

Determination of Stochastic Shear Strength Parameters of a Real landslide by Back Analysis

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Abstract

One of the most hazardous phenomena leading to enormous monetary loss and threatening human life is slope instability. The major contributors to such disasters are slope geometry, slope material strength, geohydrological condition, structural discontinuity, weathering, development of weak zones, lithological disturbance, and heavy rainfall. As the accuracy of parameters obtained from geotechnical investigations is vital for a practical understanding of the geotechnical project, the back analysis is a pragmatic approach to forecast and control landslide and slope instability. The current paper presents a stochastic back analysis of a recent landslide near a highway located in the south of Iran. Some background information has been gathered through air photos, field observations, and photographs indicating slope failure is pretty recent, and some boreholes were drilled to obtain the required geotechnical parameters of the soil media. Due to the uncertainties in these parameters, the stochastic back analysis approach was adopted. To this end, soil strength parameters have been calculated using the FEM program coded in MATLAB. Results that properly aligned with the findings of the post-event investigations showed a computationally more efficient back analysis approach. The improved knowledge of the geotechnical strength parameters gained through the stochastic back analysis better elucidated the slope failure mechanism, which provides a basis for a more rational selection of remedial measures.

Keyword: Back analysis; Finite element method; Landslide; Shear strength reduction technique; Stochastic analysis

Introduction

Recent geohazard events have increased the awareness in the slope stability analysis community about landslide hazards. Determining the conditions and establishing a suitable model of the landslide is known as back analysis. This method is preferred when there are significant limitations in using laboratory and situ test results to characterize the soil accurately.

The characterization of soil is subjected to uncertainties due to inadequate information for site characterization and inherent variability of properties within it [1–3]. Stochastic analyses provide rational means to treat the uncertainties associated with underlying parameters systematically. Based on the advantage of stochastic analysis, considerable research has been carried out in the past few years on slope failure [4,5]. Many stochastic methods have been used for slope stability analysis. These methods can be grouped into five main categories: approximate methods, Monte Carlo Simulation (MCS), numerical methods, analytical methods, and artificial intelligence methods [6,7]. Starting in the early 90s, a new technique called the Random Finite Element Method (RFEM), which combines unconditional random

field theory and the FEM, was developed in stochastic geotechnical engineering.

Griffiths and Fenton [8] investigated the probability of failure of a cohesive slope using both simple and more advanced probabilistic analysis tools (i.e., RFEM). The RFEM uses elastoplasticity combined with random field theory. This method is shown to offer many advantages over traditional probabilistic slope stability techniques because it enables slope failure to develop naturally by seeking out the most critical mechanism.

Johari and Mousavi [9] investigated the application of the Jointly Distributed Random Variables (JDRV) method as an analytical method to compare the reliability of four widely used limit equilibrium methods for slope stability analysis. In this method, the probability density functions of input variables are expressed mathematically and joined together by statistical relations. By integrating into the adopted model, a mathematical expression of the Probability Density Function (PDF) of the output parameter is derived.

The back analysis is commonly used in geotechnical engineering to calibrate relevant soil properties for modeling

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purposes [10-13]. The landslide was among the first geotechnical problems initially tackled by back analysis. This issue is becoming more critical for landslides with potentially long run-out [14-17].

Many articles discuss the shortcomings of backanalysis in landslide applications [17-18]. However, the increasing use of sophisticated mathematical models currently prompts the geotechnical community to use back analysis of reported case studies to correctly identify the corresponding model parameters. Although the interpretation of laboratory tests is commonly used for this aim, the specimens being tested are unlikely to represent real site conditions of the soil [19]. In addition, regular trial-and-error methods might be grueling processes for calibrating and validating complex constitutive models. In these cases, the use of automatic inverse modeling algorithms is indeed advantageous [20].

