Fault-tolerant Sliding Mode Controller and Active Vibration Control Design for Attitude Stabilization of a Flexible Spacecraft in the Presence of Bounded Disturbances

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Abstract

This paper concerns vibration control and attitude stabilization of a flexible spacecraft with faulty actuators. The PID-based sliding mode fault-tolerant scheme is developed to preserve the system against bounded external disturbances, rigid-flexible body interactions, and partial actuator failures. The proposed control law, which combines the advantages of the PID and SMC, is proposed to enhance the robustness and reduce the steady state errors while reducing complexity and the computational burden and preserving the great properties of the SMC controller. It has been shown that the SMC controller is effective in accommodating different actuator fault scenarios and behaves healthily. Additionally, an active vibration control (AVC) law utilizing a strain rate feedback (SRF) algorithm and piezoelectric (PZT) sensors/actuators is activated during the maneuver to compensate for residual vibrations resulting from attitude dynamics and actuator failures. Numerical simulations demonstrate the proposed schemes' superiority in fault tolerance and robustness compared to conventional approaches.

Keywords: Fault tolerant; Sliding mode control; Active vibration control; Loss of actuator effectiveness; Piezoelectric.

1. Introduction

In recent years, with the improvement of space technologies, the attitude control system has a higher role in a modern space mission, especially combined with actuator faults and external disturbances. The control system for spacecraft, regardless of their missions and attitude maneuvers, needs to be accurate and fault-tolerant [1-3].

There are passive and active approaches to fault tolerant control (FTC). Implementing an online fault detection scenario for active fault tolerant control algorithms is necessary compared to passive ones [4, 5]. Moreover, with a single fixed controller, a passive FTC algorithm can simultaneously handle many possible actuator faults [6]. So in this approach, in order to reduce the computational burden, the control algorithms are modified according to the detected fault signals.

On the other hand, many large and flexible appendages in aerospace systems can easily reduce spacecraft performance and their attitude control systems. Faults caused by the actuators, which can occur at unknown moments with unknown values and patterns, can excite high-frequency modes of flexible appendages and lead to system instability [7-10].

Many control approaches have been developed over the past few decades to minimize the effects of disturbances and actuator faults. The following are some effective FTC approaches: model predictive control [11], neural network-based control [12], adaptive fuzzy logic-based control [13], back-stepping control [14], and sliding mode control (SMC) [15].

Several studies have investigated the robustness and ease of implementation of a classical sliding mode fault-tolerant controller [15-17]. Compared with other control approaches, the sliding mode is able to resolve disturbances and/or different systems uncertainties [18]. Moreover, the conventional SMC's weakness is the chattering phenomenon associated with its high switching frequency, singularity, and inability to meet the finite time convergence. Despite the SMC's remarkable properties, it should be improved to stabilize systems quickly when faced with fault effects. In order to address these concerns, several types of SMCs are proposed, such
as terminal SMC (TSMC), Nonsingular TSMC, boundary layer method, high-order SMC, etc. The complexity of these algorithms, however, will significantly affect the system’s complexity, real-time implementation, and the required processing burden. In order to simultaneously increase the robustness and performance of the SMC, PID-based SMCs have been developed. Such an approach aims to improve the robustness of SMC by incorporating the advantages of PID controllers into its design procedure. A PID controller using a sliding mode control approach offers significantly better performance than a classical PID controller in view of the fact that the PID SMC does not rely on uncertainty [19]. Moreover, flexibility has not been considered in most of the previous studies dealing with attitude SMC fault tolerant control of systems with nonlinear fully coupled rigid-flexible bodies dynamic.

We propose a more reliable mechanism that utilizes PID-based SMC in order to overcome this weakness. Hu proposed a sliding mode fault-tolerant controller for the stability of flexible spacecraft using a redundant actuator [16].

The problem of residual vibration control has received great attention from spacecraft designers. Using PZT materials as actuators or sensors is an effective method for actively suppressing vibrations. The main characteristics of PZT materials are their high-frequency response, low power consumption, lightweight, and high stiffness. Numerous studies have been conducted on actively using PZT material to control the vibrations of flexible spacecraft structures. A flexible spacecraft’s attitude maneuvers can be suppressed using PZT patches and the component synthesis vibration suppression approach [20]. Song developed positive position feedback (PPF) control in order to dampen the vibration of the flexible structure of spacecraft [21]. In a single-axis maneuver, the SRF and SMC techniques are utilized to suppress vibrations [22].

