International Journal of Reliability, Risk and Safety:
Theory and Application / Vol. 1, No. 1, 2018
www.IJRRS.com

A Study on the Multi-state (r, s)-out-of- n Systems with Dependent Components

A. Gheisari[1*] and S. Goli[2]

1, 2.*Department of Mathematical Sciences, Isfahan University of Technology, Isfahan, Iran*

Abstract

In the study of technical systems in reliability engineering, multi-state systems play a useful role. A multi-state system is a system consisting n components that system and its components may have several level performance. In the present paper, we introduce the multi-state (r, s)-out-of-n system consisting n elements having the property that each element consists two dependent components and each component of the elements and the system can be in one of m+1 possible states: 0, 1, 2,..., m. We investigate an efficient method to compute the exact reliability by using the distribution of bivariate order statistics. Depending on the number of active components of the multi-state (r, s)-out-of-n systems at time t, the mean residual lifetime function of the system is studied. Also, an example and illustrative graph is provided.

Keywords: Multi-state (r, s)-out-of-n system; Bivariate order statistics; Reliability function; Mean residual lifetime;

Nomenclature and Units

Notation to be used in this paper listed as follows:

A_i	State component 1th element *i*th
B_i	State component 2th element *i* th
(A_i, B_i)	State element *i* th
n	Number of element
m	Perfect state
F(.)	Distridution function
F(x,y)	Joint distridution function
FGM	Farlie-Gambel-Morgenstern
$M_{1,1}(.)$	Mean residual lite conditional in all component
$M_{r,r}(.)$	Mean residual lite conditional
j	Number of state system & component
r_j	Number of component *1*th in state j or above
s_j	Number of component *2*th in state j or above
T	Lifetime of system
$T^{\geq j}$	Lifetime of multi-state k-out-of-n System in the state j or above
$T_i^{\geq j}$	Lifetime of element *i* th in the state j or above
T_i^{j}	Lifetime of element *i* th in the state j
$T_i^{1\geq j}$	Lifetime of component *1*th element *i* th in the state j or above
$T_i^{2\geq j}$	Lifetime component *2*th element *i* th in the state j or above
$T_{r:n}^{\geq j}$	R^{th} order among $T_1^{\geq j}, \dots, T_n^{\geq j}$
$R_T(t)$	Reliability function

1. Corresponding author. Tel.: +98313523275
2. E-mail address: a.gheisary@math.iut.ac.ir.

1. Introduction

Recently, interest has grown in the study of multi-state systems because some practical systems and components have several different performance levels. In addition, each component may have more than two different failure modes which each mode has a different effect on the system level performance. Thus, such systems should be investigated as multi-state systems. The multi-state system is a system with the property that the system and each component have j, (j = 0, 1, …, m) possible states, where 0 is the complete failure state and m is the perfect functioning state. In recent years, there has been some interest in the multi-state k-out-of-n systems and its properties. The system is said to have multi-state k-out-of-n structure if the system is in state j or above if there exists an integer value l (j < l < M) such that at least $k_l$components are in state l or above. Multi-state k-out-of-n systems have many applications for describing engineering systems such as oil supply system, manufacturing system and lighting system. Many authors have widely studied these systems under different conditions. Barlow and Wu (1978) discussed the coherent systems with multi-state components by using the concepts of path and cut sets from multi-state case. Yingkuiand Jing (2012); Ramirez-Marquez and David (2007) and Farsi and Najafi (2015) studied on the exact evaluation of the reliability in multi-state systems based on fault tree. Nadjafiet.al. (2017), used the concept of fault tree analysis and fuzzy failure rates to study the reliability analysis of a multi-state system and calculated the Top Event time to failure.

El-Neweihi et. al. (1978) introduced the first definition of multi-state k-out-of-n systems. According to this definition, a system has a performance level in state j or above if at least k components are in state j or above.

A generalized multi-state k-out-of-n: G system defined by Huang et. al. (2000) which in this system k can have different values for different system state levels. Zuo and Tian (2006) obtained algorithms for reliability evaluation of such system.

