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Probabilistic Assessment of Two-Unit Parallel System with Correlated Lifetime under Inspection Using Regenerative Point Technique

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Abstract

This paper deals with the reliability analysis of a complex system consisting of a two dissimilar unit' in a parallel configuration with correlated lifetime distribution. The system stops functioning when both units stop working. Both units are inspected periodically as well as being examined before assigning to repair facility. Under consideration of the system have two states: Normal and failed. Regenerative point technique has been used for the mathematical formulation of the model. The system is analyzed using Laplace transforms to solve the mathematical equations. Reliability, Availability, MTSF, Busy Period of repairmen and Cost-effectiveness of the system has been computed. The computed results have been demonstrated by tables and graphs. The repair time of both the units follows the negative exponential distribution with different parameters in a joint probability density function. The inspection times are assumed to follow the general distribution. Some particular cases of the system have also been derived from seeing the practical importance of the model.

Keywords: Reliability, MTSF, Sojourn Times, Availability, Busy time, Profit function.

Nomenclature

t	Time scale
*/ ~	Symbol for Laplace/ Laplace Stieltjes transform of a function.
N_{jo} (j=1,2)	Unit-j is operative in normal mode.
F_{ji}/F_{jwi}	Unit-j is in F-mode and under inspection/ waiting for inspection.
X_i (i=1,2)	Random variables denoting the failure times of unit-1 and unit-2 respectively for i =1, 2.
$f(X_1,X_2)$	Joint p.d.f of (X_1, X_2)
$g_i(X)$	Marginal p.d.f of Xi= $\alpha_i(1-r)\exp[-\alpha_i(1-r)x]$
$K_i(.\|x)$	Conditional c.d.f of $X_i\|X_j= X_i$, $i \neq j$; i, j=1, 2.
$k_1(x_1\|X_2=x)$	Conditional p.d.f of $X_1\|X_2=x= r_1 \exp[-\alpha_1 x_1 - \alpha_2 r x] I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x})$
$k_2(x_2\|X_1=x)$	Conditional p.d.f of $X_2\|X_1=x=r_2 \exp[-\alpha_2 x_2 - \alpha_1 r x] I_0(2\sqrt{\alpha_1\alpha_2 r x_2 x})$
$H_1(.)$	c.d.f of inspection time of unit-i (i=1, 2)
μ_i	Constant repair rate of unit-i.

Introduction

Undeniably, many existing systems in the field of production, transportation, information energy, calculation, etc., are artificial and made by human while they are natural too which indicates that they are made by man and nature and may be simple or complex. The engineering systems should be dependable enough otherwise they may be failed to serve their purpose. r Engineering systems (ES) reliability embraces an extensive range of issues intended to ensure and maintain high reliability in both indivisible elements and the entire system as a whole. From the commercial to life-critical applications, the proliferation of computing systems in everyday life has substantially increased our dependence on them. Failures in the air traffic control system, hospital patient monitoring systems, and nuclear reactors can bring disastrous consequences. To enhance reliability and adequate performances from repairable systems, it is necessary to provide regular and timely maintenance.

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Several techniques of improving the security of repairable systems are available in the literature. Redundancy is one of the favorites and well-known configuration in which some parallel paths are created with the main unit.

The two-unit redundant systems have extensively studied in the literature of reliability due to their frequent and significant use in modern business and industries. Several authors including [1-4] have analyzed two-unit passive (standby) system under different types of failure and repair namely general repair. For the purpose of maintaining the systems more reliable, efficient, and adequate, the standby unit plays a crucial role in system operation. In such type of systems, the redundant unit operates only when the operating unit fails. Furthermore, the standby unit may adjust in hot standby mode, cold standby mode and also in idle standby mode as per the situation required. In real-life systems, it can be observed that the redundant unit takes significant amount of time to be operative. Therefore, during this time the system remains in an inoperative mode which might be a tremendous and unbearable loss. To overcome this issue, numerous authors [5-8] analyzed the performance of two-unit active redundant systems presumption that a single repairman is available throughout with the system for inspection and readiness for repair when it approaches partially or entirely failed mode. In a real-life situation, it may be seen that one has to bear uncoverable severe damage if the system becomes down even for a few minutes. The breakdown for a second in a dialysis process of a patient may cause death. A sudden trip off the power during Operation in OT may cause loss of patient life. To overcome such type of problems, one has to arrange the redundant unit in the parallel form with actively standby mode that on the failure of the main unit the redundant unit ready to perform the task immediately.