Two commonly used methods to perform back analysis are deterministic and stochastic [21]. In a deterministic approach, the back analysis of geotechnical strength parameters is usually determined through a trial-and-error process. Various values for geotechnical strength parameters and slip angles are assumed and analyzed until the input values that yield $FS=1$ are obtained. However, the deterministic approach is inadequate for addressing the uncertainties in the estimated geotechnical strength parameters [22]. The stochastic method has advantages over the deterministic method that include: (1) it provides a logical way to incorporate information from other sources in the back analysis and (2) it is capable of back analyzing multiple sets of slope stability parameters simultaneously [23-24]. However, the only possible disadvantage of the stochastic back analysis method is the difficulties for implementation compared to the deterministic back analysis method. Different probabilistic approaches for geotechnical back analysis are available, including the Bayesian method and maximum likelihood method. Juang et al. [25] presented a Bayesian method to update soil parameters in a multi-stage braced excavation using field observations. It is concluded that Bayesian updating is effective in reducing the uncertainty of soil parameters. Li et al. [26] presented a method that couples the Bayesian method and Multi-output Support Vector Machine (MSVM). The results revealed that probabilistic back analysis gives further information than a deterministic one. Thus, it is capable of simulating geotechnical engineering practices closely.

Honjo [27] employed an extended Bayesian method to back analysis an embankment on soft clay, which was also based on the assumption that the number of observed data is larger than the number of updated parameters. Ledesma [28] performed geotechnical back analysis for tunnel excavation and a large underground cavern based on the maximum likelihood approach. The maximum likelihood method is applicable when the number of observed data is larger than the number of parameters to be updated. Model calibration is usually based on trial-and-error procedures that, in turn, rely on expert

judgment or previously acquired experiences for similar phenomena.

Efficient and reliable procedures for model calibration of the propagation stage of landslides are still needed. This study aims to present a practical approach for back-calculate strength parameters of a landslide via a stochastic back analysis procedure. For this purpose, a real landslide is considered. The stability analysis is carried out with a finite element-based program coded in MATLAB. The study is performed deterministically, and it is then extended to the stochastic context to consider the uncertainty of soil properties. To implement the soil parameters uncertainties, three input parameters, including the internal friction angle, cohesion, and density, have been defined as stochastic variables. The model was run for 1200 simulations to produce the developed stochastic shear strength parameters. In another part of this paper, the shape and location of the critical slip surface are assessed by taking advantage of the FEM ability to determine the maximum deviatoric strain. The location of the slip line, along with the knowledge that the slope has failed, can be used to back-calculate values for two developed shear strength parameters.

The methodology of back analysis

Several methods for back analysis ever provided. The most straightforward backanalysis is when an average shear strength is calculated from the known slope geometry and soil unit weights. This is accomplished by assuming a friction angle of zero and calculating a value of cohesion that will produce a factor of safety of 1. This practice of calculating an average strength expressed as a cohesion can lead to erroneous representations of shear strength and potentially unfavorable consequences (Cooper, 1984) [29]. In another method of back analysis called back calculating shear strength parameter based on slip surface geometry, several pairs of values of cohesion and friction angle (c and ϕ) were assumed. The pairs of values were chosen such that they represented a range in the dimensionless parameter $\lambda c\phi$, but the values did not necessarily produce a factor of safety of one [30]:

$$\lambda c\phi = (\gamma H \tan \phi) / c \quad (1)$$

where H is the slope height, and c and ϕ represent the appropriate total stress or effective stress shear strength parameters. The critical circles and corresponding minimum factors of safety were calculated for each pair of values of the strength parameters. Values of the developed shear strength parameters (c_d and ϕ_d) were calculated for each pair of strength. Parameters from the following equations using the assumed cohesion and friction angle and the computed factor of safety [31]:

$$c_d = \frac{c}{SRF} \quad (2)$$

$$\phi_d = \arctan(\tan \phi / SRF) \quad (3)$$

The developed cohesion and friction angle represent back-calculated values required to produce a factor of safety of 1.

The depth of the critical slip surface for each pair values of strength parameters was calculated.

The determined back-calculated cohesion and friction angle was plotted versus the depth of the slip surface.

The cohesion and friction angle corresponding to the observed slide depth was determined using the plotted results.

