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Stochastic Analysis of Complex System with Auto Changeover Switch and Advert Environment Employing Copula Approach

V. V. Singh[1*] and M. Gahlot[2]

1. Mathematics, Yusuf Maitama Sule University, Kano, Nigeria
2. Department of Mathematics, Mewar University, Chittorgarh, Rajasthan, India

Abstract

This paper studies the reliability measures of a system consisting of two subsystems in a series configuration for different types of failure and two types of repair. The subsystem-1 has four identical units in a parallel configuration operating under 3-out-of-4: G policy and this has connected to subsystem-2 which has three identical units arranged in a parallel configuration and working under 2-out- of-3: F, scheme. The units of subsystem-1 are controlled by a controller for preventing failure effect and safety purposes. It is assumed that units of each subsystem have different types of failure and repair rates. The unsuitability of the environmental conditions such as overheating as a well-known natural cause of failure of any system and also weather conditions like heavy rain, thunderstorm, and catastrophic shakeups, etc. have treated as a complete failure of the system. This study considers the environmental causes of failure in the proposed repairable system as complete failure by which the system stops functioning. Human failure in the system is trickled as complete failure and repair employing copula (Gumbel- Hougaard family copula distribution) like another complete failed states of subsystems. To analyze the proposed system, the supplementary variable technique are used and some measures of system reliability like availability, reliability; MTTF and incurred profit function for different values of parameters are derived. Some particular cases for different values of failure rates that have explicit are also studied.

Keywords: Environment failure, Human failure, Availability, MTTF, Reliability, Profit analysis, Gumbel-Hougaard family copula.

Nomenclature

λ_1/λ_2:	The failure rates of units of the subsystem-1/ failure rate of subsystem-2.
λ_E/λ_h:	Failure rates of system due to environmental failure/ human failure.
λ_{C_1}:	Failure rate of the controller of the subsystem-1.
$\varphi_1(x)$, $\varphi_2(x)$:	Repair rate of partial failed states for the subsystem-1, subsystem-2.
$P_0(t)$:	State transition probabilities of the system in state S_0.
$\overline{P}(s)$:	Laplace transform of state transition probability P(t).
P_i (x, t):	State transition probability that the system is in state S_i, i=1, 2, 3…..,8 with state transition probability P_i (x, t) in which the system is under repair with repair variable x, t.
$C_\theta(u_1, u_2(x))$:	The expression for joint probability distribution (failed state S_i to perfect state S_0);conferring to the Gumbel-Hougaard family is: $\mu_0(x) = C_\theta(u_1(x), u_2(x)) = exp[x^\theta + log\{\varphi(x)\}^\theta]^{1/\theta}$, Where, $u_1 = \varphi(x)$ and $u_2 = e^x$, $1 \le \theta \le \infty$
Ep(t):	Expected profit in the interval [0, t)
K_1/K_2	Revenue generation/ service cost of the system per unit time in the interval [0, t).

1. Corresponding Author Email: singh_vijayvir@yahoo.com

Introduction

Industrial systems have become very compact and more complicated due to the excessive use of automation and miniaturization. Therefore, detecting and repairing faults in various units or components used in the system, sometimes, becomes imperatively challenging and time consuming as well. Technology advancement coupled with the complexity of networks is a great option for proper performance of system. In other words, it is crucial to keep the system failure-free. Hence, scientists and engineers have developed systems configurations that can perform better than conventional systems. The quality of the performance of the systems is tested by employing probability rules and distributions for failure and repair policies. Redundancy is another suitable

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technique which has widely recognised to improve the reliability of the system. In the case of redundancy of the system, some additional units or paths are created together with the central unit to support the complex industrial system to improve the reliability of the system. The k-out-of-n configuration structure is another proper structure which is widely applied to improve industrial systems performance. The k-out-of-n system configuration works, if and only if at least k of the n units/ components works. Thus, the k-out-of-n system plays a significant role in operations of industries system, which have attracted the attention of researchers.

Researchers working in the field of reliability and related fields have extensively developed and investigated systems particularly redundant systems, and evaluated the performances based on reliability measures of with a significant degree of satisfaction. Refereeing to a few works wise, M. Ram et al. (2013) examined the stochastic analysis of a redundant standby system with waiting for repair strategy. The study demonstrated the effect of waiting time to repair the system to restore to operations mode. Rekha et al. (2013) has studied different reliability parameters for a complex redundant system under head-of-line repair. Ibrahim et al. (2012) studied a redundant system with three types of failure and emphasized on the comparative analysis of different situations. In this work, they observed that the preventive maintenance of a system is far better than without preventive maintenance of the system. Singh et al. (2013) discussed a system having two units in a series configuration with a controller for availability, MTTF and cost analysis. In continuation of the study of repairable systems, Singh et al. (2013) examined the reliability characteristics of the complex system consisting of two subsystems in the series configuration under human and controller failure. Singh and Ram (2014) studied the operational behaviour of a, multi-state- state k- out- of- n: G; system and analysed for 2-out-of-3: G; the system as an exceptional case for computations. Dalah and Singh (2014) examined a two-unit standby system with the concept of switch failure. Eryilmaz et al. (2011) studied signature-based analysis of m-Consecutive-k-out-of-n: F system with exchangeable components.

