IJRRS
International Journal of Reliability, Risk Saftey: Theory and Application

DOI : 10.30699/IJRRS.3.1.8 Vol. 3/ Issue 1/ 2020
pp. 69-76

Clustering of Condition-Based Maintenance Considering Perfect and Imperfect actions

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Abstract

Recent developments in condition monitoring technology have delivered important opportunities for condition-based maintenance. Although condition-based maintenance allows for more effectively planned maintenance actions, its relative performance depends on the behavior of the deterioration process. The objective of this paper is to develop a clustering model of maintenance activities and analyze the effect of perfect, imperfect, and hybrid maintenance on the cost. We consider a two-component system that experiences three degradation states before a complete failure. The components are equipped with a monitoring system that signals before each state change, on which our clustering is based. Actually, we have three types of clustering aiming at cost minimization. The results provide a general insight into when and how the activities are clustered and what kind of maintenance is selected such that the cost is minimized. Moreover, The results showed that clustering with a more degree of the clusters is more appropriate and produced cost savings about 70%, if the fixed cost exceeds a certain amount.

Keywords: Imperfect maintenance, Condition-based maintenance, Clustering, Condition monitoring, Prediction signal.

Introduction

Many studies have been conducted on maintenance strategies, which determine the right time for maintenance activities. While failure-based maintenance is always performed late, just after the failure, preventive maintenance (PM) strategies, such as age-based maintenance (ABM), are usually conservative and result in the planning and performing of redundant maintenance actions. In between, condition-based maintenance (CBM) can be an effective strategy in that while it postpones the maintenance actions as much as possible, it restricts the number of failures by continuously monitoring of special states such as vibration and temperature. This is while most of the related studies have focused on PM rather than CBM. However, recent advances in sensor technology have led to special attention to CBM. CBM is a PM based on condition monitoring. In most CBM models, it is assumed that the system has some operational states and one failure state. In a failure state, the system stops, and thus it is easily distinguished from the operational states. The difference between the operational states is not completely clear and the inference about the actual operating status of the system is based on condition monitoring. Optimized maintenance policies seek an optimal situation in terms of system accessibility and secure performance with minimum maintenance cost. By definition, a perfect maintenance restores the system to the as-good-as-new state. Performing a perfect maintenance seems simple but it is almost expensive and, in real conditions, the system usually does not restore to the as-good-as-new state, unless a replacement is made. Imperfect maintenance, which is considered as a real maintenance activity, restores the system to a state between as-good-as-new and as-bad-as-old. Due to the importance of an appropriate maintenance strategy in the industry, different maintenance strategies have been proposed by researchers. Recently, many studies have been conducted on combining CBM and imperfect maintenance, wherein an optimized PM action is performed based on the observed condition of the system. In most of these maintenance policies, only repair or imperfect preventive maintenance activities have been considered and it has been assumed that the system can undergo infinite imperfect preventive repair. However, this assumption may not be always feasible. Because of economic or technical considerations indifferent service or engineering plans, the system can be repaired only a limited number of times [1-4].

Besides, each imperfect maintenance may make it more prone to the next failure. Accordingly, in [5] and [6], a model with a limited number of imperfect maintenance actions was proposed; however, this number was considered as a decision variable and the effect of imperfect maintenance was not addressed. In

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Received: 2020.05.25, Accepted: 2020.11.14, Online ISSN: 2676-3346

[7], an optimization model was proposed for imperfect maintenance and non-periodic inspections in a multi-component system. In [8], an optimized replacement strategy was proposed for a multi-state system under imperfect maintenance to determine the optimal number of failures before a complete replacement that the maintenance is time-based. Although most of the studies have used perfect maintenance, in reality, due to the resource, technological, and time limitations, imperfect maintenance is used instead of complete replacement [9]. Therefore, employing imperfect maintenance, they provided a new policy for a three-state system and proposed an algorithm minimizing the cost per unit time by finding an optimal interval for performing inspections and preventive repairs. In works on imperfect maintenance, [10] and [11] have based maintenance decisions on the age of facility and statistical information, not on the real condition of the facility. To overcome this deficiency, the literature has introduced CBM. Monitoring equipment provides real information about the system's condition over time. Most of the CBM studies have been considered perfect maintenance and only a few studies have addressed imperfect maintenance. Besides, in most CBM models, periodic inspections, which are not economical due to their high costs, are performed. In [12] proposed a condition-based maintenance (CBM) policy with dynamic thresholds and multiple maintenance actions for such a system subject to periodic inspection. The hazard rate is described by the proportional hazards model with a continuous-state covariate process. At each inspection epoch, appropriate action is selected from no maintenance, imperfect maintenance, and preventive replacement based on two dynamic thresholds.