Afterwards, Tian et al (2009) studied a new multi-state k-out-of-n system, and made a algorithm for its reliability evaluation. Kolowrocki and Soszyska (2006), Kolowrocki and Kwiatus zewska-Sarnecka (2008), and Eryilmaz (2010) considered a multi-state k-out-of-n system with M + 1 possible states, then the lifetime of the system in state j or above is actually the time spent by the system with a minimum performance level of jforj= 1, …, M and investigated the time spent by the system in specific state subsets or specific states. Eryilmaz (2013) investigates the lifetimes of multi-state systems. In particular, the lifetimes of two different multi-state k-out-of-n system models are represented in terms of order statistics, and bounds and approximations are presented using these representations. The results are illustrated for a multi-state system whose components' degradation occurs according to a Markov process. Tian et al. (2009) proposed a new multi-state k-out-of-n model, Zhao and Cui (2010), and Chatuverdi et al (2012) developed recursive algorithm for its reliability evaluation for such systems. Several authors have extensively discussed the multi-state k-out-of-n systems and its properties. See for example Proschanet al (1978)**,** Eryilmaz (2013), Zuoet al (2011) and the references therein.

The mean residual life function (MRL) is a useful measure to the maintenance policies. Suppose T be the lifetime of a system with distribution function F(t) and the system is still alive at time t. The MRL system represents the expected value of the remaining lifetime of the system given that the system is functioning at time *t* and is defined as

$M(t)=E(T-t|T\geq t);$

for all t provided that $F(t)>0$.

Recently, the mean residual life function of the multi-state system depending on the number of active components at time *t* have been studied by some authors. See for example Bairamovet al. (2002), and Eryilmaz (2010) defined and studied mean residual and mean past lifetime functions for multi-state systems.

In many practical systems, the systems consist independent element that each element has two or more dependent components. In addition, this system and its components may have several level performance, therefore, such systems that are called the multi-state (r, s)-out-of-n system should be studied as multi-state systems. The aim of the present paper is to concentrate on the reliability functions of such systems. The rest of the paper is organized as follows. In Section 2, we introduce the multi-state (r,s)-out-of-n system and prepare the assumptions and notations that will be needed throughout the paper. The reliability function of the system obtained depends on the distributions of bivariate order statistics in Section 3. Section 4 is an investigation on the mean residual lifetime of multi-state (r,s)-out-of-n systems under different scenarios. First, we obtain the MRL of the multi-state (r,s)-out-of-n system under the assumption that all of the components are in state j or above at time t. Then the mean residual lifetime of a multi-state (r, s)-out-of-n system with the property that it operates as long as at least (n - r + 1), (r < n - $\mathbf{s_j}$ + 1), components operate in state j or above at time *t* is obtained. Also an example is provided in Section 5.

2. Description of the multi-state (r, s)-out-of-*n* system

Consider a system consisting independent elements that each element has two dependent components (A_i, B_i),i= 1, 2 ,…, n and thesystemandeach component have j, (j = 0, 1, …, m) possible states, where 0 is the complete failure state and *m* is the perfect functioning state. A multi-state (r, s)-out-of-*n* structure is said to be in state j, (j = 0, 1,…, m) or above if at least *r* of first components A_1, …,$A_n$are instate l, (j ≤ l ≤m), or above and at least so second components B_1,…, $B_n$are in statel, (j ≤ l ≤m), or above.