The human failure plays a major role in the evaluation of repairable systems performance during installation, production, and maintenance of the complex system. A slight negligence during operations of a complex system may have a cause of significant damage, which can destroy the whole system and might be rendering for a substantial loss in the sense of safety of human life. In the context of human failure, Surbhi et al. (2013) studied the operational behaviour of primary part assembly system of an automobile incorporating human error in maintenance and also intensive on environmental failure. Occasionally ecological failure in the system can damage the whole system and stop functioning of the system instantly. Unsuitability of the environmental condition may be one of the leading causes of failure of a system. Singh et al. (2014) studied availability, MTTF and cost analysis of the complex system under pre-emptive resume repair policy using copula distribution approach. Suitability of environment is essential for proper operations of a complex system. In contrast to the study of human failure in the repairable systems, Dhillon et al. (1993), Vanderperre (1990) and Ram et al. (2010) studied reliability features of the complex system with common cause failure and reliability of duplex standby system by supplementary variable technique and Laplace transform. Rawal et al. (2014) studied the functioning of internet data centre with redundant server together with the primary mail server for different failure and repair facility using copula. Singh and Rawal. (2015) have studied a complex system consisting three units as superiority, priority, and ordinary under primitive resume repair policy using two types of repair. Jyoti Gulati et al. (2016) have examined performance of a system having three subsystems in series configuration employing copula linguistic approach and have concluded that copula repair is more beneficial over general repair. Lado et al. (2018, 2019) have studied a two units series system configuration under general repair policy and copula approach and conclude that copula repair improves performance of the system. In a competitive business world warranty on the newly launched products have played a significant role. The warranty for the replacement/repair of a product during a specific period for any equipment is an important policy factor, which frequently attracts the attention of customers. During the warranty period, the particular inferior part of the system is either replaced or repaired without any extra charge, and after the expiry of the warranty period, it would be charged for repair or replacement. In this context, Ram Niwas and M. S. Kadyan (2015) studied reliability characteristics of the maintained system with warranty and degradation using supplementary variable technique D.R Cox (1995).

R. B. Nelson (2006) showed that Copula distributions play a crucial role in the study of repairable systems. Let us recall that a d-dimensional copula is a distribution function defined on the hypercube $[0, 1]^d$ with standard uniform marginal distributions. A mapping C: $[0, 1]^d \rightarrow [0.1]$, i.e., a mapping C of unit hypercube into the unit interval is called Copula if the following three properties hold:

1. $C(u_1, u_2, u_3 \ldots\ldots u_d)$, is increasing in each component u_i.
2. $C(1, \ldots. 1, u_i, 1, \ldots. 1) = u_i$ for all $i \in \{1, \ldots., d\}, u_i \in [0,1]$.
3. For all, $(a_1, \ldots\ldots, a_d), (b_1, \ldots\ldots, b_d) \in [0,1]^d$ $with\ a_i \le b_i$ we have, $u_1 = a_i\ and\ u_2 = b_i\ for\ all\ i \in \{1, \ldots.. d\}$.

Though the various types of copulas available in the literature, in this paper, we have applied the Gumbel-Hougaard copula for the study of the analytical part which couples two kinds of distribution functions, namely, general distribution and exponential distribution.

Gumbel-Hougaard family Copula function is defined as; $C_\theta(u_1, u_2) = exp[\,(-(-\,logu_1)^\theta + (-\,logu_2)^\theta)^{\frac{1}{\theta}}]$, $1 \le \theta \le \infty$, for $\theta = 1$ the Gumbel-Hougaard copula models areindependent, for $\theta \to \infty$it converges to comonotonicity?

The present paper studies a system, which consists of two subsystems (subsystem-1 & subsystem-2) in a series configuration. In addition, essential class of faults, i.e., the environmental failure which many a time is also the cause of damage to the system is considered beside other types of failures. Hence, a mathematical model which consists of two subsystems (subsystem-1 and subsystem-2) in a series configuration is devised. In subsystem-1, four identical units are arranged in a parallel configuration which is connected with subsystem-2 which has three identical units in parallel configuration. Initially, in state $S_{0,}$ both the subsystems are in good operational condition, i.e., the system is in perfect state. During operations, if any one unit of subsystem-1 fails then, the system approaches to state S_1. Further failure of any unit in the subsystem-1 the system approaches to state S_2 and further failing any unit in the subsystem-1 it approaches to state S_3 which is a complete failed state. Failing any one unit in the subsystem-2 it will be in state S_4 and again failing second unit of the subsystem-2, the system will approach to complete failed state S_5 as per policy 2-out-of-3: F. Failure due to the adverse effect of environmental conditions, i.e., ecological failure and human failure is indicated by the states S_6 and S_7 respectively. The system is repaired by employing general time distribution in the degraded status and using copula distribution in the complete fail state. The state S_8 is a complete failed state which brings the system in un-operational state. The system is analyzed using the supplementary variable technique and various measures of reliability especially availability, MTTF, cost analysis through profit evaluations' and some other particular cases are investigated to highlight the results.

The paper has organized in the following manner as prescribed format as Nomenclature, Introduction, State description, Assumptions, and state transition diagram of the model. The mathematical modelling and solution with computational analysis as Availability, Reliability, mean time to system failure (MTTF) and profit analysis of the present model have investigated. Results discussion and concussions is last section of this analysis.

State Description of The Model

State:	State Description
$S_{0:}$	In the state $S_{0,}$ both subsystems are in good condition.
$S_{1:}$	The state S_1 represents the state due to failing one unit in subsystem-1, which is a minor partial failed situation in the subsystem-1.
$S_{2:}$	State S_2 represents a major partial failed state in subsystem-1.
$S_{3:}$	State S_3 is a complete failed state in system due to which system stop working and it need urgent repair. A copula distribution has employed for repair the failed system.
$S_{4:}$	The state represents manifestation failure in subsystem-2. The system is in degraded but working state with minor partial failure. The system is under general repair and elapsed repair time is (x, t).
$S_{5:}$	The system is the complete failed state due to failure in subsystem-2 as 2-out-of-3: F scheme.
$S_{6:}$	State S_6 represent complete failed state due to human failure.
$S_{7:}$	The state S_7 represents the presence of environmental failure in the system due to unfavorable environmental conditions. The system is the complete failed state. The system is in repair and elapses repair time is (x, t).
$S_{8:}$	The state S_8 represents the advent of controller failure in the system. The system is the complete failed state. The system is in repair and elapses repair time is (x, t).

Assumptions

The following assumptions have been made throughout the discussion of the model.