The approach used in this research is the developed model of back calculating shear strength parameter based on slip surface geometry method. Instead of using the dimensionless parameter $\lambda_{c\phi}$, the depth of the slip surface is determined by using FEM-SSR. Then the developed cohesion and internal friction angle be obtained.

FEM-SSR technique

Backanalyzing a failed slope usually involves trying to establish what conditions existed at the time of failure. In the back analysis of failure, the assumption is made that the safety factor is 1.0 so that the forces equal the driving forces. Successful back analyzing requires accurate information regarding geometry and material properties. The FEM-SSR technique has shown increased promise over the past few years as a reasonable methodology for performing slope stability analysis. While the FEM-SSR method typically takes more computational power to perform, it avoids the complex searching algorithms required to determine the critical slip surface. To reach the state of limiting equilibrium, the SRF is gradually increased. This means that the soil shear strength becomes weaker until it is no longer possible for the FEM analysis to reach convergence. At this stage, it can be said that failure of the slope occurs and $FOS = SRF$. Non-convergence within a specified number of iterations and tolerance is an indicator of slope failure because of the absence of force equilibrium. Several pairs of values of cohesion and friction angle were assumed. The pairs of values that were chosen did not necessarily produce a factor of safety of 1. The critical slip line and the corresponding minimum factor of safety (SRF) were calculated for each pair of c and ϕ .

Case study

Landslides are mass movements that present a well-defined failure surface. According to the failure surface geometry being respectively circular, polygonal, or complex, they are classified as rotational, translational, or complex slides. The slides can be classified as shallow or deep. It is according to the relative depth of the failure surface to the longitudinal length of the landslides. November 2018, a landslide occurred on Yasuj-Babamidan Road in the south of Iran. The Ministry of Roads and Urban Development Technical & Soil Mechanics Lab investigated the landslide, and the results of this investigation are reported. Figure 1 shows the air photos of the landslide location in 2017 before the occurrence of the landslide. Figure 2 shows air photos of the landslide site after the landslide occurred; the slip lines created by the landslide can be seen in this

figure. According to Figs. 1 and 2, the construction of a new road line can be considered a factor that triggered the landslide.



Figure 1. Pre-landslide air photos captured in 2017



Figure 2. Post-landslide air photos in 2018

The geotechnical exploration program consisted of four boreholes (BH.1 to BH.4) are located in critical locations. The average boreholes spacing was considered 30m. Boreholes 1 and 2 were drilled to a maximum depth of 30 meters, and boreholes 3 and 4 reached a depth of 14 meters. The characteristics of the boreholes are given in Table 1. Various in-situ (i.e., the Standard Penetration Test (SPT)) and geotechnical laboratory tests were carefully conducted as part of the investigation program to assess the subsurface conditions. The boreholes database BH.1 to BH.4 is summarized in Tables 2–5, respectively.

Table 1. Site boreholes characteristics

Borehole Number	UTM		Depth (m)
	X	Y	
1	30.596048	51.527802	30
2	30.597239	51.528456	28
3	30.596357	51.528483	14
4	30.595752	51.528751	14

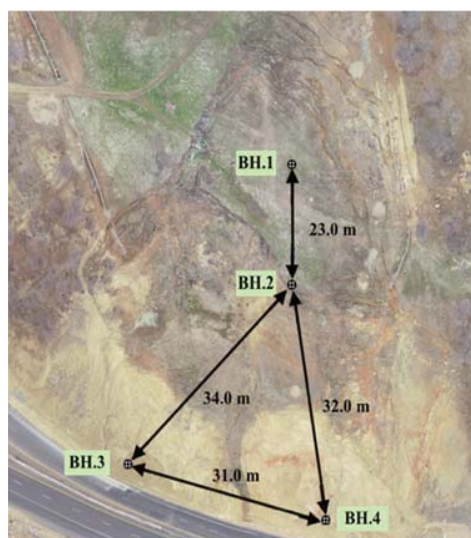


Figure3. The site and boreholes locations

In this site investigation, undisturbed soil samples were taken every two meters from borings, and

the grain size analysis, water content, and the Atterberg limits were determined. The shear strength parameters were measured at various depths. Test borings drilled across the site encountered one type of soil texture; a fine-grained soil. According to Tables 2–5, the fine-grained soil, which generally consisted of low plasticity clay (CL), was encountered from the soil surface to a depth of 30 m.