Performing maintenance activities on different units jointly are known as clustering or opportunistic maintenance. Usually, opportunistic maintenance on multi-unit systems with economic dependence leads to considerable saving [13]. An economic dependency between the units is an incentive for the combination (or clustering) of maintenance activities. In the case of economic dependence, optimizing maintenance decisions for each unit separately does not mean a desirable policy at the system level, and the maintenance policy should provide the possibility of clustering the activities. This clustering is performed seeking cost savings, not technical considerations. The simplest form of maintenance clustering is the one in which all activities are clustered. [14] formulates a novel dynamic planning framework that captures economic dependence in both preventive and opportunistic replacement. A flexible dynamic programming algorithm is developed to optimize the maintenance grouping, and the strategy framework is further extended to condition-based maintenance scenarios. In [15], they present an opportunistic maintenance scheduling methodology considering the stochastic nature of opportunities duration in a predictive maintenance strategy. The prognostic information is used to select opportunities coming before the failure. The proposed maintenance scheduling methodology is based on an optimal stopping problem algorithm known as Bruss algorithm. In [16] developed a mathematical framework to integrated the optimization of a dependence-based model with different maintenance policies and a clustering method for maintenance actions in order to decrease the total cost was proposed. [17] introduced a hybrid multi-component opportunistic maintenance policy that consist of two thresholds on the level of degradation. [18] to utilize the economic dependence between two-component, was proposed a replacement opportunistic maintenance policy. A replacement is carried out to the component of which the degradation exceeds a preventive maintenance threshold at an inspection. In [19] and [20], a combination of CBM and ABM has been considered. Under these policies, as soon as a unit needs to be repaired, a complete replacement is performed. This replacement can be done when a component reaches a degradation level beyond its specified threshold for preventive replacement or reaches a certain age. Both the preventive and age-based replacement thresholds are decision variables that should afford setup costs for different maintenance activities. Although [19] and [20] reported considerable savings in comparison to separate maintenance activities for each unit, their model was not responsive to the degree of dependence. Clustering all maintenance activities is generally optimal for a system between whose components exists a heavy dependence. Otherwise, maintenance activities are performed too much in a complete clustering, yielding higher costs. Do et al. [3] proposed a CBM policy with both perfect and imperfect maintenance actions for a degrading system. Their first objective was to evaluate the effect of imperfect maintenance actions. The second objective was to provide a maintenance policy that, at each inspection, specifies the type of maintenance, perfect or imperfect. The inspection was periodic and the time between two inspections was based on the remaining useful life of the system. Besides the periodic and costly inspections, their proposed model can be applied only to a single-unit system and provides no insight about multi-unit systems.

The present research proposes a clustering model for CBM activities of a degrading system with imperfect, perfect, and hybrid maintenance actions. In this model, the condition of the system is identified by continuous monitoring, and there is no need for inspection, which is a time consuming and costly activity. Herein, each state represents a level of degradation that due to deterioration, the system can only to higher degradation level, leading to poorer performance. Maintenance is applied to bring the system to a probably better state. At each time a system state is detected by signaling, and decision must be made based on the system’s detected condition. The decision can be either to replace the degraded system by an as-good-as-new one, to maintain it to a less degraded level, or to continue operating. By consideration the cost of maintenance a best clustering selected to optimize the policy so that the mean long-run cost rate is minimized over an infinite time horizon. Contribution of this article include: (a) we determine the clustering CBM activities of three-state system, taking imperfect maintenance and inspection into consideration; (b) we combines both inspection and continues monitoring to reduce unnecessary thorough inspection; (c) rather than analyzing complex CBM policies that rely on detailed condition information, we consider systems with just three signals that can be further extended to CBM problems; (d) unlike conventional approaches that restrict all maintenance activities into finite planning horizon, our proposal focuses on activities clustering based on predication signal without specifying the horizon; (e)In the proposed model, the effects of imperfect, perfect, and hybrid maintenance actions on the cost in the clustering are compared and selected the most appropriate type of clustering CBM activities.

General problem description

We consider a system that consists of two identical components. Each component has sensors that generate three signals before a failure. These signals are based on the deterioration level of the system. When the measured parameter exceeds a certain threshold value, the sensor gives a signal. In fact, the sensors use three threshold values for the measured parameter. At the lowest level, it produces the first signal, at the next level it raises an alert, and before the complete failure it raises an alarm. At a fixed period time D after the third warning, the maintenance is performed to prevent a complete failure. In this system, the alarm level must be selected so that the time D between an alarm and maintenance time is longer than the time required for performing a maintenance activity. We assume the deterioration process is a stochastic variable with exponential distribution with failure rates $\lambda_i (i = 1,2,3)$.The maintenance costs in our systems consist of a fixed and a variable part. We denote the fixed cost of the maintenance action by C, and the variable cost per unit on which maintenance is performed by c.Furthermore we let $R = \frac{C}{c}$ denote the relative fixed costs of performing maintenance. We consider three clustering policies and compare them with the policy without the coordination of maintenance activities, called No clustering. de Jonge (2016) used this clustering model for one and two signals and here we apply it for the case of three signals for system with perfect, imperfect and hybrid maintenance. In order to reduce the probability of failures, an imperfect preventive maintenance is applied and the clustering model assumes perfect and imperfect maintenance actions. The clustering model assumes imperfect. The maintenance action cost is thus a fraction of c, which is related to the level of deterioration reduction after a maintenance action.We consider three modes of clustering based on symptom levels, and compare them with No clustering mode. The Cost Saving Rate (CST) is used as a measure of performance in all three modes of clustering. We let $MC_N$and MC_A denote cost per unit of time in the state of without clustering and cost per unit of time in the state of without clustering. The percentage cost savings from clustering are:

$$CST = \frac{MC_N - MC_A}{MC_N} \ \% \qquad (1)$$

In our study of the system, we let λ_1,λ_2,λ_3 denote the Signal, Alert, and Alarm rates, respectively, where the maintenance activity is required after $\lambda_3$with an interval D. The clustering is based on the Signal, Alert, andAlarm warnings. The system has four states as follows: state 1, the system is as-good-as-newstate2, if the system has given Signal; state3, if the system has given Alert; state 4^1, if the system has given Alarm (but maintenance is not yet needed); and state 4^2, when maintenance is needed.