3. The Reliability function of the system

In this section we obtain the reliability function of a multi-state(r,s)-out-of-n system in terms of the reliability functions of ordered lifetimes of its components. Consider a multi-state (r,s)-out-of-n system consists of n elements and the lifetimes of the each element are denoted by ($\mathbf{T_i^1}$, $\mathbf{T_i^2}$),i = 1, 2 , …, n that $\mathbf{T_i^1}$ and $\mathbf{T_i^2}$ represent the lifetime of the component $\mathbf{A_i}$and $\mathbf{B_i}$ (i = 1, 2 ,…, n) which are distributed according to a common continuous distribution *F*. It is well known that $\mathbf{T_i^1}$ and $\mathbf{T_i^2}$ are dependent random variables with joint cdf $F(\boldsymbol{T^1},\boldsymbol{T^2})$, that can be use FGM copula for joint distribution function of dependent random variables with bottom for me,

$$F(\boldsymbol{T^1},\boldsymbol{T^2})= \boldsymbol{F_{T^1}(t^1)F_{T^2}(t^2)}\{1+\boldsymbol{\alpha}(1-\boldsymbol{F_{T^1}(t^1)})(1-\boldsymbol{F_{T^2}(t^2)})\},$$

where is $\propto\in[\mathbf{-1},\mathbf{1}]$.

Let Tdenote the lifetime of the system then we have

$$T_i^{\geq j} = min\left(T_i^{1^{\geq j}}, T_i^{2^{\geq j}}\right) \tag{1}$$

$$= T_i^j + T_i^{j+1} + \cdots + T_i^m, \text{ and}$$

$$T_{r:n}^{\geq j} = min\left(T_{r:n}^{1\ \geq j}, T_{r:n}^{1\ \geq j}\right). \tag{2}$$

Hence

$$T = T^{\geq j} = max(T^{\geq j}_{n-(r_j,s_j)+1:n}, \dots, T^{\geq j}_{n-(r_m,s_m)+1:n}) = max(min) (T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}), \dots, min(T^{1}_{n-r_m+1:n}{}^{\geq j}, T^{2}_{n-s_m+1:n}{}^{\geq j})). \quad (3)$$

$$T = T^{\geq j} = max(T^{\geq j}_{n-(r_j,s_j)+1:n}, \dots, T^{\geq j}_{n-(r_m,s_m)+1:n})$$

$$max(min(T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}), \dots, min(T^{1}_{n-r_m+1:n}{}^{\geq j}, T^{2}_{n-s_m+1:n}{}^{\geq j})).$$

Remark 1 Suppose $(r_j, s_j) \leq (r_{j+1}, s_{j+1}) \leq \dots \leq (r_m, s_m)$ and $r_j \leq r_{j+1} \leq \dots \leq r_m$, $s_j \leq s_{j+1} \leq \dots \leq s_m$, $j = 1, \dots, m$ then the life time of a multi-state (R, S)-out-of-n system

$$T = T^{\geq j} = min\left(T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}\right), \quad (4)$$

that R, S is the vectors of r_j and s_j, respectively j=0,1,…,m, and $r_j, s_j \in \{0,1,\dots,n\}$.

It can be obtained that

$$R_T(t) = P\left(min\left(T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}\right) > t\right)$$
$$= \bar{F}_{T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}}(t,t)$$
$$= 1 - F_{T^{1}_{n-r_j+1:n}{}^{\geq j}}(t) - F_{T^{2}_{n-s_j+1:n}{}^{\geq j}}(t) + F_{T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}}(t,t) = \sum_{i=n-r_j+1}^{n} \sum_{i=n-s_j+1}^{n} \sum_{r=a}^{b} \pi_{11}^{r} \pi_{12}^{i-r} \pi_{21}^{s-r} \pi_{22}^{n-i-s+r} \quad (5)$$

Where

$a = max(0, i + s_j - n)$, $b = min(i, s_j)$,

$c = \binom{n}{r_j, i-r_j, n-i, n-i-s_j+r_j}$ and

$\pi_{11} = \bar{F}(t,t)$,

$\pi_{12} = \bar{F}_{T^1}(t) - \bar{F}(t,t)$,

$\pi_{21} = \bar{F}_{T^2}(t) - \bar{F}(t,t)$,

$\pi_{22} = F(t,t)$.

Where

$\bar{F}_{T^1}(t) = 1 - F_{T^1}(t)$,

$\bar{F}_{T^2}(t) = 1 - F_{T^2}(t)$,

$\bar{F}(t,t) = P(T^1 > t, T^2 > t)$.