Among the presented methods, the SRF law has a wider active damping region and the ability to stabilize more than one vibration mode. In addition, it is easy to implement. In this method, the structural velocity coordinate (strain rate) is multiplied by a negative gain and feedback to the structure.

This paper is focused on a passive fault tolerant PID-based SMC approach to stabilize the system with fully coupled rigid-flexible body dynamics interactions equipped with three faulty momentum generation actuators. The SRF control approach is also contributed to suppressing the residual vibrations during and after the attitude maneuver. A comparative study with healthy systems validates the proposed model and algorithm, and the overall system stability is proven by applying the Lyapunov theorem. Following is a summary of the main contributions:

The proposed method has demonstrated superior performance and robustness compared to conventional methods [23-25]. By utilizing AVC to compensate for extra vibrations, the proposed method is superior to FTC approaches without AVC [26, 27]. Following is the rest of the paper’s organization. The dynamics of the flexible spacecraft and actuator faults are described in Section 2. SMC-based PID fault tolerant control and SRF-based vibration suppression for flexible panels are discussed in Section 3. Section 4 presents the simulation results of the FTC system with and without AVC in the presence of external disturbances, and the last section concludes the paper.

2. Mathematical modelling of flexible spacecraft and actuator faults

The flexible spacecraft dynamic modelling is considered a rigid-flexible body dynamic, consisting of a main rigid hub and two PZT mounted flexible appendages attached symmetrically. Elastic deformation of panels during multi-axis attitude maneuvers has been modelled using the Euler-Bernoulli beam theory. A fixed reference frame is used to separate the attitude motions of the spacecraft from its translational motions. Figure 1 shows the flexible spacecraft consisting of a rigid hub and two flexible panels equipped with piezoelectric sensor/actuator patches.

![Flexible spacecraft model](image)

The equations of the motion of the flexible spacecraft with PZT patches are given as [22]:

\[
\begin{align*}
M_R \ddot{\omega} + M_{RP} \dot{\eta}_k + C_A \omega + C_{RF} \dot{\eta}_k &= u \\
M_{PR} \dot{\omega} + M_{PR} \dot{\eta}_k + C_{PR} \omega + C_{RF} \dot{\eta}_k + K_F \eta_k &= -P \eta_k A_d s - d_s \\
A_d^p &= g N^{-1} p \eta_k
\end{align*}
\]

where \( u, \dot{u}, \dot{\eta} \in \mathbb{R}^{3 \times 1} \) \( (\text{with } u = u_s + u_p), \)
\( d_s, \eta = [\eta_1, \eta_2, ..., \eta_n] \) and \( g \) denote control torque, external disturbances on the hub, external disturbances on flexible panels, the \( k \)th modal components for appendages, PZT sensor/actuator gain amplifier, respectively. The \( M, C, K \) are the mass, damping, and stiffness matrices, \( A_d, A_d^p \) subscripts \( R, F \) and \( RF \) represent the rigid, flexible and rigid-flexible dynamics, and superscripts \( a \) and \( s \) denote as PZT sensor and actuator, respectively.
The unit quaternions (which are also called Euler's symmetric parameters), as state parameters, describes the body frame's attitude orientation of flexible spacecraft $q = [q_0 \ q_1:3] \in \mathbb{R}^{4 \times 1}$, are defined as:

$$\begin{cases} q_{1:3} = [q_1 \ q_2 \ q_3]^T = e(t)sin \frac{\Phi(t)}{2} \\ q_0 = cos \frac{\Phi(t)}{2} \end{cases}$$

(2)

where $0 \leq \Phi(t) \leq 2\pi$ refers to a rigid body’s rotation around the Euler axis $e(t)$. The quaternions are related to the vector of angular velocities $\omega = [\omega_x \ \omega_y \ \omega_z]^T$ by the following relation:

$$\dot{q}(t) = \frac{1}{2}\left( [q_0 q_{1:3} + q_{1:3}^T q_0] \omega - q_{1:3}^T \omega q_0 \right)$$

(3)

The dynamic modelling of actuator faults is as follows:

$$u_c = [I_3 - E(t)]u$$

(4)

where $E(t) = \text{diag}(f_1(t), f_2(t), f_3(t)) e \in \mathbb{R}^{3 \times 3}$ denotes the matrix of reducing the effectiveness of spacecraft actuator faults with $0 \leq f_i(t) \leq 1 \ (i = 1, 2, 3)$. It should be noted that if $f_i(t)$ is equal to zero, it means that the actuator works normally. If $f_i(t)$ is between zero and one, the actuator loses its effectiveness partially and has not yet completely failed. Therefore, by rewriting the dynamics according to Eq. (4), we have:

$$M_R \ddot{\omega} = -M_{RF} \ddot{\eta}_k - C_R \dot{\omega} - C_R \dot{\eta}_k + [I_3 - E(t)]u + d$$

(5)

3. Controller design

This section presents two control approaches; the fault-tolerant PID-based SMC and the SRF for AVC. Initially, we discussed a simple SMC with actuator faults. Next, the fault-tolerant property is added to the sliding mode structure, followed by the AVC algorithm.