The slope based on receiving coordinate in Figure 1 is plotted in Figure 4. It was tried to model the road excavation.

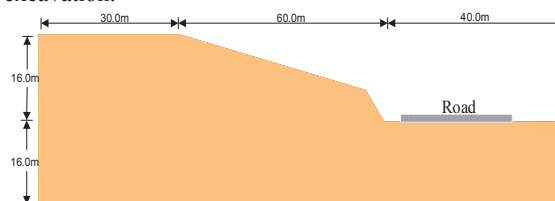


Figure4. Assessed slope geometry

Table 2. Geotechnical parameters of borehole No. 1.

Depth (m)	SPT	Plasticity index	Internal friction angle (Deg.)	Cohesion(kPa)	Fine content (%)
2	25	15	24	22	85
4	18	17	-	-	87
6	26	19	25	24	86
8	25	15	-	-	88
10	33	14	25	23	93
12	48	16	-	-	90
14	55	15	27	26	94
16	51	19	-	-	91
18	53	18	28	25	92
20	52	18	-	-	90
22	61	19	29	27	90
24	58	23	-	-	93
26	62	22	30	28	91
28	62	21	-	-	93
30	64	19	30	27	94

Table 3. Geotechnical parameters of borehole No. 2.

Depth (m)	SPT	Plasticity index	Internal friction angle (Deg.)	Cohesion(kPa)	Fine content (%)
2	22	17	26	24	88
4	21	16	-	-	90
6	24	15	24	23	91
8	27	19	-	-	88
10	32	17	28	25	94
12	38	18	-	-	92
14	36	16	30	27	93
16	41	20	-	-	92
18	44	17	27	26	90
20	46	19	-	-	93
22	57	20	29	29	92
24	61	22	-	-	89
26	58	24	30	31	93

Table 4. Geotechnical parameters of borehole No. 3.

Depth (m)	SPT	Plasticity index	Internal frictionangle (Deg.)	Cohesion(kPa)	Fine content (%)
2	30	16	24	22	87
4	36	19	-	-	89
6	38	18	25	24	93
8	39	19	-	-	91
10	44	20	25	23	92
12	48	19	-	-	94

Table 5. Geotechnical parameters of borehole No. 4.

Depth (m)	SPT	Plasticity index	Internal frictionangle (Deg.)	Cohesion(kPa)	Fine-grained content (%)
2	32	15	24	26	91
4	34	17	-	-	93
6	38	19	25	27	92
8	41	15	-	-	90
10	43	14	25	31	92

Modeling and verification

One of the main advantages of the FEM to slope stability analysis is that no assumption is needed to be made in advance about the shape or location of the failure surface. Failure occurs naturally through the zones within the soil mass in which the soil shear strength is unable to sustain the applied shear stresses [32].

In this research, a finite element-based program was coded in MATLAB to predict the landslide based on slope geometry. The model was provided for two-dimensional, plane strain conditions using eight-noded quadrilateral elements of elastic visco-plastic soil with the Mohr-Coulomb failure criterion and a non-associated flow rule. The boundary conditions are defined by fully restraining the bottom and horizontally restraining the left and the right side of the soil domain. The soil model initially consisted of 2904 quadrilateral elements.

Slope failure in the finite element model occurs 'naturally' through the zones in which the shear strength of the soil is insufficient to resist the shear stresses [32]. Each input parameter is initially considered from a series of data with a normal distribution. The mean values of these data are obtained from test pits that were excavated in the area to investigate subsurface soil conditions. The critical slip line and the slip depth associated with the data distribution assessed using Shear Strength Reduction (SSR). Each pair of shear strength parameters (c and ϕ) corresponds to a unique slip line. The results of critical slip lines are shown in Figure 8. The location of the slip line, along with the knowledge that the slope has failed, can be used to back-calculate values for two developed shear strength parameters.