If the first component is in condition (a) and the second component is in condition (b), the general status of the system is (a, b), which is equivalent to (b, a). Figure 1 displays the system probable states, the time distribution between one state, transition to another state, and the transition probabilities. Appendix A provides the details of the probabilities calculation. Assume that the maintenance is complete, i.e. each component returns to its as-good-as-new condition after the repair.

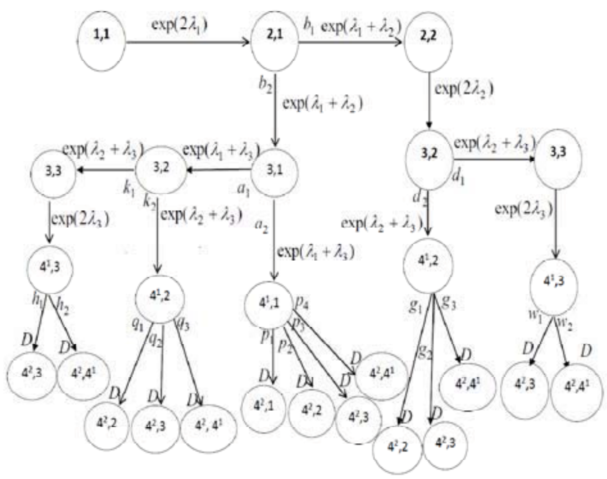


Figure 1. The state space of the system with three warning levels.

No clustering

In No clustering mode, each component is maintained separately. We have the cost of inspection and repair if a component warns and needs repair and we only have the cost of inspection if it does not require repair. If the system does not warn and needs repair, it will incur a sudden failure cost, and if it does not warn and does not require repair, there is no cost. The average time between two maintenance activities for each specified component equals $\lambda_1^{-1} + \lambda_2^{-1} + \lambda_3^{-1} + D$. So the average cost per unit of time for two units is as follows:

$$MC_N = 2 * \frac{C+c}{\lambda_1^{-1}+\lambda_2^{-1}+\lambda_3^{-1}+D} \quad (2)$$

Alarm clustering

In the alarm clustering, the cost of maintenance on a component that needs repair is equal to the sum of the fixed and variable costs. If the system reaches one of the conditions $(4^2, 4^1)$, maintenance is also performed on the second unit which in this case may or may not warn. The system reaches one of the states labeled as $(4^2, 4^1)$ with probability:

$$b_2a_2p_4 + b_2a_1k_2q_3 + b_2a_1k_1h_2 + b_1d_2g_3 + b_1d_1w_2 \quad (3)$$

With a 1-β probability, the system warns and needs repair, so we will have the cost of inspection and repair. It warns with the probability of α and we are in a situation other than $(4^2, 4^1)$ where we only have inspection costs. Also, with the probability of β, the system may not warn, but it may need repair, which may result in the cost of sudden failure. Therefore the cost of any maintenance is equal to:

$$A_{Alarm} = C + cr + \alpha c_i + (1-\beta) * (c_i + (b_2a_2p_4 + b_2a_1k_2q_3 + b_2a_1k_1h_2 + b_1d_2g_3 + b_1d_1w_2)cr + \beta(b_2a_2p_4 + b_2a_1k_2q_3 + b_2a_1k_1h_2 + b_1d_2g_3 + b_1d_1w_2)c_f(4) \quad (4)$$

If the state $(4^2, 1)$ or one of the states of $(4^2, 4^1)$ occurs, the system will reach state (1,1) after complete repair, otherwise it may return to state (1,3) or (1,2) because in situations other than $(4^2, 4^1)$ repair is only done on the first component. The probability of the system returning to state (1,1) equals:

$$b_2a_2p_1 + b_2a_2p_4 + b_2a_1k_2q_3 + b_2a_1k_1h_2 + b_1d_2g_3 + b_1d_1w_2 \quad (5)$$

With the same probability, the time between two maintenance actions involves an exponential distribution with the parameter $2\lambda_1$between state (1,1) and (1,2). In addition, the time between two repairs always involves an exponential distribution with parameter of$\lambda_1 + \lambda_2$and a constant time D. For the rest of the situations, we consider their times according to their probabilities. So the time between two maintenance actions is:

$$B_{Alarm} = \frac{1}{2\lambda_1}(b_2a_2p_1 + b_2a_2p_4 + b_2a_1k_2q_3 + b_2a_1k_1h_2 + b_1d_2g_3 + b_1d_1w_2) + \frac{1}{2\lambda_2}b_1 + \frac{1}{2\lambda_3}(b_2a_1k_1 + b_1d_1) + \frac{1}{\lambda_2+\lambda_3}(b_1 + a_1b_2) + \frac{1}{\lambda_1+\lambda_3}b_2 + \frac{1}{\lambda_1+\lambda_2} + D + h \quad (6)$$