1. Initially, the system is inthe perfect state S_0, and both subsystems are in good working conditions.
2. The subsystem-1 works successfully until at least 3- units of its are in good condition.
3. The subsystem-2 work successfully when two units are in good condition. When more than two units fail, it approached to a complete failed state.
4. Only one transition is allowed at a time between two adjacent transition states.
5. Both human and environmental failures bring the system in a complete failed state.
6. Controller failure of the system brings the entire system in complete failed state.
7. The partially failed/ degraded state in the system is repaired using general time distribution.
8. The complete failed states need fast repairing, and hence these states are repaired-using Gumbel- Hougaard family copula.
9. The repaired system is assumed to work like a new one and repair did not damage anything.

State transition diagram of a model

The schematic diagram of changes in the various states during the operation of any system is known as state transition diagram. In the present mathematical model total nine states (S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8) are possible. Among nine states S_0 is a perfect state, (S_0, S_1, S_2, S_4) operational and (S_3, S_5, S_6, S_7, S_8) non - operational states representing in the state transition diagram in Figure 1. The partially failed states are repaired employing general repair rates $\varphi_1(x), \varphi_2(x)$but the complete failed states by Gumbel-Hougaard family copula distribution

$\mu_0(x) = C_\theta(u_1(x), u_2(x)) = exp[x^\theta + \varphi(x)\}^\theta]^{1/\theta}$,
Where, u_1 = φ(x) and u_2 = e^x.

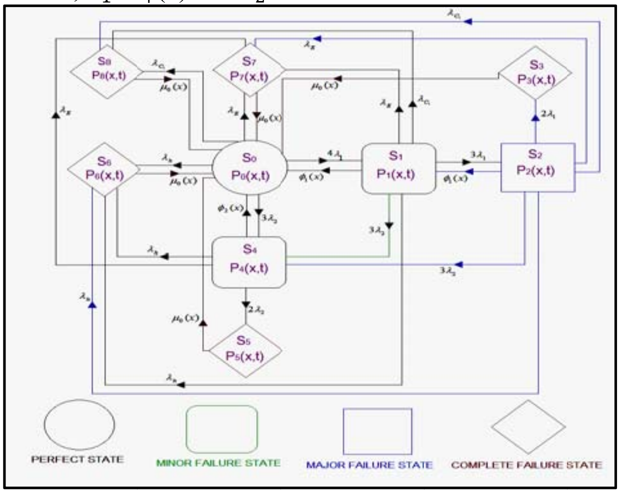

Fig.1. State transition diagram of the model

Formulation of mathematical model

By probability of considerations and continuity arguments, as state transtions accordence to a Markov model,the following set of differential equations governing the present mathematical model are obtained: (See Appendix 1).

$$\left(\frac{\partial}{\partial t} + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right) P_0(t) = \int_0^\infty \varphi_1(x) P_1(x,t)\,dx + \int_0^\infty \varphi_2(x) P_4(x,t)\,dx + \int_0^\infty \mu_0(x) P_3(x,t)\,dx + \int_0^\infty \mu_0(x) P_5(x,t)dx + \int_0^\infty \mu_0(x) P_6(x,t)dx + \int_0^\infty \mu_0(x) P_7(x,t)dx + \int_0^\infty \mu_0(x) P_8(x,t)dx \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 3\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1} + \varphi_1(x)\right) P_1(x,t) = 0 \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1} + \varphi_1(x)\right) P_2(x,t) = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) P_3(x,t) = 0 \quad (4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda_2 + \lambda_h + \lambda_E + \varphi_1(x)\right) P_4(x,t) = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) P_5(x,t) = 0 \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) P_6(x,t) = 0 \quad (7)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) P_7(x,t) = 0 \quad (8)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x)\right) P_8(x,t) = 0 \quad (9)$$

Conditions are:

$$P_1(0,t) = 4\lambda_1 P_0(t) \quad (10)$$
$$P_1(0,t) = 4\lambda_1 P_0(t) \quad (11)$$
$$P_2(0,t) = 12{\lambda_1}^2 P_0(t) \quad (12)$$
$$P_4(0,t) = 3\lambda_2(1 + 4\lambda_1 + 12{\lambda_1}^2) P_0(t) \quad (13)$$
$$P_5(0,t) = 6{\lambda_2}^2 P_0(t) \quad (14)$$
$$P_6(0,t) = \lambda_h\left(1 + 4\lambda_1 + 12{\lambda_1}^2 + 3\lambda_2\right) P_0(t) \quad (15)$$
$$P_7(0,t) = \lambda_E\left(1 + 4\lambda_1 + 12{\lambda_1}^2 + 3\lambda_2\right) P_0(t) \quad (16)$$
$$P_8(0,t) = \lambda_{C_1}(1 + 4\lambda_1 + 12{\lambda_1}^2) P_0(t) \quad (17)$$

Laplace transformations of the (Eq. 1- 17) with initial conditions $P_0(0)=1$ and $P_j(x,0)=0$ for j=1,2….8 one can get.

$$\left(s + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)\bar{P}_0(s) = 1 + \int_0^\infty \varphi_1(x)\bar{P}_1(x,s)\,dx + \int_0^\infty \varphi_2(x)\bar{P}_4(x,s)\,dx + \int_0^\infty \mu_0(x)\bar{P}_3(x,s)\,dx + \int_0^\infty \mu_0(x)\bar{P}_5(x,s)dx + \int_0^\infty \mu_0(x)\bar{P}_6(x,s)dx + \int_0^\infty \mu_0(x)\bar{P}_7(x,s)dx + \int_0^\infty \mu_0(x)\bar{P}_2(x,s)dx \quad (18)$$

$$\left(s + \frac{\partial}{\partial x} + 3\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1} + \varphi_1(x)\right)\bar{P}_1(x,s) = 0 \quad (19)$$

$$\left(s + \frac{\partial}{\partial x} + 2\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1} + \varphi_1(x)\right)\bar{P}_2(x,s) = 0 \quad (20)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_3(x,s) = 0 \quad (21)$$

$$\left(s + \frac{\partial}{\partial x} + 2\lambda_2 + \lambda_h + \lambda_E + \varphi_1(x)\right)\bar{P}_4(x,s) = 0 \quad (22)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_5(x,5) = 0 \quad (23)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_6(x,s) = 0 \quad (24)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_7(x,s) = 0 \quad (25)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_8(x,s) = 0 \quad (26)$$