3.1. Conventional sliding mode control

A sliding surface can be defined by taking angular velocity and quaternions vectors [29]:

$$S = \omega + k q_{1:3}$$

(6)

where $k$ is a positive constant. Convergence of state variables requires satisfaction of the sliding surface equation ($S = 0$). The convergence property is given in the following lemma.

**Lemma 1:** If a proper sliding mode controller can be designed to persuade the sliding mode condition $S^T S < 0$, then desired maneuver can be realized. As a result, the spacecraft state parameters $\omega$ and $q$ will converge to zero.

**Proof:** By satisfying the sliding conditions from sliding mode theory, the system is lastly forced to be in sliding mode:

$$S = \omega + k q_{1:3} = 0$$

(7)

A candidate Lyapunov function is as follows:

$$V_1 = \frac{1}{2}(1 - q_0)$$

(8)

Using Eq. (3), the time derivative of the Lyapunov function becomes:

$$\dot{V}_1 = -2q_0 \omega' q_{1:3}$$

(9)

Substituting the sliding surface Eq. (7) in Eq. (9), it follows that:

$$\dot{V}_1 = -q_{1:3}^T k q_{1:3}$$

(10)

It can be shown $\dot{V}_1 = 0$ only if $q_{1:3} = 0$. Subsequently, $V_1$ is a Lyapunov function such that the $q \to 0$. Therefore, we can easily derive $\omega \to 0$ from Eq. (7) and can guarantee the system’s stability by introducing the sliding vector Eq. (6). Then, this completes the proof of lemma 1.

The time derivative of the sliding surface along the attitude dynamics model Eq. (5) leads to:

$$\dot{S} = \omega + k q_{1:3} = M_{R}^{-1}(-M_{RF} \ddot{\eta}_k - C_R \dot{\omega} - C_R \dot{\eta}_k + [I_3 - E(t)]u + d + 0.5k(q_0 \omega + q_{1:3}^T \omega)$$

(11)

The sliding surface's time derivative should be zero to derive the equivalent control, which gives:

$$u_{eq} = M_{RF} \ddot{\eta}_k + C_R \dot{\omega} + C_R \dot{\eta}_k - 0.5k M_R (q_0 \omega + q_{1:3}^T \omega)$$

(12)

The nominal controller is proposed as [30]:

$$u_{nom} = u_{eq} - K_p q_{1:3} - K_d \text{tanh} \left( \frac{\omega}{\rho^2} \right)$$

(13)

where $K_p$, $K_d$, and $K_I$ are positive constants, and $\rho^2$ is a non-zero sharpness function.

Consider $V_1 = \frac{1}{2} S^T S$ as a candidate Lyapunov function. Introducing the first time derivative of $V_1$ and substitution Eq. (11) into the sliding condition leads to:
Using the closed-loop control in the nominal controller Eq. (14), the sliding condition in Eq. (13) can be rewritten as:

\[
S^T \dot{S} = S^T \left( M_R^{-1} \left( -M_R \dot{q}_{1:3} - C_R \omega - C_{RF} \eta_k + \left[ I_3 - E(t) \right] u + d \right) + 0.5k (q_0 \omega + q_1 \omega) \right) - K_t \int \dot{q}_{1:3} dt < 0
\]  

(15)

Using the unit quaternion property in combination with the hyperbolic tangent function, Eq. (15) may have an upper bound as:

\[
|K_R q_{1:3}| + |K_d \tanh \left( \frac{\omega}{p} \right)| + |K_t \int \dot{q}_{1:3} dt| \leq K_p + K_q + K_t
\]  

(16)

Accordingly, from Lemma 1, assumption 1.5, the control objective \( q_0 \to 0 \), \( q_{1:3} \to 0 \) and \( \omega \to 0 \) as \( t \to \infty \) can be achieved.