Therefore, the average cost per unit time for Alarm clustering is:

$$MC_{Alarm} = \frac{A_{Alarm}}{B_{Alarm}} \quad (7)$$

Alert clustering

In alert clustering, the maintenance cost of the unit that needs to maintained equals to the sum of the fixed and variable costs. If one of the states $(4^2, 4^1)$ or one of the states $(4^2, 3)$ is reached, the second unit also needs maintenance. After maintenance, the system either returns to (1,1) or (2,1), because in the state of$(4^2, 2)$, only the first unit is repaired and the second unit remains in state (2, 1). So the probability that the second unit will need maintenance is equal to:

$$(1 - p_1a_2b_2 - p_2a_2b_2 - q_1k_2a_1b_2 - g_1d_2b_1) \quad (8)$$

In this case, if the system warns and needs repair, we will bear both inspection and repair costs, and if it does not require repairing, we will only have the cost of inspection. The system may not warn with the probability β but it will need to be repaired in which case if it is in one of the state $(4^2.3)$, an additional cost will be charged to the system and in the state $(4^2.4^1)$, we will have the cost of a sudden failure. Therefore, the average cost of each maintenance activity is:

$$A_{Alert} = C + cr + \alpha c_i + (1-\beta) * (c_i + (1 - b_1d_2g_1 - b_2a_2p_2 - b_2a_1k_2q_1 - b_2a_2p_1)\, cr + \beta(b_1d_2g_2 + b_1d_1w_1 + b_2a_1k_1h_1 + b_2a_1k_2q_2 + b_2a_2p_3)cr_3 + \beta\,(b_1d_2g_3 + b_1d_1w_2 + b_2a_1k_1h_2 + b_2a_1k_2q_3 + b_2a_2p_4)c_f \quad (9)$$

If the state $(4^2,1)$ or one of the states $(4^2,3)$, $(4^2,4^1)$ occurs, the system returns state (1,1) after repair, otherwise it returns to state (2,1) and the probability of returning to state (1,1) equals to:

$$(1 - b_2a_2b_2 - b_2a_1k_2q_1 - b_1d_2g_1) \quad (10)$$

With the same probability, the interval between the two repairs involves an exponential distribution with the parameter $2\lambda_1$. In addition, the time interval between the two repairs always includes an exponential distribution with parameter $\lambda_1 + \lambda_2$ and constant time D. So the average time between two repairs is:

$$B_{Alert} = \frac{1}{2\lambda_1}(1 - p_2a_2b_2 - q_1k_2a_1b_2 - g_1d_2b_1) + \frac{1}{2\lambda_2}b_1 + \frac{1}{2\lambda_3}(b_1d_1 + b_2a_1k_1) + \frac{1}{\lambda_1+\lambda_3}b_2 + \frac{1}{\lambda_2+\lambda_3}(a_1b_2 + b_1) + \frac{1}{\lambda_1+\lambda_2} + D + h \quad (11)$$

So the average cost per unit time for Alert clustering is:

$$MC_{Alert} = \frac{A_{Alert}}{B_{Alert}} \quad (12)$$

Signal clustering

The cost of maintenance of the first unit is equal to the total variable and fixed costs. If one of the states of $(4^2,3)$, $(4^2,4^1)$, $(4^2,2)$ occurs, in other words, in any state different from the state $(4^2,1)$, the second unit is also repaired. Therefore, with$(1 - b_2a_2p_1)$ probability, the system needs repair and if the system warns, we will bear the inspection and repair costs. If the second unit is in condition 3 and does not warn, an additional cost of Cr_3 is imposed on the system and if itis in condition 2 and does not warn we will have the cost of Cr_2 that is $Cr_3>Cr_2$, while in the condition of 4^1 we have the cost of a sudden failure. Therefore, the average cost of each maintenance activity is:

$$A_{\mathrm{Signal}} = C + cr + \alpha c_i + (1-\beta)(c_i + (1 - b_2a_2p_1)cr) + \beta(b_1d_2g_1 + b_2a_1k_2q_1 + b_2a_2p_2)cr_2 + \beta(b_1d_1w_1 + b_2a_1k_1h_1 + b_2a_1k_2q_2 + b_2a_2p_3)cr_3 + \beta(b_2a_1k_1h_2 + b_2a_1k_2q_3 + b_2a_2p_4 + b_1d_2g_3 + b_1d_1w_2)c_f \quad (13)$$