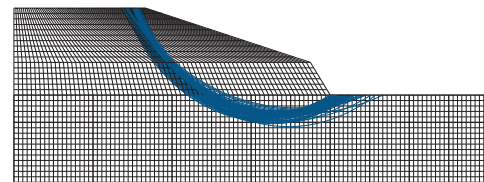


Figure 8. Location of slip lines obtained by simulation

Field inspection indicated that the sliding mass had a triangular shape but with a rather uniform thickness. The average thickness over the entire sliding surface was estimated to be about 7.0m. To validate the results obtained from the back analysis.

A deterministic slope stability modeling in GeoStudio was performed. By examining the results obtained from the back analysis and the results obtained from the GeoStudio, the concordance of these two analyses can be concluded.



Figure 9. The PDF of the depth of slip line

Stochastic analysis

To implement the soil parameters uncertainties in slope stability, three input parameters, including the internal friction angle, cohesion, and density, have been defined as stochastic variables. The mean and standard deviation (Std.) values of the stochastic parameters were summarized in Table 6. Figure 5-7 show the distribution

of internal friction angle, cohesion, and density. Based on 1200 simulations, the histograms of the stochastic input variable are plotted in these figures.

Table 6. The stochastic parameters

Density (kg/cm ³)		Cohesion (kPa)		Internal friction angle (Deg.)	
Mean	Std.	Mean	Std.	Mean	Std.
19.50	2.0	25.0	2.0	27.0	2.0

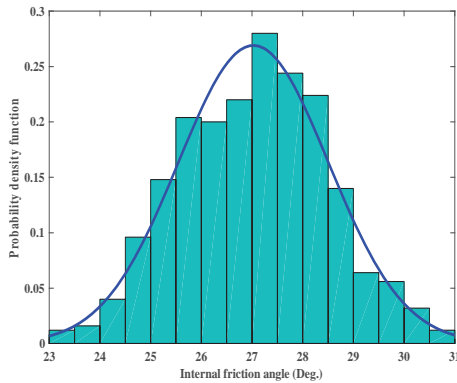


Figure 5. Normal distribution of internal friction angle

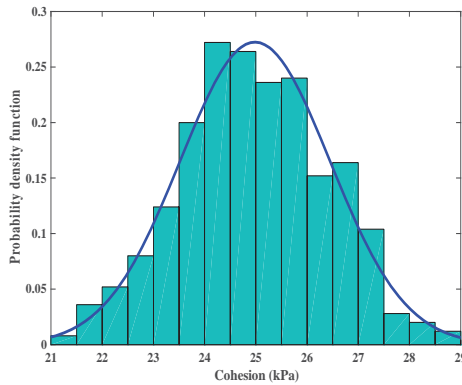


Figure 6. Normal distribution of cohesion

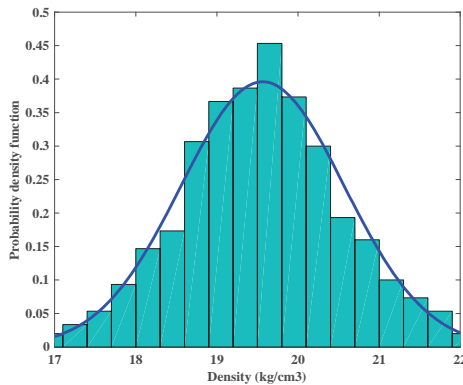


Figure 7. Normal distribution of density

Many researchers have studied the statistical distributions of these stochastic parameters. Numerous researchers emphasized that the normal, truncated normal, and lognormal distributions are more compatible with the behavior of soil parameters [33]. However, other distributions, such as triangular, Gumbel, Weibull, versatile beta, and generalized gamma, are also reported. In this paper, the truncated normal distributions are used to model stochastic soil parameters. The PDFs of truncated normal distributions for the stochastic parameters are as follows[9]:

$$f_{\gamma}(\gamma) = \frac{1}{\sigma_{\gamma}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\gamma - \gamma_{mean}}{\sigma_{\gamma}}\right)^2\right) \quad (4)$$