Remark 2 If $(r_j, s_j) \leq (r_{j+1}, s_{j+1}) \leq \dots \leq (r_m, s_m)$ and $r_j \leq r_{j+1} \leq \dots \leq r_m$, $s_j \leq s_{j+1} \leq \dots \leq s_m$, $j = 1, \dots, m$. and

$$T = T^{\geq j} = max(T^{\geq j}_{n-(r_j,s_j)+1:n}, \dots, T^{\geq j}_{n-(r_m,s_m)+1:n}) = T^{\geq j}_{n-(r_j,s_j)+1:n} = min(T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}) = T^{2}_{n-s_j+1:n}{}^{\geq j}. \quad (6)$$

Then the reliability function of a multi-state (r, s)-out-of-n system is obtained as

$$R_T(t) = P(T > t) = P(T^{2}_{n-s_j+1:n}{}^{\geq j} > t) = 1 - F_{n-s_j+1:n}(t) = \sum_{r=0}^{n-s_j} \binom{n}{r} F(t)^r (1 - F(t))^{n-r}. \quad (7)$$

4. The Residual Lifetime function of the system

In this section, firstly, we derive the residual lifetime of the multi-state (r, s)-out-of-n system with lifetime *T* under the condition that at time *t* the system sin state j or above and at least n - k_j + 1elements are in state *j* or above at time *t*.

$$T_t = (T - t \mid T > t). \quad (8)$$

The reliability function (8) can be expressed as

$$P(T - t > x \mid T > t) = \frac{P(T > x+t)}{P(T > t)} = \frac{\bar{F}_{T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}}(x+t, x+t)}{\bar{F}_{T^{1}_{n-r_j+1:n}{}^{\geq j}, T^{2}_{n-s_j+1:n}{}^{\geq j}}(t,t)}. \quad (9)$$

Now consider a multi-state (r, s)-out-of-*n* system has the property that all of the components are in state j or above at time *t*, that is

$$T_{t,n} = (T - t \mid T^{1}_{1:n}{}^{\geq j} > t, T^{2}_{1:n}{}^{\geq j} > t). \quad (10)$$

The reliability function $T_{t,n}$ can be obtained as:

$$P(T_{t,n} > x) = P(T - t > x \mid T^{1}_{1:n}{}^{\geq j} > t, T^{1}_{1:n}{}^{\geq j} > t)$$
$$= 1 - \frac{1}{\bar{F}(t,t)^n} \times \sum_{i=r_j}^{n} \binom{n}{i} [\bar{F}(t,t) - \bar{F}(x+t,t)]^i \times \bar{F}(x+t,t)^{n-i} - \frac{1}{\bar{F}(t,t)^n}$$
$$\times \sum_{i=r_j}^{n} \binom{n}{i} [\bar{F}(t,t) - \bar{F}(t,x+t)]^i \times \bar{F}(t,x+t)^{n-i} + \frac{1}{\bar{F}(t,t)^n} \quad (11)$$
$$\times \sum_{i=n-r_j+1}^{n} \sum_{j=s_j-r_j+1}^{n} \sum_{k=a}^{b} c(n,k,i,j) \times q_{11}^{k} q_{12}^{i-k} q_{21}^{j-k} q_{22}^{n-i-j+k},$$

where

$q_{11} = \bar{F}(x+t, x+t) - \bar{F}(t, x+t) - \bar{F}(x+t, t) + \bar{F}(t,t)$,

$q_{12} = \bar{F}(x+t, t) - \bar{F}(x+t, x+t)$,

$q_{21} = \bar{F}(x, x+t) - \bar{F}(x+t, x+t)$,

$q_{22} = \bar{F}(x+t, x+t)$.