$$\bar{P}_1(0,s) = 4\lambda_1\bar{P}_0(s) \quad (27)$$

$$\bar{P}_2(0,s) = 12{\lambda_1}^2\bar{P}_0(s) \quad (28)$$

$$\bar{P}_3(0,s) = 24{\lambda_1}^3\bar{P}_0(s) \quad (29)$$

$$\bar{P}_4(0,s) = 3\lambda_2(1 + 4\lambda_1 + 12{\lambda_1}^2)\bar{P}_0(s) \quad (30)$$

$$\bar{P}_5(0,s) = 6{\lambda_2}^2\bar{P}_0(s) \quad (31)$$

$$\bar{P}_6(0,s) = \lambda_h\left(1 + 4\lambda_1 + 12{\lambda_1}^2 + 3\lambda_2\right)\bar{P}_0(s) \quad (32)$$

$$\bar{P}_7(0,s) = \lambda_E\left(1 + 4\lambda_1 + 12{\lambda_1}^2 + 3\lambda_2\right)\bar{P}_0(s) \quad (33)$$

$$\bar{P}_8(0,s) = \lambda_{C_1}(1 + 4\lambda_1 + 12{\lambda_1}^2)\bar{P}_0(s) \quad (34)$$

Employing separation of variables the solutions of the differential equations from (19) - (26) can be obtained implications of the boundary conditions (27)-(34) as;

$$\bar{P}_1(x,s) = \bar{P}_1(0,s)\left\{e^{-(s+3\lambda_1+3\lambda_2+\lambda_h+\lambda_E+\lambda_{C_1})x}e^{-\int_0^x \varphi_1(x)dx}\right\}$$
$$\bar{P}_2(x,s) = \bar{P}_2(0,s)\left\{e^{-(s+2\lambda_1+3\lambda_2+\lambda_h+\lambda_E+\lambda_{C_1})x}e^{-\int_0^x \varphi_1(x)dx}\right\}$$
$$\bar{P}_3(x,s) = \bar{P}_3(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}$$
$$\bar{P}_4(x,s) = \bar{P}_4(0,s)\left\{e^{-(s_1+2\lambda_2+\lambda_{\square}+\lambda_E)x}e^{-\int_0^x \varphi_2(x)dx}\right\}$$
$$\bar{P}_5(x,s) = \bar{P}_5(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}$$
$$\bar{P}_6(x,s) = \bar{P}_6(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}$$
$$\bar{P}_7(x,s) = \bar{P}_7(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}$$
$$\bar{P}_8(x,s) = \bar{P}_8(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}$$

Using solutions $\bar{P}_1(x,s)$,$\bar{P}_2(x,s)$,$\bar{P}_3(x,s)$, $\bar{P}_4(x,s)$,$\bar{P}_5(x,s)$,$\bar{P}_6(x,s)\bar{P}_7(x,s) and \bar{P}_8(x,s)$ in equation (18) one can have

Equation (35) by $\overline{S}_{\varphi_i}(s) = \frac{\varphi_i}{s+\varphi_i}$, $i = 1,\ 2$ and $\frac{1-\overline{S}_{\varphi_i}(s)}{s} = \frac{1}{s+\varphi_i}$, $i = 1,\ 2, \overline{S}_{\mu_0}(s) = \frac{\mu_0}{s+\mu_0}$,

$$\begin{aligned}
&\left(s + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)\bar{P}_0 = 1 + \bar{P}_1(0,s)\left\{e^{-(s+3\lambda_1+3\lambda_2+\lambda_h+\lambda_E+\lambda_{C_1})x}e^{-\int_0^x \varphi_1(x)dx}\right\}dx + \\
&\int_0^\infty \varphi_2(x)\bar{P}_4(0,s)\left\{e^{-(s_1+2\lambda_2+\lambda_h+\lambda_E)x}e^{-\int_0^x \varphi_2(x)dx}\right\} + \\
&\int_0^\infty \mu_0(x)\bar{P}_3(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}dx + \\
&\int_0^\infty \mu_0(x)\bar{P}_5(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}dx + \\
&\int_0^\infty \mu_0(x)\bar{P}_6(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}dx \\
&+\int_0^\infty \mu_0(x)\bar{P}_7(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}dx + \\
&\int_0^\infty \mu_0(x)\bar{P}_8(0,s)\left\{e^{-sx}e^{-\int_0^x \mu_0(x)dx}\right\}dx
\end{aligned} \quad (35)$$

$$\begin{aligned}
&\left(s + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)\bar{P}_0(s) = 1 + \bar{P}_1(0,s)\overline{S}_{\varphi_1}(s + 3\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E) \\
&+\bar{P}_4(0,s)\overline{S}_{\varphi_2}(s + 2\lambda_2 + \lambda_h + \lambda_E) + \bar{P}_3(0,s)\overline{S}_{\mu_0}(s) + \bar{P}_5(0,s)\overline{S}_{\mu_0}(s) + \bar{P}_6(0,s)S_{\mu_0}(s) + \bar{P}_7(0,s)\overline{S}_{\mu_0}(s) + \bar{P}_8(0,s)\overline{S}_{\mu_0}(s)
\end{aligned}$$

$$\left(s + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right) - \left[\frac{4\lambda_1\varphi_1}{(s+3\lambda_1+3\lambda_2+\lambda_h+\lambda_E+\lambda_{C_1}+\varphi_1)} + \frac{3\lambda_2(1+4\lambda_1+12{\lambda_1}^2)\varphi_2}{(s+2\lambda_2+\lambda_h+\lambda_E+\varphi_2)} + \frac{24{\lambda_1}^3\mu_0}{(s+\mu_0)} + \frac{6{\lambda_2}^2\mu_0}{(s+\mu_0)} + \frac{\lambda_h(1+4\lambda_1+12{\lambda_1}^2+3\lambda_2)\mu_0}{(s+\mu_0)} + \frac{\lambda_E(1+4\lambda_1+12{\lambda_1}^2+3\lambda_2)\mu_0}{(s+\mu_0)} + \frac{\lambda_{C_1}(1+4\lambda_1+12{\lambda_1}^2)\mu_0}{(s+\mu_0)}\right]\bar{P}_0(s) = 1$$

