability is restricted and so, the customers identify that they could not improve their reliability by changing their actions through rational decision-making.

![Figure 2. Customer cost and social cost of the system by changing $\alpha$ and $\beta$ (solid line: no constrain on SAIDI, dash line: SAIDI < 1.3, dash-dot line: SAIDI < 1.27, dot line: SAIDI < 1.24)](image)

Then, as is expected the output of the model is close to the output of conventional SCMA. Table 4 shows such a thing.
In Table 4 (in parameter profile of S1), the tie line capacity is gradually restricted to the more upstream parts of the feeder, and as the results show SAIDI and social cost are much closed to each other.

4. Conclusion

In this paper, a competitive framework based on the Cournot game model was analyzed to explain customer interactions for desired reliability values in distribution systems. The Nash Cournot determines the output of the game and the results show that the model adds much flexibility over the distribution system to control and manage customer strategic behaviors in a reliability enhancement program. The model can also be useful for regulatory purposes to design and analyze customer behaviors in a competitive framework for reliability. The results show that distribution utilities can utilize the framework to provide customer preferences for reliability by conventional means of enhancing reliability.

For future work, it is also valuable to consider different tools for enhancing reliability in distribution systems. Distributed generation and demand response are effective in reliability improvement over the system. Using them in the proposed framework can provide more flexibility for distribution companies to manage load point reliability and control free riding. Also, this paper uses pure strategies in the game-theoretic framework to model customers’ competition for reliability enhancement. Other game theoretic models also can provide more information for modeling customer behavior in a competitive manner. Further, other reliability indices are also valuable to see in the proposed framework as some of them might need reliability enhancement requirements other than outage times.

<table>
<thead>
<tr>
<th>Load points restored by tie line</th>
<th>SAIDI</th>
<th>Social Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 3, 4, 5</td>
<td>10.8%</td>
<td>2.49%</td>
</tr>
<tr>
<td>3, 4, 5</td>
<td>7.32%</td>
<td>1.55%</td>
</tr>
<tr>
<td>4, 5</td>
<td>0.8%</td>
<td>0.15%</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 4, the percentage of increase in output of CBRP comparing with SCMA.

5. References