In the sequel, we focus on the residual lifetime of a multi-state (r, s)-out-of-n system with the property that, with probability 1, it operates as long as at least (n - r + 1), (r < n - s_j + 1), components operate instate j or above at time *t*. The residual lifetime of a such system is

$$(T - t \mid T^{1}_{r:n}{}^{\geq j} > t, T^{2}_{r:n}{}^{\geq j} > t). \quad (12)$$

It can be shown that the reliability function of the conditional random variable (12) of a multi-state (r, s)-out-of-n system is

$$P(min({T^1_{n-r_j+1:n}}^{\geq j}, {T^2_{n-s_j+1:n}}^{\geq j}) > t + x \Big| {T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big) = P\Big({T^1_{n-r_j+1:n}}^{\geq j} > x + t, {T^2_{n-s_j+1:n}}^{\geq j} > x + t \;|{T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big) = \frac{1}{P\Big({T^1_{1:n}}^{\geq j} > t, {T^2_{1:n}}^{\geq j} > t\Big)} \times P({T^1_{n-r_j+1:n}}^{\geq j} > x + t, {T^2_{n-s_j+1:n}}^{\geq j} > x + t, {T^1_{r:n}}^{\geq j} > t, \; {T^1_{r:n}}^{\geq j} > t), \quad (13)$$

where

$$P({T^1_{n-s_j+1:n}}^{\geq j} > x + t, {T^2_{n-s_j+1:n}}^{\geq j} > x + t, {T^1_{r:n}}^{\geq j}, \; {T^1_{r:n}}^{\geq j}) = \bar{F}_{{T^1_{n-r_j+1:n}}^{\geq j}, {T^2_{n-s_j+1:n}}^{\geq j}}(t,t) - P({T^1_{n-r_j+1:n}}^{\geq j} > x + t, {T^2_{n-s_j+1:n}}^{\geq j} > x + t, {T^1_{r:n}}^{\geq j}, \; {T^1_{r:n}}^{\geq j}). \quad (14)$$

This implies that the mean residual lifetime of a multi-state (r, s)-out-of-n system described above, which we denote by, can be written as

$$M_{r,r}(t) = E\Big(T - t|\; {T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big) = \int_0^\infty P(T - t > x\; |{T^1_{r:n}}^{\geq j} > \; t, {T^2_{r:n}}^{\geq j} > t)\, dx = \int_0^\infty P\Big(min\Big({T^1_{n-r_j+1:n}}^{\geq j}, {T^2_{n-s_j+1:n}}^{\geq j}\Big) > t + x \,\Big|\, {T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big)\, dx. \quad (15)$$

Remark 3 If $(r_j, s_j) \leq (r_{j+1}, s_{j+1}) \leq \ldots \leq (r_m, s_m)$ and $r_j \leq r_{j+1} \leq \ldots \leq r_m$, $s_j \leq s_{j+1} \leq \ldots \leq s_m$, $j = 1, \ldots, m$, it is easily seen that

$$M_{1,1}(t) = E\big(T - t|\; {T^1_{1:n}}^{\geq j} > t, {T^2_{1:n}}^{\geq j} > t\big) = \int_0^\infty P(T - t > x|{T^1_{1:n}}^{\geq j} > t, {T^1_{1:n}}^{\geq j} > t)dx = \int_0^\infty P({T^2_{n-s_j+1:n}}^{\geq j} > x + t| \; {T^1_{1:n}}^{\geq j} \geq t, {T^2_{1:n}}^{\geq j} > t) \quad (16)$$

$$P({T^2_{n-s_j+1:n}}^{\geq j} > x + t|{T^1_{1:n}}^{\geq j} > t, \; {T^2_{1:n}}^{\geq j} > t) = \frac{1}{P\Big({T^1_{1:n}}^{\geq j} > t, {T^2_{1:n}}^{\geq j} > t\Big)} \times P({T^2_{n-s_j+1:n}}^{\geq j} > x + t, {T^1_{1:n}}^{\geq j} > t, \; {T^1_{1:n}}^{\geq j} > t) = \sum_{i=s_j}^{n} \binom{n}{i} \big(1 - \varphi(t)\big)^{n-i} \varphi(t)^i, \quad (17)$$