$$D(s)\bar{P}_0(s) = 1 \Rightarrow \bar{P}_0(s) = \frac{1}{D(s)} \quad (36)$$

$$\bar{P}_1(s) = \frac{4\lambda_1}{D(s)}\left\{\frac{1 - \overline{S}_{\varphi_1}\left(s + 3\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)}{\left(s + 3\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)}\right\} \quad (37)$$

$$\bar{P}_2(s) = \frac{12{\lambda_1}^2}{D(s)}\left\{\frac{1 - \overline{S}_{\varphi_1}\left(s + 2\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)}{\left(s + 2\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)}\right\} \quad (38)$$

$$\bar{P}_3(s) = \frac{24{\lambda_1}^3}{D(s)}\left\{\frac{1 - \overline{S}_{\mu_0}(s)}{s}\right\} \quad (39)$$

$$\bar{P}_4(s) = \frac{3\lambda_2(1+4\lambda_1+12\lambda^2{}_1)}{D(s)}\left\{\frac{1-\overline{S}_{\varphi 2}(s+2\lambda_1+\lambda_h+\lambda_E+\lambda_{C_1})}{(s+2\lambda_1+\lambda_h+\lambda_E+\lambda_{C_1})}\right\} \quad (40)$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_4(s) \quad (41)$$

The Laplace transformations of the probabilities that the system is in failed state or down state can be obtain as follows:

$$\overline{P}_{Down}(s) = 1 - \overline{P}_{up}(s) \quad (42)$$

Particular cases

Availability analysis: When a regular repair is employed to the system then system reliability is arbitrated as availability:

When repair follow exponential distribution: Setting $\overline{S}_{\mu_0}(s) = \frac{\mu_0}{s+\mu_0}$, $\mu_0(x) = exp[x^{\theta+}\{log\,\varphi\,(x)\}^\theta]^{\frac{1}{\theta}}$, $\overline{S}_{\varphi_i}(s) = \frac{\varphi_i}{s+\varphi_i}$, $i = 1,\ 2$ in equation (41) and setting the different values of the parameters as:

(a). λ_1 =0.02, λ_2=0.015, λ_h=0.01, λ_E=0.015,λ_{C_1}=0.03, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

(b). λ_1=0.02, λ_2 =0.015, λ_h =0.01, λ_E=0.015,λ_{C_1}=0.03, $\varphi = 1,\ \theta = 0, x = 1,\ z = 1$

(c). λ_1=0.02, λ_2 =0.015, λ_h =0.01, λ_E=0, λ_{C_1}=0, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

(d). λ_1=0.025, λ_2 =0.04, λ_h=0.03, λ_E=0, λ_{C_1}=0.03, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

On taking inverse Laplace transform, we obtain the following expressions

a.$P_{up}(t)$= $-0.10840986e^{-1.0850000t}+0.024153019e^{-2.7866372t}-0.02095566e^{-1.2461829t}+0.11394594e^{-1.0794339t}+0.99233436e^{-0.001045778t}-0.001067804e^{-1.1400000t}$

b.$P_{up}(t)$= $0.12095378e^{-1.0850000t}+0.007054546e^{-1.281297}-0.09801777e^{-1.0907525t}+0.01315904e^{-1.021941t}+0.95807131e^{(-0.0010097t}-0.001220903e^{-1.140000t}$ (43)

c.$P_{up}(t)$=$-0.000704405e^{-1.242652125t}+0.00478928e^{-1.078289t)}+0.01056409e^{-1.011209t}+0.9866133e^{-0.00285197t}-0.00126232e^{-1.11000t}$

d. $P_{up}(t)$=$0.15170119e^{-1.070000t}+0.0015534e^{-1.260220t}-0.1311845e^{-1.0744506t}+0.00798227e^{1.013981t}+0.97114397e^{-0.0013483t}-0.001196234e^{-1.125000t}$

For, t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.... , we obtain different values of Pup (t) as shown in Table 1.

Table 1. The values of Availability concerning time.

Time t	Availability (a)	Availability (b)	Availability (c)	Availability (d)
0	1.000	1.000	1.000	1.000
10	0.982	0.948	0.959	0.958
20	0.971	0.939	0.932	0.945
30	0.961	0.929	0.906	0.933
40	0.951	0.920	0.880	0.920
50	0.941	0.911	0.856	0.908
60	0.932	0.902	0.831	0.896
70	0.922	0.893	0.808	0.884
80	0.913	0.884	0.785	0.872
90	0.903	0.875	0.763	0.860

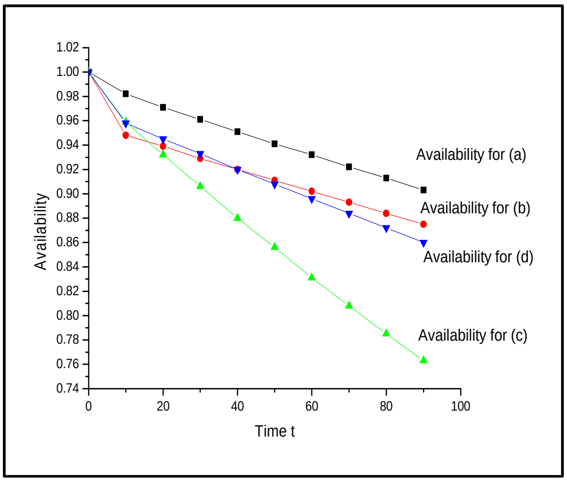

Fig. 2. Time v/s Availability

Reliability analysis: Taking all repair rates equal to zero in (42) and then taking inverse Laplace transform, we get an expression for the reliability of the system after taking the failure rates as:

λ_1 =0.02, λ_2 =0.015, λ_h=0.01, λ_E=0.015, λ_{C_1}=0.03 $\varphi = 0$ and now consider the same cases like availability, we have Reliability of entire system.