The interval between the two maintenance activities always includes an exponential distribution with parameter $2\lambda_1$, an exponential distribution with parameter $\lambda_1+\lambda_2$ and a constant duration D. Other exponential distributions are considered with their related probabilities. So the average time between two maintenance activities is:

$$B_{\mathrm{Signal}} = \frac{1}{2\lambda_1} + \frac{1}{2\lambda_2}b_1 + \frac{1}{2\lambda_3}(b_1d_1 + b_2a_1k_1) + \frac{1}{\lambda_1+\lambda_3}b_2 + \frac{1}{\lambda_2+\lambda_3}(b_1 + a_1b_2) + \frac{1}{\lambda_1+\lambda_2} + D + h \quad (14)$$

And the average cost per unit time clustering based on the first warning is:

$$MC_{\mathrm{Signal}} = \frac{A_{\mathrm{Signal}}}{B_{\mathrm{Signal}}} \quad (15)$$

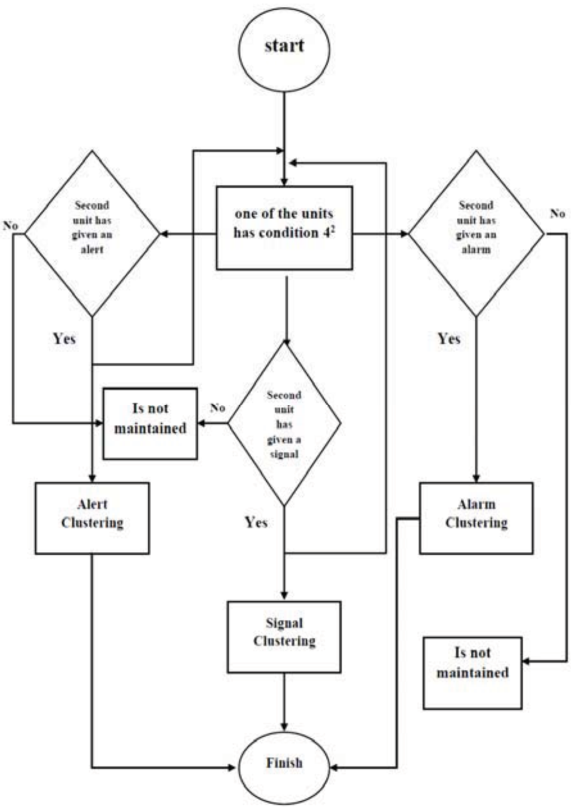


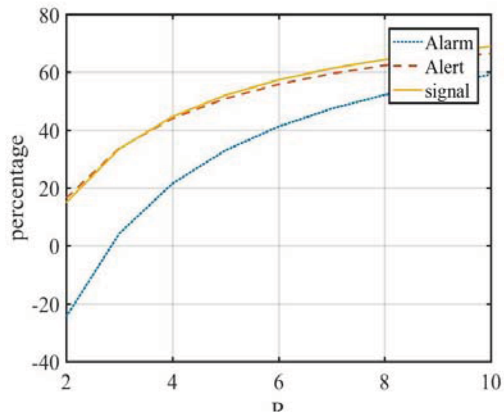
Figure 2. the procedure of clustering

Numerical study

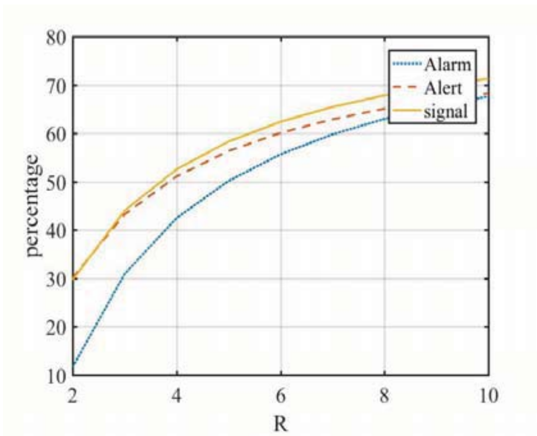
In this section we explore the relative performance of these policies. Our computations are made using Matlab. Figure 3 shows the percentage decrease in cost for the three modes of clustering compared to the No clustering mode as a function of the relative constant maintenance cost (R) for each maintenance activity and the different values of the parameters.

Figure 3(a) shows that if the fixed part of the cost is considerable, the clustering is more economical. Among the three different types of clustering, Alarm clustering has less savings, which is natural because it includes less degree of the clusters and approaches the No Clustering mode. For Alert and Signal clustering, there is a threshold for the fixed maintenance cost beyond which Signal clustering becomes more efficient. This result is also expectable because Signal clustering includes more percentage of the clusters, making natural its more savings. In Figure 3(b), the saving has risen with the increase of signal time. It can be said that as the system signals later, the maintenance action is performed later, and consequently, the costs reduce.

In the first state, we assumed that the maintenance action is perfect, i.e. the system restores the as-good-as-new state after each maintenance action. For the imperfect maintenance, we assumed that the system restores (2,2), (2,3), or (3,3) states after each maintenance action. In this case, the required time is calculated and the corresponding results are shown in Figure 4.