and

$$M_{r,r}(t) = E\Big(T - t|\; {T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big) = \int_0^\infty P(T - t > x\; |{T^1_{r:n}}^{\geq j} > \; t, {T^2_{r:n}}^{\geq j} > t)\, dx = \int_0^\infty P({T^2_{n-s_j+1:n}}^{\geq j} > x + t|\, {T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t),$$

$$P({T^2_{n-s_j+1:n}}^{\geq j} > x + t|{T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t) = \frac{1}{P\Big({T^1_{r:n}}^{\geq j} > t, {T^2_{r:n}}^{\geq j} > t\Big)} \times P({T^2_{n-s_j+1:n}}^{\geq j} > x + \; t, {T^1_{1:n}}^{\geq j} > t, \; {T^1_{1:n}}^{\geq j} > t) = \frac{1}{\sum_{i=0}^{r-1}\binom{n}{i}\psi(t)^i} \times \sum_{s=n-r+1}^{n} \sum_{k=s_j}^{s} \frac{n!}{k!(s-k)!(n-s)!} \times \psi(t)^{n-s}(1 - \varphi(t))^{s-k} \times \varphi(t)^k, \quad (18)$$

where

$$\varphi(t) = \frac{\bar{F}(x+t)}{\bar{F}(t)}, \; \psi(t) = \frac{F(t)}{\bar{F}(t)}.$$

5. A numerical example

A factory unit producing medical equipment containing n lines which each line has two devices A_i and B_i, i=1,2,…,n, sequential. To execute the final product, it is necessary to do at least *r* numbers of working devices A_1, A_2,…, A_n in state l, (j ≤ l ≤ m), and at least *s* of working devices B_1, B_2,…, B_n in statel, (j ≤ l ≤m). Suppose Tdenotes the life time of this system consists of n elements described in Remark 1. Let (T_i^1, T_i^2),i = 1, 2,…, n denote the life times of the each element. The life span of the components follows the exponential distribution. There liability function of such *a* systems:

$$P(T > t) = P({T^2_{n:n}}^{\geq j} > t) = \sum_{r=0}^{n-1} (1 - exp(-t))^r \, exp(-t)^{n-r}$$

Table 1. The reliability function for n=50

t	P(T>t)
1	1.000000000
2	0.999304356
3	0.922187956
4	0.603179936
5	0.286831437
6	0.116700957
7	0.044590180
8	0.016636013
9	0.006151870
10	0.002267473

Table 2. The relaibility function for n=100

t	P(T>t)
1	1.000000000
2	0.999999516
3	0.993945286
4	0.842533837
5	0.491390600
6	0.219782800
7	0.087192075
8	0.032995269
9	0.012265895
10	0.004529805

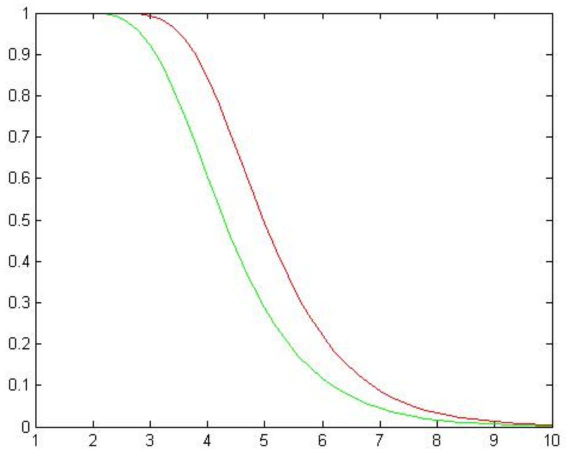

Figure 1. The relaibility function for n=50,100

Using Matlab, Figure 1 shows the graph of the reliability function of system for t = 1,…, 10 and different values of n = 50, 100 respectively. As you can see, the reliability function decreases in time and increases in n except for values less than 1 and more than 10, the two functions converge for both values n, some numerical values of the reliability function for n = 50, and n = 100 and t = 1, …, 10 are obtained in Tables 1 and 2.