(a). R(t)= 4.0 $e^{-.160000t}$+0.5138526$e^{-0.085000t}$+0.1200$e^{-.140000t}$ -3.63385$e^{-.180000t}$

(b). R(t)= 0.5138526$e^{-0.055000t)}$-3.633853$e^{-.150000t)}$ +4.0$e^{-.130000t)}$+0.120000$e^{-.110000t}$ (44)

(c). R(t)= 0.513853$e^{-0.070000t}$+0.12000$e^{-0.125000t}$+4.0$e^{-0.145000t}$ -3.633853$e^{0-.165000t}$

For different values of time variable $t = 0,10,20,30,40,50,60,70,80,90$ and 100 units of time, in (44) one may get different values of reliability $R(t)$with the help of (44) as shown in table-2 and the corresponding figure-3.

Table 2. Variation of Reliability respect to time t

Time t	Reliability (a)	Reliability (b)	Reliability (c)
0	1.000	1.000	1.000
10	0.456	0.616	0.530
20	0.165	0.301	0.223
30	0.058	0.144	0.092
40	0.022	0.072	0.039
50	0.008	0.037	0.018
60	0.003	0.020	0.008
70	0.001	0.011	0.004
80	0.001	0.006	0.002
90	0.000	0.003	0.001

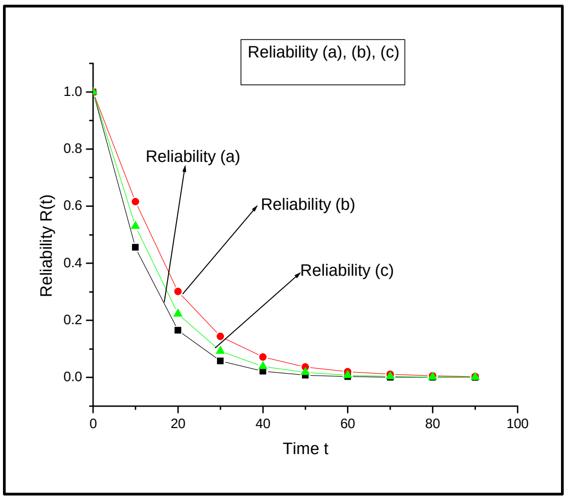

Fig.3. Variation of Reliability respect to time t

Mean time to system failure (MTTF) analysis

Mean time between failures (MTBF) is the predicted elapsed time between inherent system, during normal system operation. MTBF can be calculated as the arithmetic mean (average) time between failures of a system. The term is used for repairable systems, while mean time to failure (MTTF) denotes the expected time to failure for a non-repairable system. If R(t) is reliability function obtained by taking inverse Laplace transform of $P_{up}(s)$ than average time to system failure for a continuous valued function

MTTF =E(t)=$\int_0^{\infty} R(t)dt = \lim_{s\to 0} R(s)$

Setting, and taking all repairs to zero in equation (41). Taking limit,$\lim_{s\to 0} R(s)$ one can obtain the MTTF as:

$$MTTF = \frac{\left\{1+\frac{4\lambda_1}{3\lambda_1+\lambda} + \frac{12\lambda_1{}^2}{2\lambda_1+\lambda} + \frac{3\lambda_2\mu}{2\lambda_1+\lambda_E+\lambda_{C_1}}\right\}}{4\lambda_1+\lambda} \quad (45)$$

Where,

$\lambda = 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}, \mu = 12\lambda_1{}^2 + 4\lambda_1 + 1$

Setting, λ_2 =0.015, λ_h=0.01, λ_E=0.015, λ_{C_1}=0.03 and varying λ_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (45) one may obtain Table 3 whose column 2 demonstrates variation of MTTF with respect to λ_1.

Setting λ_1 =0.02, λ_h=0.01, λ_E=0.015, λ_{C_1}=0.03 and varying λ_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (45) we obtain Table 3, whose column 3 demonstrates variation of MTTF with respect to λ_2.

Setting λ_1 =0.02, λ_2 =0.015, λ_E=0.015, λ_{C_1}=0.03 and varying λ_h as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07,0.08, 0.09 in (45) one may obtain Table 3, whose column 4 shows variation of MTTF with respect to λ_h.

Setting λ_1 =0.02, λ_2 =0.015, λ_h=0.01, λ_{C_1}=0.03 and varying λ_E as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (45) one may obtain Table 3, which reveals variation of MTTF with respect to λ_E in column 5.

Setting λ_1 =0.02, λ_2 =0.015, λ_h=0.01, λ_E=0.03 and varying λ_{C_1} as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (45) one may obtain Table 3, which reveals variation of MTTF with respect to λ_{C_1} in column 6.

Table 3. The values of MTTF on failure rates.

Failure Rate	MTTF λ_1	MTTF λ_2	MTTF λ_3	MTTF λ_E	MTTF λ_{C_1}
0.01	14.56	11.96	11.71	12.35	14.77
0.02	11.71	11.56	10.93	11.14	13.07
0.03	9.93	11.40	10.24	10.14	11.71
0.04	8.69	11.35	9.64	9.29	10.61
0.05	7.77	11.35	9.10	8.58	9.70
0.06	7.071	11.38	8.61	7.96	8.92
0.07	6.51	11.43	8.18	7.43	8.26
0.08	6.06	11.47	7.78	6.96	7.69
0.09	5.68	11.52	7.42	6.54	7.19
0.10	5.36	11.57	7.09	6.17	6.74

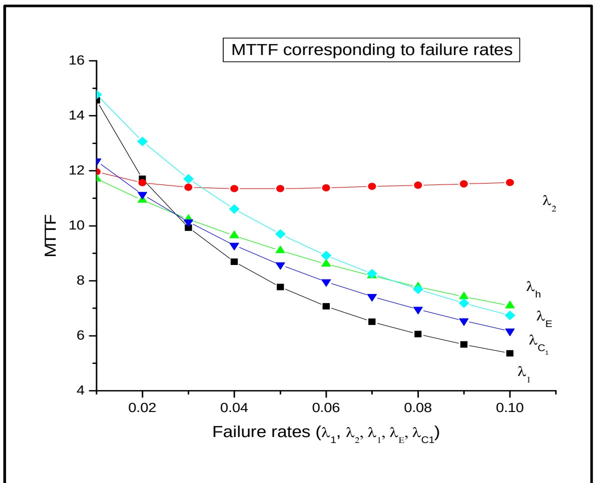

Fig.4. Variation of MTTF on Failure Rate.