### 3.2. Fault-tolerant SMC

In order to compensate for actuator faults as well as external disturbances, the SMC law is designed in such a way that the sliding surface can always be reached. The proposed fault-tolerant SMC is considered to be [31]:

\[
u = \nu_{\text{nom}} + \nu_{\text{FTC}}
\]

(17)

where \( \nu_{\text{nom}} \) regulates the nominal system behavior bounded to the sliding manifold. The discontinuous fault tolerant control component that compensates for the possible actuator fault effect \( \nu_{\text{FTC}} \) is selected as:

\[
u_{\text{FTC}} = \begin{cases} -K_S S - \beta \frac{(DM_R^{-1})^T S}{\left\| (DM_R^{-1})^T S \right\|} & \text{if } S \neq 0 \\ 0 & \text{otherwise} \end{cases}
\]

(18)

where \( \beta \) and \( K_S \) are positive constant, and \( D \in \mathbb{R}^{3 \times 3} \) is a constant matrix.

**Theorem 1**: Assume that the attitude control systems of flexible spacecraft described by Eqs. (1) - (3) are affected by partial actuator faults and external disturbances. Then the sliding manifold \( s = 0 \) is reachable by employing the controller in Eq. (17) with \( \nu_{\text{nom}} \) and \( \nu_{\text{FTC}} \) given in Eqs. (13) and (18), respectively.

**Proof**: Consider the following Lyapunov function as:

\[
V_2 = \frac{1}{2} S^T S
\]

(19)

Introducing the first time derivative \( V_2 \) for \( s \neq 0 \), applying FTC law in Eq. (17) leads to:

\[
\dot{V}_2 = S^T \dot{S} = S^T \left( M_R^{-1} \left( -M_R \dot{q}_{1:3} - C_R \omega - C_{RF} \eta_k + \left[ I_3 - E(t) \right] u + d \right) + 0.5k (q_0 \omega + q_1 \omega) \right) - K_t \int \dot{q}_{1:3} dt - K_S S - \beta \frac{(DM_R^{-1})^T S}{\left\| (DM_R^{-1})^T S \right\|} \leq - \beta - K_S M_R^{-1} S^2 < 0
\]  

(20)

This indicates that even with external disturbances and partial loss of actuator effectiveness, the sliding motion can be maintained; this concludes the proof.

### 3.3. Active vibration control

In order to create high-precision maneuvers, in this section, an active vibration control algorithm has been designed using PZT patches. The output current of the piezoelectric sensor is converted to the voltage of the sensor \( V_e \) using a signal regulator with a gain of \( G_c \) and applied to the piezoelectric actuators with the proportional gain factor of the controller. The following equation can represent the output voltage of piezoelectric sensors [32]:

\[
V_e(t) = G_c i(t) = G_c e^{31} \left( \frac{h_p}{2} + h_p \int_0^L \frac{a^2}{ax^2} \psi^k(x) \psi^k(t) dx \right)
\]

(21)

where \( \psi^k(x) \) is the element shape function, \( i(t) \) is the circuit current, \( e^{31} \), \( h_p \), and \( L_p \) represent the PZT charge/voltage constant, width, length and thickness respectively. The input voltage to the actuator \( V_a \) is given by:

\[
V_a = K_p z(t)
\]

(22)

where \( K_p \) is the controller gain matrix. The relative control force \( f_{ctrl} \) applied to the patches is as follows:

\[
f_{ctrl} = E_p d_{31} z_p \left( \frac{h_p + h_p}{2} \right) \int_0^L \frac{a}{ax} \psi^k(x) dx V_a(t)
\]

(23)

where \( E_p \) and \( d_{31} \) is the young’s modulus and strain constant of PZT layer, respectively.

### 4. Numerical Simulation and Results

This section presents numerical simulations for flexible spacecraft systems to illustrate the performance of the proposed fault-tolerant sliding mode and AVC law and compare the dynamics without the control laws under the actuator faults and external disturbances. All the simulations have been carried out by using the Newmark-Beta numerical integration method on the MATLAB/Simulink software.