Due to the lifetime distribution function of components is exponential, the $M_{1,1}(t)$ for different values of 0 < t < 10 is constant and is equal to 4.499205 and 5.187378 for n=50,100, respectively which means that it does not change with respect to t, also this value towards n is increasing.

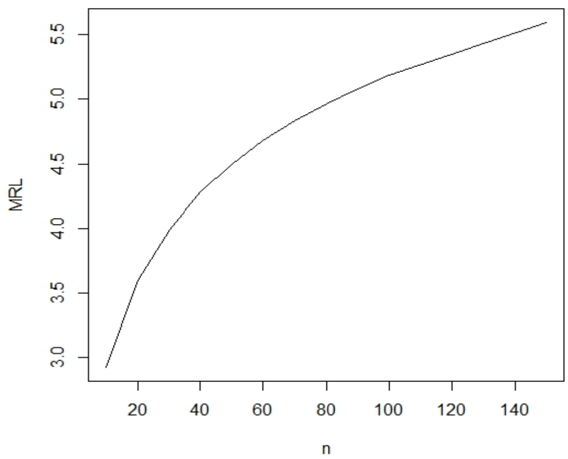

Figure 2 . The $M_{1,1}(t)$ function

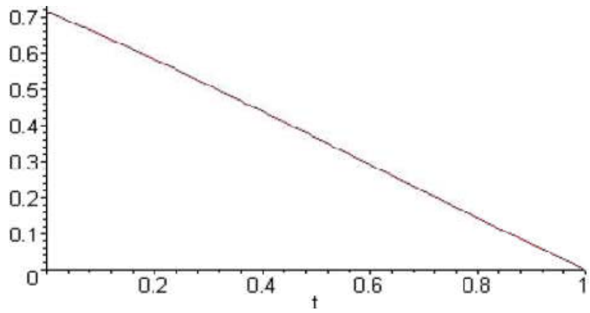

Figure 3 . Graph of $M_{1,1}(t)$, $\alpha = 0.5$, $n = 4$

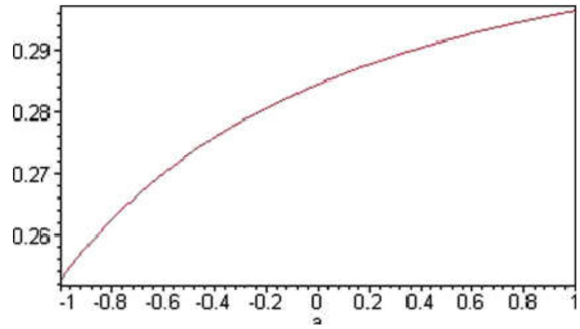

Figure 4 . Graph of$M_{1,1}(t)$, $t = 0.6, n = 4$, $\alpha \in [-1,1]$

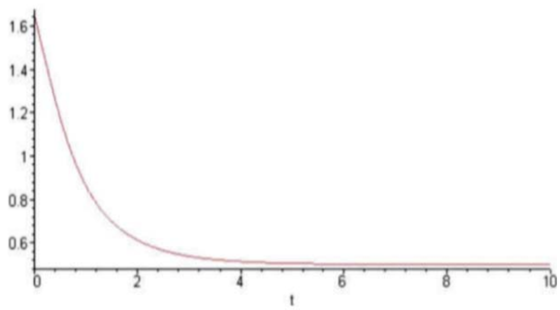

Figure 5. The model of MRL for system with n=5 element

6. Conclusions

In this paper, we have introduced the multi-state (r, s)-out-of-n system consisting n elements having the property that each element consists two dependent components and each component of the elements and the system can be in one of m+1 possible states: 0, 1, 2,..., m. Under the definition a multi-state (r, s)-out-of-*n* structure is said to be in state j, (j = 0, 1,…, m) or above if at least r of first components $\mathbf{A_1}, \ldots, \mathbf{A_n}$are in state l, (j ≤ l ≤ m), or above and at least so f second components $\mathbf{B_1}, \ldots, \mathbf{B_n}$are in statel, (j ≤ l ≤ m), or above. Based on this definition, an efficient method to obtain the exact reliability of the multi-state (r, s)-out-of-*n* system have been proposed. The mean residual life function of the multi-state (r, s)-out-of-*n* system have been obtained when all of the components are in state j or above at or at least (n - r + 1), ($r < n - s_j + 1$), components operate in state j or above at time t. Last mentioned an example to illustrate the method. Some copula models can be used to extend the numerical results but we have chosen the FGM copula for showing relationship component in this paper.