Profit analysis

When the system is in operational mode and the manufacture is being done and let the profit of per unit item is $K_1$and the production cost per unit item is K_2 in the interval [0, t) than the net profit for the system at any time in interval [0, t) can be obtain by equation (46) for given values of failure rates.

(a) Let the failure rates of the system be $\lambda_1 = 0.02,\ \lambda_2 = 0.015,\ \lambda_h = 0.01,\ \lambda_E = 0.015,\ \lambda_{C_1} = 0.03$, mean time to repair of be $\varphi_1 = 1, \varphi_2 = 1,\ x = 1,\ \theta = 1$, and Setting,

$$\overline{S}_{\mu_0}(s) = \frac{exp[x^\theta+\{log\,\varphi(x)\}^\theta]^{\frac{1}{\theta}}}{s+exp[x^\theta+\{log\,\varphi(x)\}^\theta]^{\frac{1}{\theta}}},\ \overline{S}_{\varphi_i}(s) = \frac{\varphi_i}{s+\varphi_i},$$

$i = 1,\ 2,$ in equation (42) and taking inverse Laplace transform, we obtain (42).

Let the service facility is available all the time, then the expected profit during the interval [0, t) is

$$E_p(t) = K_1 \int_0^t P_{up}(t)\,dt - K_2 t \quad (46)$$

Where K_1 and K_2 are revenue and service, cost per unit time in the range [0, t). Hence the case (a) and (b) presents the expected profit of operation of the system when repair follow Gumbel- Hougaard family copula and general repair.

(a). $Ep(t) =K_1(-0.120953787e^{-1.085000t}+0.00705455e^{-1.281297t}-0.09801778e^{-1.09075t}+0.013159036e^{-1.021941t}+0.95807131e^{(-0.0010097t}-0.0012209e^{-1.1400000t}t+0.099916924e^{-1.085000t}-0.008667443e^{-2.78663}+0.016815883e^{-1.246183t}-.10556083e^{-1.079434t}+948.892) - K_2t$ (47)

(b). $Ep(t)= K_1(-0.111478e^{-1.08500t)}-0.0055057e^{-1.281299tt}+0.089862e^{-1.0907525t}-0.01287e^{-1.0219408t}-948.8533e^{-0.0010097\,t}+0.00107e^{-1.140000t}+948.892)-K_2(t)$ (48)

Setting K_1= 1and K_2= 0.60, 0.50, 0.40, 0.30, 0.20, 0.10 respectively in table (47), (48) and varying t =0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100… one gets Tables. (4a & 4b) which shows the expected profit on operation of system when the system follows copula repair and general repair. Conclusively the copula repair is more beneficial and profitable over general repair.

Table 4(a). Expected profit in interval [0, t) when repair follow Gumbel Hougaard family copula

Time (t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)
	K2=0.6	K2=0.5	K2=0.4	K2=0.3	K2=0.20	K2=0.10
0	0	0	0	0	0	0
10	3.87	4.87	5.87	6.87	7.87	8.87
20	7.64	9.64	11.64	13.64	15.64	17.64
30	11.30	14.30	17.30	20.30	23.30	26.30
40	14.87	18.87	22.87	26.87	30.87	34.87
50	18.34	23.34	28.34	33.34	38.34	43.34
60	21.71	27.71	33.71	39.71	45.71	51.71

Time (t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)
	K2=0.6	K2=0.5	K2=0.4	K2=0.3	K2=0.20	K2=0.10
70	24.98	31.98	38.98	45.98	52.98	59.98
80	28.15	36.15	44.15	52.15	60.15	68.15
90	31.23	40.23	49.23	58.23	67.23	76.23
100	34.22	44.22	54.22	64.22	74.22	84.22

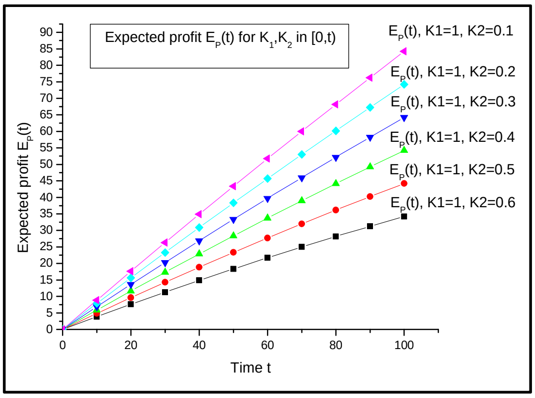

Fig.5(a). Expected profit in interval [0, t) when repair follow Gumbel Hougaard family copula

Table 4(b). Expected profit in interval [0, t) when repair follow general distribution

Time t	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)	Ep(t)
	K2=0.6	K2=0.5	K2=0.4	K2=0.3	K2=0.20	K2=0.10
0	0	0	0	0	0	0
10	3.57	4.57	5.57	6.57	7.57	8.57
20	7.01	9.01	11.01	13.01	15.01	17.01
30	10.35	13.35	16.35	19.35	22.35	25.35
40	13.60	17.60	21.60	25.60	29.60	33.60
50	16.75	21.75	26.75	31.75	36.75	41.75
60	19.82	25.82	31.82	37.82	43.82	49.82
70	22.80	29.80	36.80	43.80	50.80	57.80
80	25.67	33.67	41.67	49.67	57.67	65.67
90	28.46	37.46	46.46	55.46	64.46	73.46
100	31.17	41.17	51.17	61.17	71.17	81.17