The parameters for the main body and panels of flexible spacecraft are considered to be: density \( \rho_A = 2 \frac{kg}{m^3} \), Young modulus \( E_I = 35 \frac{Gpa}{m} \), length \( L_m = 2 \frac{m}{m} \), width \( m = 0.3 \frac{m}{m} \), hub dimension \( a = 0.3 \frac{m}{m} \), spacecraft moment of inertia \( J_z = 7.31 \frac{kg \cdot m^2}{m} \), \( J_x = 13.44 \frac{kg \cdot m^2}{m} \), and \( J_y = 11.72 \frac{kg \cdot m^2}{m} \), and the PZT parameters: strain constant \( d_{31} = 125 \times 10^{-12} \frac{m/V}{m} \), stress constant \( e_{31} = 10.5 \times 10^{-3} \frac{V/m}{N} \), density \( \rho_A = 0.906 \frac{kg}{m^3} \), width \( z_p = 0.0635 \frac{m}{m} \), length \( L_p = 0.0635 \frac{m}{m} \), thickness \( h_p = 1.905 \times 10^{-3} \frac{m}{m} \), and Transmission coefficient \( e_r^2 = 1.5 \times 10^{-8} \frac{F/m}{m} \) are considered. In addition, external disturbances are applied to the rigid body and flexible panels of the spacecraft, respectively \( \tau_e = 0.04 \sin(0.07t) \) and \( d_a = 0.0083 \sin(12t) \). The fault scenario is as follows:

\[
f_i = \begin{cases} 0 & \text{if } t < 10 \\ 0.8 & \text{if } t \geq 10 \end{cases}
\]
In the simulation, the initial condition of angular velocity and quaternions are set as \( \omega = [0 \ 0 \ 0] \) and \( q(t_0) = [0.174 \ -0.0263 \ 0.789 \ -0.526] \). The first three flexible modes, \( k = 3 \), are considered to discretize elastic deformation. The fault-tolerant sliding mode control parameters are selected as nominal control gains \( K_c = 0.5, \ K_p = 0.5 \) and \( K_1 = 0.0001 \), sharpness parameters \( P^T = 0.1 \), fault-tolerant control gains \( K_s = 0.0001 \), \( D = I_{3 \times 3} \) and \( \beta = 0.01 \). Also, for implementation of the active vibration control algorithm, design parameters are considered for elements 1, 3, 5, and 7 as \( G_c = 127 \) and \( K_{P_{ST}} = [32 \ \ 27 \ \ 19 \ \ 7] \).

Figures 1-4 show the time history of control actions, attitude quaternions, angular velocities, and modal displacement in response to the actuators torques, respectively. Furthermore, Figures 5 and 6 illustrate the performance achieved by the SRF algorithm in reducing the effects of rigid-flexible coupling based on the output torque of the attitude actuators and modal displacements.

As shown in Figs. 2 and 3, the fault-tolerant SMC law (17) achieved a good attitude stabilization performance despite partial loss of actuator effectiveness and external disturbances approximately in 40 seconds.

Control performance in the missions with pointing accuracy and system agility requirements is also greatly influenced by considering elastic vibrations during the control design procedure. Figure 4 illustrates the first three flexible modes for systems with and without the FTC. As can be seen, actuator failures lead to extra vibration on flexible parts, which can lead to the fracture of large flexible structures. However, the proposed PID-based SMC considering \( u_{FTC} \) Eq. (18), can attenuate the extra vibrations.
Using SRF simultaneously with attitude control has also significantly reduced the vibration caused by flexible body dynamics. Figures 5 and 6 illustrate the time history of control effort and vibration modes with and without AVC for the fault-tolerant SMC algorithm, respectively. As can be seen, AVC has led to smooth attitude control commands, resulting in more accurate maneuvers (the oscillations settle within 25 seconds).

It is noteworthy that several factors contribute to the excitation of high-frequency modes in the flexible parts, including structural couplings, external disturbances, and actuator faults and failures. It has been shown that active vibration control algorithms significantly reduce control effort as well as residual vibrations, which can interact with one another.

5. Conclusion

This paper develops fault-tolerant PID-based sliding mode and SRF control schemes for the attitude stabilization problem of a flexible spacecraft in the presence of partial loss of actuator effectiveness and external disturbances simultaneously. Introducing PID into SMC increases robustness and transient response while preserving the great features of the SMC. Using SRF in conjunction with attitude control can help eliminate residual vibrations both during and after the maneuver and reduce vibration-induced effects on rigid body dynamics. Initially, a PID-based SMC law is designed for asymptotic attitude stabilization. Next, the proposed FTC scheme fully compensates for the actuator faults' effects from the maneuver's beginning. In the proposed control design approach, no system identification procedure was required to identify faults, nor was a fault detection and isolation procedure required. Furthermore, a lower bound is not required for the actuator's effectiveness. The Lyapunov criterion is used to ensure the stability of the entire hybrid system, and numerical simulations are given to confirm the proposed FTC in the presence of actuator fault scenarios and external disturbances.

6. Reference


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