Acknowledgment

We would like to express our sincere thanks to an assistant editor for their helpful comments and suggestions which improved the presentation of the paper.

References

[1] Bairamove, I., "Reliability and mean residual life of complex system with two dependent components per element," *IEEE Transfactions on Reliability*, Vol. 62, No. 1, 2013, pp. 276-285.

[2] Bairamove, I., Ahsanullah, M., Akhundov, I., "A residual life function of a k-out-of-n structure at the system level," *IEEE Transfactions on Reliability*, Vol. 55, No. 2, 2002, pp. 119-132.

[3] Chaturvedi, S., Basha, S., Amari, S., and Zuo, M.J., "Reliability analysis of generalized multi-state k-out-of-n system," *Journal of Risk and Reliability*, Vol. 226, 2012, pp. 327-336.

[4] Farsi, M.A. and Najafi, M., "Reliability estimation of multi-statesystem based on fault tree analysis," *Modares Mech Eng,* Vol. 15, No. 1, 2015, pp. 257–266.

[5] El-Neweihi, E., Proschan, F., Sethuraman, J., "*Multistate coherent system*," *Journal of Applied Probability*, Vol. 15, 1978, pp. 675–688.

[6] Eryilmaz, S., "Mixture Representations for Three-State Systems with Three State Component," *IEEE Transaction action son Reliability*, Vol. 64, No. 2, 2015, pp. 829-834.

[7] Eryilmaz, S., "Mean r esidual and mean past lifetime of multi-state systems with identical components," *IEEE Transactions on Reliability*, Vol. 59, 2010, pp. 644–649.

[8] Erylmaz, S., "Lifetime of Multistate k-out-of-n Systems," *Quality and Reliability Engineering International*, 2014, pp. 1050-1022.

[9] Huang, J., ZuoMJ., Wu, Y., "Generalized multi- state k-out-of-n:G systems," *IEEE Transactions on Reliability*, Vol. 49, 2000, pp. 105-111.

[10] Nadjafi, M., Farsi, M.A. and Jabbari, H., "Reliability analysis of multi-stateemergency detection system usingsimulation approach based on fuzzy failurerate," *International Journal of System Assurance Engineering and Management*, Vol. 8, No. 3, 2017, pp. 532-541.

[11] Natvig, B., *Multistate systems reliability theory with applications*, Wiley, New York, 2010.

[12] Ramirez-Marquez, JE., David, WC., "Multi- state componentcriticality analysis for reliability improvement inmulti-state systems," *Reliability Engineering System Safety*, Vol. 92, No. 12, 2007, pp. 1608–1619.

[13] Tian, Z., Yam, R., Zuo, MJ., and Huang, HZ., "Reliability bounds for multi-state k-out-of-n systems," *IEEE Transactions on Reliability*, Vol. 57, 2008.

[14] Tian, Z., Zuo, MJ., Yam, R., "Multi-statek-out-of-n systems and their performance evaluation," *IEEE Transactions on Reliability*, Vol. 41, 2009, pp. 32-44,.

[15] Yingkui, G. and Jing, L., "Multi-state system reliability: anew andsystematic review," *ProcEng*, Vol. 29, 2012, pp. 531–536.

[16] Zio, E., "Reliability engineering: old problems and new Challenges," *Reliability Engineering System Safety*, Vol. 94, No. 2, 2009, pp. 125–141.

[17] Zuo, MJ., Tian, Z., "Performance evaluation for generalized multi-state k-out-of-n systems," *IEEE Transactions on Reliability,* Vol. 55, 2006, pp. 319–327.