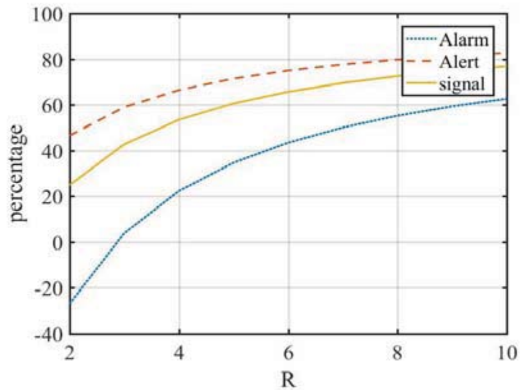


(a)

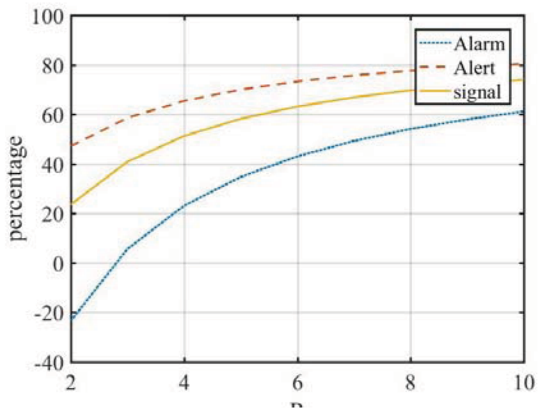


(b)

Figure 3. Percentage of reduction cost of alarm, alert, and signal clustering in comparison to No clustering in perfect maintenance.(a)$\lambda_1^{-1} = 8, \lambda_2^{-1} = 4, \lambda_3^{-1} = 3$ ، (b) $\lambda_1^{-1} = 9, \lambda_2^{-1} = 6, \lambda_3^{-1} = 5$.



(c)

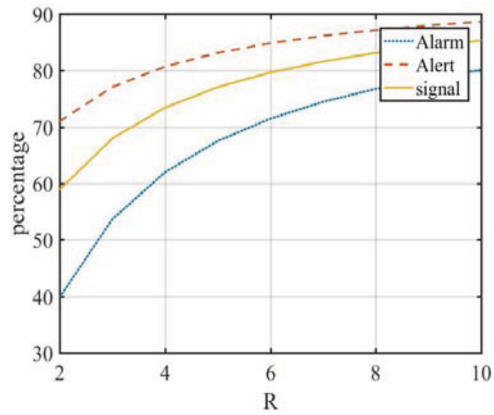


(d)

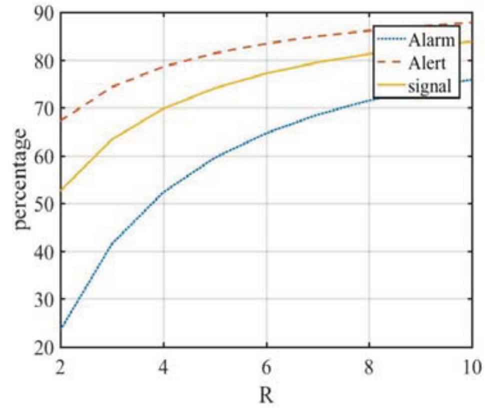
Figure 4. The percentage of cost reduction for the clustering based on Signal, Alert, Alarm clustering in comparison to No-Clustering in imperfect maintenance.

Figure 4(c) shows the cost reduction for the imperfect maintenance. In imperfect maintenance action, the Alert clustering produces more savings compared to the two other clustering. As can be seen in Figure 4(d), with the increase of average time to signal, the amount of savings change not much. The amount of savings for the imperfect maintenance is generally more than that of the perfect one, with the difference that in the perfect maintenance, the Signal clustering is preferred while in the imperfect maintenance, Alert clustering is superior to the signal clustering and Alarm clustering.

In the hybrid maintenance, we assume that one component restores to the healthy state and the other component restores to a state between the as good as new state and bad as old. Thus, if the system after a repair restores to (2,1) and (3,1) states, the hybrid maintenance has taken place. Accordingly, the related times are calculated and their corresponding chart, comparing the amount of savings for this case with those of the perfect and imperfect maintenance actions, is plotted. Figure 4 shows the percentage of cost reduction for this type of maintenance.



(e)



(f)

Figure 5. The percentage of cost reduction on Signal, Alert and Alarm clustering in comparison to No-Clustering in hybrid maintenance.

As can be seen in Figure 5(e), the amount of savings for the hybrid case is more than that of the other two types. In this case, like for the imperfect maintenance, the clustering based on the Alert clustering is better than the others. It can be seen in Figure 5(f) that with the increase of the average time to signal, the savings has increased when the proportion of the fixed cost to the variable cost is not considerable. Table 1. Reports these percentage the cost saving of these policies for values (a)$\lambda_1^{-1} = 8, \lambda_2^{-1} = 4, \lambda_3^{-1} = 3$، (b) $\lambda_1^{-1} = 9, \lambda_2^{-1} = 6, \lambda_3^{-1} = 5$.

Table1.The amount cost saving of these policies.