$$\gamma_{min} \leq \gamma \leq \gamma_{max}$$

$$f_c(c) = \frac{1}{\sigma_c\sqrt{2\pi}} \exp\left(-0.5\left(\frac{c - c_{mean}}{\sigma_c}\right)^2\right) \quad (5)$$

$$c_{min} \leq c \leq c_{max}$$

$$f_{\varphi}(\varphi) = \frac{1}{\sigma_{\varphi}\sqrt{2\pi}} \exp\left(-0.5\left(\frac{\varphi - \varphi_{mean}}{\sigma_{\varphi}}\right)^2\right) \quad (6)$$

$$\varphi_{min} \leq \varphi \leq \varphi_{max}$$

where φ_{min} , φ_{max} are the minimum and maximum values of soil internal friction angle with standard deviation σ_{φ} and c_{min} , c_{max} are the minimum and maximum values of soil cohesion with standard deviation σ_c . γ_{min} , γ_{max} are minimum and maximum values of soil unit weight and σ_{γ} is standard deviation of soil unit weight [9].

$$\begin{cases} \gamma_{min} = \gamma_{mean} - 4\sigma_{\gamma} \\ \gamma_{max} = \gamma_{mean} + 4\sigma_{\gamma} \\ c_{min} = c_{mean} - 4\sigma_c \\ c_{max} = c_{mean} + 4\sigma_c \\ \varphi_{min} = \varphi_{mean} - 4\sigma_{\varphi} \\ \varphi_{max} = \varphi_{mean} + 4\sigma_{\varphi} \end{cases} \quad (7)$$

Considering the stochastic variables within the range of their mean plus or minus four times the standard deviation [Eq. (4)], 99.994% of the area beneath the normal density curve is covered. It should be noted that, for choosing the initial data, the following conditions must be observed for the angle of shearing resistance, cohesion intercept, and unit weight of soil in the sliding line[9].

$$\begin{cases} \gamma_{min} - 4\sigma_{\gamma} > 0 \\ c_{min} - 4\sigma_c > 0 \\ \varphi_{min} - 4\sigma_{\varphi} > 0 \end{cases} \quad (8)$$

Results

The model was run for 1200 simulations to produce the developed stochastic shear strength parameters. Figure 10 and 11 show the PDF and Cumulative Distribution Function (CDF) of the depth of slip line achieved from SSR analysis. As it is clear, the mean value

of the depth of the slip line is about 7.0m which is agreed closely with field observation of slip surface depth.

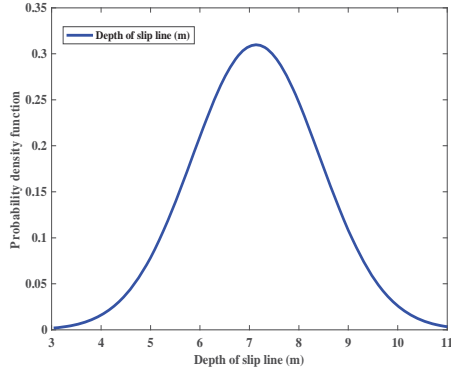


Figure 10. The PDF of the depth of slip line

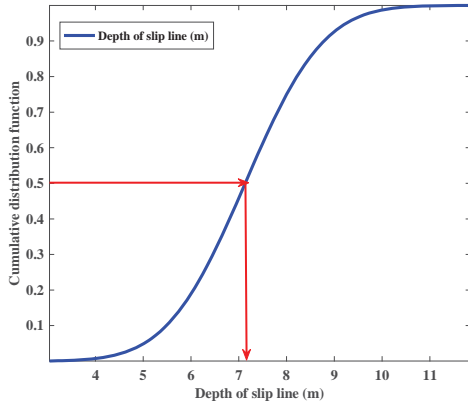


Figure 11. The CDF of the depth of slip line

Figure 12 and 13 illustrate the results of the cohesion and developed cohesion variations with respect to the depth of the slip line, respectively. The same as the cohesion, the variation of friction angle and developed friction angle with respect to the depth of the slip line are plotted in Figure 14 and 15.