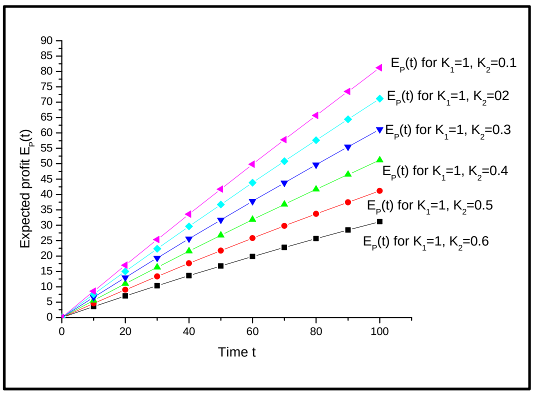

Fig.5(b). Expected profit in interval [0, t) when general distribution

Result and conclusion

The availability of the complex repairable system changesconcerning thetime when the failure rates are kept fixed at different values (see Table 1 and Fig. 1). When the failure rates are set at lower valuesas

(a)λ_1 =0.02,λ_2=0.015, λ_h=0.01, λ_E=0.015, λ_{C_1}=0.03, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

(b).λ_1=0.02λ_2=0.015, λ_h=0.01, λ_E=0.015, λ_{C_1}=0.03, $\varphi = 1,\ \theta = 0$

(c).λ_1 =0.02, λ_2 =0.015, λh =0.01, λE=0, λ_{C_1}=0, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

(d).λ_1=0.025, λ_2=0.04, λ_h =0.03, λ_E=0, λ_{C_1}=0.03, $\varphi = 1,\ \theta = 1,\ x = 1,\ z = 1$

The availability of the system decreases and the probability of failure increase, with the passage of the time and ultimately becomes steady to the value zero after a sufficiently long time. Hence, one can safely predict the future behavior of the complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the mathematical model.

Table 2 and the figure 2 presents reliability of the system when all repair are assume to zero. The table 1 and table 2 clearly explain when repair is employed the system performance is better than non repairable system. The yield the mean-time-to-failure (MTTF) of the system on variation in failure rates λ_1 =0.02, λ_2 =0.015, λ_h=0.01,

λ_E=0.015, λ_{C_1}=0.03, respectively when the other parameters are kept constant (see Table 3). The change in the values of MTTF corresponding to failure rates shown in Table 3 and corresponding graphs in Fig. 4.

If the revenue cost per unit time K1fixed at 1, service cost K_2 = 0.50, 0.40, 0.30, 0.20, 0.10, profit has been calculated and results obtained demonstrated in Fig. 5. One can easily conclude that as the service cost decreases profit increases.

Researchers can further discuss the comparative study of copula for the particular system. This system can be used to analyze by the help of other types of copula like, Archimedean copula Carleton copula and Franklin copula etc.

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Appendix 1

By probability consideration and arguments, the following difference-differential equation will be associated with the present mathematical model.

The state transition probability that the system which in the state S_0 will be remain in the state S_0 during the time [t, t+Δt] if it will be not move to any other state and if it in failed state then after repaired it be approaches to S_0 state. If failure rate to move the state

S_1, S_3, S_4, S_6, S_7, S_8 during time [t, t+Δt] is $4\lambda_1\Delta t$, $3\lambda_2 \Delta t$, $\lambda_h\Delta t$, $\lambda_E\Delta t$, $\lambda_{C1}\Delta t$, then the rate not to move to the state will be $(1-4\lambda_1\Delta t)$, $(1-3\lambda_2\Delta t)$,$(1-\lambda_h\Delta t)$, $(1-\lambda_E\Delta t)$. $(1-\lambda_{C1}\Delta t)$ the system will be in state S_0 during the time t and [t+ Δt] and if it is in another failed state than after repair it must come to state S_0 is given as;

$$P_0(t+\Delta t) = (1-4\lambda_1\Delta t)(1-3\lambda_2\Delta t)(1-\lambda_h\Delta t)(1-\lambda_E\Delta t)\left(1-\lambda_{C_1}\Delta t\right)P_0(t) + \int_0^\infty \emptyset_1(x)P_1(x,t)dx\Delta t + \int_0^\infty \emptyset_2(x)P_4(x,t)dy\Delta t + \int_0^\infty \mu_0(x)P_3(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_5(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_6(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_7(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_8(x,t)dx\Delta t.$$

$$P_0(t+\Delta t) = [1-(4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1})(\Delta t) + (Pr\,o\,duct\ of\ two\ terms)(\Delta t)^2 + \cdots . + ...]P_0(t) + + \int_0^\infty \emptyset_1(x)P_1(x,t)dx\Delta t + \int_0^\infty \emptyset_2(x)P_4(x,t)dy\Delta t + \int_0^\infty \mu_0(x)P_3(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_5(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_6(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_7(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_8(x,t)dx\Delta t.$$

$$lim_{\Delta t\to 0}\frac{P_0(t+\Delta t) - P_1(x,t)}{\Delta t} + \left(4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)P_0(t) = \int_0^\infty \emptyset_1(x)P_1(x,t)dx\Delta t + \int_0^\infty \emptyset_2(x)P_4(x,t)dy\Delta t + \int_0^\infty \mu_0(x)P_3(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_5(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_6(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_7(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_8(x,t)dx\Delta t.$$

$$\left(\frac{\partial}{\partial t} + 4\lambda_1 + 3\lambda_2 + \lambda_h + \lambda_E + \lambda_{C_1}\right)P_0(t) = \int_0^\infty \emptyset_1(x)P_1(x,t)dx\Delta t + \int_0^\infty \emptyset_2(x)P_4(x,t)dy\Delta t + \int_0^\infty \mu_0(x)P_3(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_5(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_6(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_7(x,t)dx\Delta t + \int_0^\infty \mu_0(x)P_8(x,t)dx\Delta t \qquad \ldots(1)$$