Signal Clustering			Alert Clustering			Alarm Clustering		
Perfect	imperfect	hybrid	Perfect	imperfect	hybrid	Perfect	imperfect	Hybrid

a	70%	78%	70%	68%	82%	89%	60%	62%	80%
b	71%	72%	82%	69%	80%	86%	67%	60%	75%

Conclusion

In this paper, we proposed a clustering model for the maintenance activities of a system with perfect, imperfect, and hybrid maintenance actions and evaluated it in terms of cost savings. The previous studies on the clustering of CBM and imperfect maintenance have focused more on complex models and the solution methods and provided no general insight into this problem. We proposed a general model that can be applied to every system and evaluated that which maintenance strategy produces more savings. The results showed that in the case of perfect maintenance if the fixed cost exceeds a certain amount, clustering with a more degree of the clusters (signal clustering) is more appropriate. Accordingly, Alarm clustering produced the lowest savings, less than 60%, for the high proportions of the fixed cost to the variable cost. In the imperfect and hybrid maintenances, Alert clustering produced more savings. Generally, imperfect maintenance incurs less cost in comparison to the perfect one because it takes less time and cost for repair. In the hybrid case, one component restores to the as-good-as-new state at the first early signals and the other component restores to a working state using a less time and cost. Thus, its cost is less than the other two cases. Moreover, when the system is repaired at the early signals, serious failures are prevented. Therefore, disregarding the negative effects of the imperfect maintenance (such as accelerating the degradation process), it can be considered as the more appropriate action compared to the perfect one. To develop this model, it may be suggested to consider a system with non-identical components that do not behave like each other in the deterioration process. We can also consider delay time as a random variable in the model. Subsequent development relates to considering another deterioration process in which the alert rate is not constant.

References

[1] Olde Keizer, M., Flapper, S., Teunter, R., "Condition-based maintenance policies for systems with multiple dependent components: a review, "*European Journal of Operational Research*, 261(2): pp. 405-420, 2017.

[2] de Jonge, B., Klingenberg, W., Teunter, R., Tinga, T., "Reducing cost by clustering maintenance activities for multiple critical units, "*Reliability Engineering and System Safety*,145: pp. 93-103, 2016.

[3] Do, P., Voisin, A., Levrat, E., Iung, B.,"A proactive condition-based maintenance strategy with both perfect and imperfect maintenance actions," *Reliability Engineering and System Safety,* 133, pp. 22–32, 2016.

[4] Rasay, h., Fallahnezhad, M.S., zaremehjerdi, Y., "Application of multivariate control charts for condition based maintenance, "*International Journal Of Engineering*, 31(4): pp. 597-604,2018.

[5] Zhang, X., Zeng, J., "A general modeling method for opportunistic maintenance modeling of multi-unit systems, "*Reliability Engineering and System Safety*, 140: pp. 176–190, 2015.

[6] Ahmadi, R., Newby, M., "Maintenance scheduling of a manufacturing system subjected to deterioration, "*Reliability Engineering & System Safety*, 96: pp. 1411-1420, 2011.

[7] Zhao, J., Chan, A.H.C., Roberts, C., Madelin, K.B., "Reliability evaluation and optimization of imperfect inspections for a component with multi-defects, " *Reliability Engineering & System Safety*, 92: pp. 65-73, 2017.

[8] Liu, Y., Huang, H.Z.,"Optimal replacement policy for multi-state system under Imperfect Maintenance, "*IEEE Transactions on Reliability*, 59: pp. 483-495, 2011.

[9] Zhang, Z., Shen, J., Ma, Y., "Optimal maintenance policy considering imperfect repairs and non-constant probabilities of inspection errors," *Reliability Engineering & System Safety*,193: pp. 106-118, 2020.

[10] Pham, H., Wang, H.,"Imperfect maintenance*, " European journal operation research*, 94: pp. 425-38, 1996.

[11] Wu, S., Zuo, J.M.,"Linear and nonlinear preventive maintenance models, *"IEEE Transactions on Reliability,* 59(1): pp. 242-9, 2010.

[12] Wang, L.,"A survey of maintenance policies of deteriorating systems, *" European Journal of Operational Research*, 139 (3), pp. 469-489, 2002.

[13] Shafiee, M., Finkelstein, M., "A proactive group maintenance policy for continuously monitored deteriorating systems: Application to offshore wind turbines, *" Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability,* 229 (5): pp. 373-384, 2015.

[14] Shafiee, M., Finkelstein, M., Berenguer, C., " An opportunistic condition-base maintenance policy for offshore wind turbin blades subjected to degradation and environmental shocks, *"Reliability Engineering and System Safety*, 142: pp. 463-471, 2015.

Appendix

Consider Figure 1. The system starts with condition (1,1) in which none of the units give the first warning and do not need to be repaired. The time to first warning for each unit is an exponential distribution with the$\boldsymbol{2\lambda_1}$parameter. After that time until the next alarm is an exponential distribution with parameter$\boldsymbol{\lambda_1 + \lambda_2}$. Therefore, the probability that the unit in the second condition send the second alarm is equal to:

$$b_2 = \frac{\lambda_2}{\lambda_1 + \lambda_2}$$

Which based on this probability the system changes into state (3.1) and if the first alarm relates to another unit, the system will reach state $(2, 2)$and its probability is equal to:

$$b_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

In state (2, 2) the time to the next alarm is an exponential distribution with the parameter $2\lambda_2$ that brings the system to condition (3, 2). The time to next alarm is an exponential distribution with parameter $\lambda_2 + \lambda_3$ which is the probability that the second unit gives alarm that equals to:

$$d_1 = \frac{\lambda_2}{\lambda_2 + \lambda_3}$$

And the system reaches state (3, 3) and the probability that the second unit gives third warning is equal to:

$$d_2 = \frac{\lambda_3}{\lambda_2 + \lambda_3}$$

In state (3, 3) where the time to next alarm is an exponential distribution with the parameter $2\lambda_3$, when the system reaches state (4^1, 3), during time D it has the opportunity to be repaired. During the time D, the other unit with the following probability does not give a third warning and reaches the state (4^2, 3):

$$w_1 = e^{-\lambda_3 D}$$

If during this time another unit gives the third warning and reaches the condition (4^2,4^1) its probability is:

$$w_2 = 1 - e^{-\lambda_3 D}$$

The probabilities of h_1 and h_2 are also likewise calculated.

$$h_1 = e^{-\lambda_3 D}$$

$$h_2 = 1 - e^{-\lambda_3 D}$$

The system with probability d_2 reaches the state (4^1, 2) then during time D the system has the opportunity to be repaired and during this time, three different conditions occur for the system. With the following Probability

$$g_1 = e^{-\lambda_2 D}$$

The other unit does not give a second warning and reaches the state (4^2, 2). The probabilities $g_3$and g_2 depend on the sum of $x_2 + x_3$ of the two exponential distributions of the random variables $x_2 \sim \exp(\lambda_2)$ and $x_3 \sim \exp(\lambda_3)$. g_3 is equal to the probability that the second unit has sent the second and third alarms during period D after the first alert so we have:

$$g_3 = F_{X_2+X_3}(D) = 1 - \frac{1}{\lambda_2 - \lambda_3}(\lambda_2 e^{-\lambda_3 D} - \lambda_3 e^{-\lambda_2 D})$$

And the system reaches state(4^2, 4^1). g_2 is also the probability that the second unit gives the second warning but not the third warning that is equal to:

$$g_2 = 1 - g_1 - g_3 = \frac{\lambda_2}{\lambda_2 - \lambda_3}\left(e^{-\lambda_2 D} - e^{-\lambda_1 D}\right)$$

The probabilities q_1 و q_2 و q_3 are likewise computed.

$$q_1 = e^{-\lambda_2 D}$$

$$q_2 = 1 - q_1 - q_3 = \frac{\lambda_2}{\lambda_2 - \lambda_3}\left(e^{-\lambda_2 D} - e^{-\lambda_1 D}\right)$$

$$q_3 = F_{X_2+X_3}(D) = 1 - \frac{1}{\lambda_2 - \lambda_3}(\lambda_2 e^{-\lambda_3 D} - \lambda_3 e^{-\lambda_2 D})$$

The system with probability b_2 reaches state (3, 1) after which the time to next alarm is an exponential distribution with parameter $\lambda_1 + \lambda_3$that if the first unit gives the third signal, its probability equals to:

$$a_2 = \frac{\lambda_3}{\lambda_1 + \lambda_3}$$

And the system reaches the state $(4^1, 1)$ and if another unit gives the first warning the probability is equal to:

$$a_1 = \frac{\lambda_1}{\lambda_1 + \lambda_3}$$

In the state (3, 2) also the time to next alarm is an exponential distribution with parameter $\lambda_2 + \lambda_3$ which reaches state (3, 3) if it gives a second alarm with the following probability:

$$k_1 = \frac{\lambda_2}{\lambda_2 + \lambda_3}$$

And if it gives a third warning it reaches a state $(4^1, 2)$ which its probability is equal to:

$$k_2 = \frac{\lambda_3}{\lambda_2 + \lambda_3}$$

In the condition of $(4^1, 1)$during time D maintenance activities should done. During this period, for the system four conditions are possible and the other unit does not give the first warning with the following probability:

$$P_1 = e^{\lambda_1 D}$$

With the probability of p_4 the other unit gives first, second, and third alarms, so:

$$P_4 = F_{X_1+X_2+X_3}(D) = 1 - e^{-\lambda_3 D} + \frac{\lambda_1 \lambda_3}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)}\left(e^{-\lambda_3 D} - \right.$$
$$e{-}\lambda 2D{-}\lambda 2\lambda 3\lambda 2{-}\lambda 1\lambda 1{-}\lambda 3e{-}\lambda 3D{-}\ e{-}\lambda 1D$$

With probability $p_3$the other unit gives first and second but not the third alarm during D time and reaches the condition $(4^2, 3)$:

$$P_3 = 1 - (P_1 + P_2 + P_4)$$

Also with probability $p_2$during D time the unit gives the first but not the second and third alarms and reaches the condition but does not $(4^2, 4^1)$:

$$P_2 = \int_0^D \int_{D-x_1}^{\infty} \lambda_2 e^{-\lambda_2 x_2} \lambda_1 e^{-\lambda_1 x_1}\, dx_2 dx_1 = \frac{\lambda_1}{(\lambda_1 - \lambda_2)}\left(e^{-\lambda_2 D} - e^{-\lambda_1 D}\right)$$