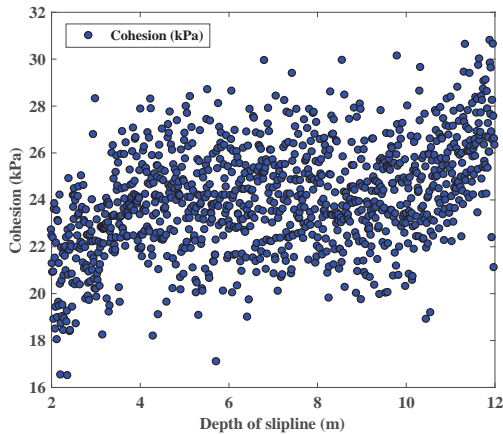


Figure 12. Variation of cohesion with the depth of the slip line

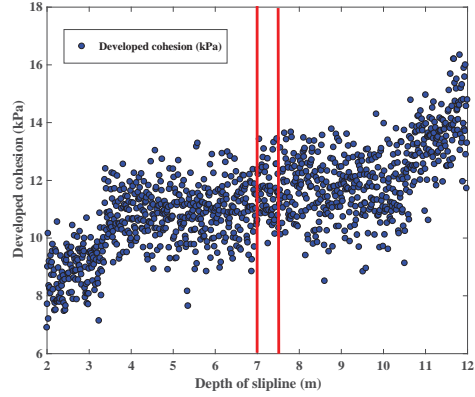


Figure 13. Variation of developed cohesion with the depth of the slip line

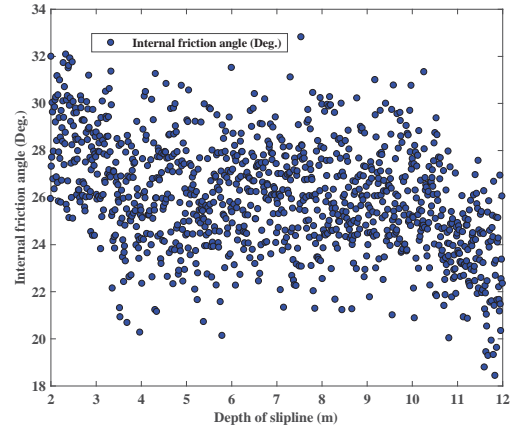


Figure 14. Variation of friction angle with the depth of the slip line

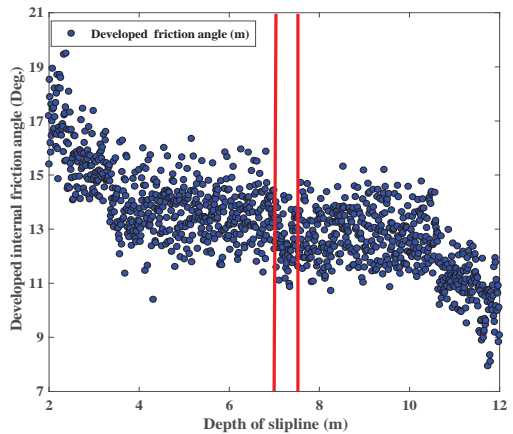


Figure 15. Variation of developed friction angle with the depth of the slip line

Using this information, the amount of corresponding mobilized cohesion with a depth of slip line about 7-7.5 m can be deduced from Figure 13. The PDF of developed cohesion at depth 7.0-7.5m are presented in Figure 15. It can be seen the mean value of the developed cohesion is 10.5 kPa. This procedure was carried out for the developed friction angle. The values of the mobilized friction angle corresponding to the depth of the slip line are shown in Figure 15. Figure 17 shows the probability density function of developed friction angle at depth 7.0-7.5 m. It can be seen the mean value of the developed friction angle is 12.8 degrees.

Figure 19 shows the CDF of developed cohesion, the probability that the developed cohesion is less than or equal to 10.5 kPa is 50%. Therefore, the cohesion on the site is more than 10.5kPa

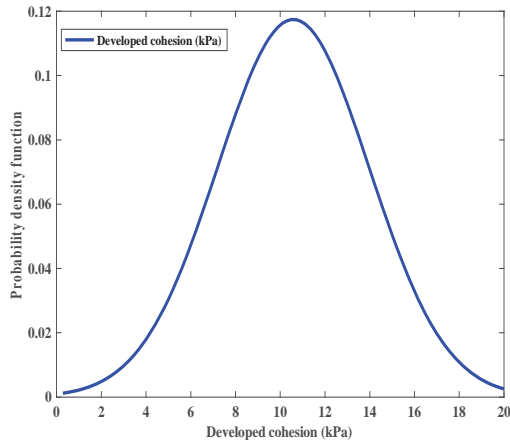


Figure 16. The PDF of developed cohesion at depth 7.0-7.5 m

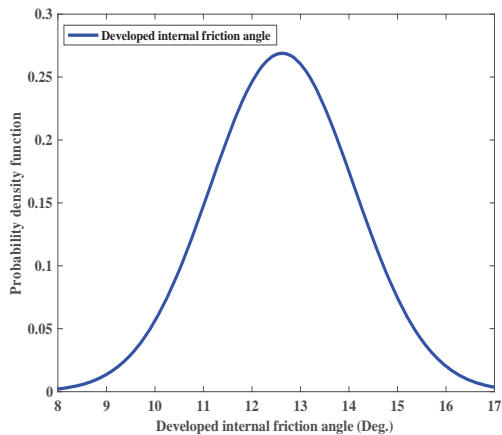


Figure 17. The PDF of developed friction angle at depth 7.0 -7.5m

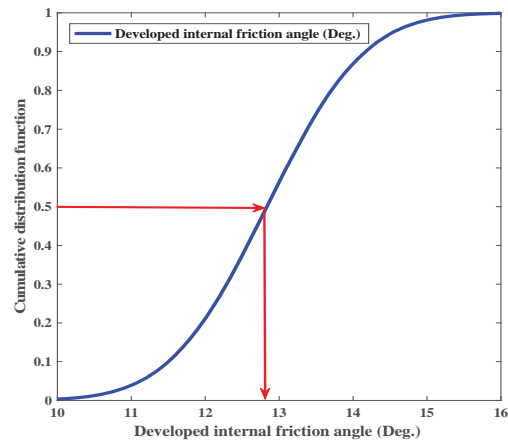


Figure 18. The CDF of developed internal friction angle at a depth of 7.0m

Figure 18 shows the CDF of developed internal friction angle, the probability that the developed cohesion is less than or equal to 12.8 (Deg.) is 50%.

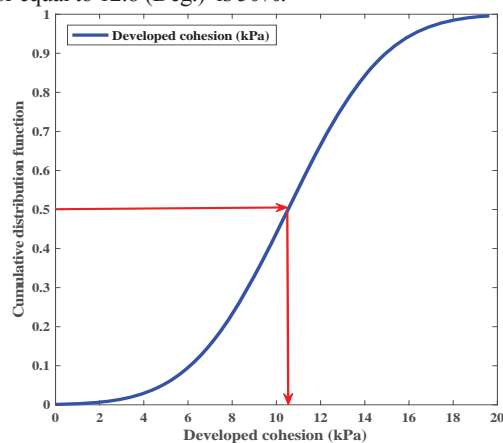


Figure 19. The CDF of developed cohesion at depth 7-7.5 m

Conclusion

In November 2018, a landslide occurred in the south of Iran. This paper investigates the setting and primary characteristics of this landslide and analyzed its failure behavior using a stochastic back analysis method. Soil strength parameters have been calculated using the FEM codes program in MATLAB based on the shear strength reduction technique. The probability density function, the cumulative distribution function of the cohesion, and friction angle were determined through back analysis. As a result, the probable soil shear strength parameters of the slope were obtained. Results, which agreed closely with the post-event investigations, showed a computationally more efficient back analysis approach. The improved knowledge of the

geotechnical strength parameters gained through the stochastic back analysis better elucidated the slope failure mechanism, which provides a basis for a more rational selection of remedial measures.

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