The researchers have examined the performance of standby complex systems under different types of failure and one kind of repair by employing various techniques. Authors Pandey, Tyagi and Kumar [9] have analyzed the reliability of series and parallel network using triangular intuitionistic Fuzzy sets. Munday and Malik [10] examined a computer system software redundancy with priority repair to hardware failure over the software failure using semi Markov process and regenerative point technique. Tiwari and Singh [11] have studied two units' system in a series configuration in which the second unit consists of a standby unit under the different types of failure and two types of repair using copula. Lado et al. [12] have evaluated the reliability measures (Availability, reliability, and MTTF, sensitivity and profit analysis) of the repairable complex system with two subsystems connected in a series configuration using the supplementary variable and Laplace transforms. The preventive maintenance of the system has done before it fails. Referring to the preventive maintenance, Ibrahim Yusuf [13] have studied a system with two types of repair online and offline repair using Kolmogorov forward equation method. Ibrahim Yusuf et al. [14] have investigated reliability characteristics of a linear consecutive 2- out- of- 4 supporting device for operation using Chapman Kolmogorov equation method. Permila and Malik [15] have analyzed a, 2-out of- 2: G system with single cold standby unit with priority to repair and arrival time of the server using semi Markov and regenerative point technique. Singh et al. [15] studied reliability measures (Availability, MTTF, and cost of a system which have two subsystems in a series configuration with a controller. Singh et al. [16,17] have studied the reliability measures of a standby complex system under the concept of switch failures and controllers using copula distribution. Singh and Ayagi [18] have analyzed a complex system under preemptive resume repair using Gumbel- Hougaard family copula. Researchers Singh and Ayagi [19] have studied Reliability measures of a system consisting of two subsystems in the series configuration using copula.

Reliability index is a standard universally used indicator to assessing safety parameter for electrical and mechanical systems. Ghasemi and Nowak [20, 21] studied target reliability for bridges with consideration unlimited limit state, reliability index of a non- normal distribution of limited state function. It has been conferred that the mechanical properties of the bones are random variables. Treating fact in view Ghasemi et al. [22] have studied fatigue reliability analysis of medial tibial stress syndrome using the finite element method to determine the damage states of the tibias. Ghasemi and Nowak [23] have analyzed the circular tunnels with strength limit state by calibrating load resistance factor of design code tunnels. In the consistent study of reliability parameters Ghasemi and Nowak [24], computed the mean maximum value of non- normal distributions for different periods. Design of highway bridges requires the assessment of live loads, which can comprise a substantial degree of uncertainties. To perform a reliability analysis, it is necessary to consider live load as a random variable. Multiple truck presence (MTP) and headway distance also are two random variables that should be deliberated for the assessment of live weight. S. H. Ghasemi et al. [25] have studied statistical parameters in a lane multiple truck presence and a new procedure to analyze the lifetime of the bridge. Yanaka et al. [26] have developed recommendations for durability design of structures in marine environments from the reliability point of view, taking into consideration the life cycle cost of a structure. Monika Gahlot et al. [27] have analyzed the performance of the repairable system in the series configuration under different types of failure and two types of repair using Gumbel-Hougaard family Copula distribution. Yusuf. I [28], Kumar [29, 30] and Barak [31] have studied the reliability characteristics of complex systems consisting of the main unit with supporting unit and repair facility.

The researchers have presented an excellent work in the field of system reliability and have proclaimed the better performance of the repairable systems by their operations, but still, it needs further study of the new type of models with a justified and satisfactory assessment. Keeping this fact in view, we in this present paper have analyzed a system with two distinct unit's parallel system assuming that the sort of correlation exists between the lifetimes of units. The repair time distributions of both the units are taken to be negative exponential with different parameters, and inspection units are assumed to follow the general distribution. The following essential measures of the system effectiveness obtained by using regenerative point techniques:

(i) Reliability and mean time to system failure (MTSF)
(ii) Point wise and steady-state availabilities of the system,
(iii) An expected busy period of the repairman during [0, t).
(iv) Net expected profit earned by the system in the interval [0, t] and steady state.

Assumptions

The following assumptions have been made throughout the study of the model:

1. Initially, the system is in the state S_0 , and both the units are operative in a normal mode having the parallel configuration.
2. The operation of only one unit is enough to perform the taskadequately.
3. The system fails when both units stop functioning.
4. Each unit of the system has two modes -Normal (N) and total failure (F).
5. Each failed unit goes for inspection before entering a repair facility.
6. A single repairman is available with the system to repair a failed unit and inspection work. The service discipline of the repairman is FCFS.
7. As soon as the failed unit repaired, it is ready to perform the task as good as new. No damage has reported due to repair of the system.
8. The repair time distributions of both groups are taken to be negative exponential with different parameters.
9. When both the units are operative, then their failure times are assumed to be correlated random variables having their joint distribution as B.V.E with density function as given below:
   $$f\left(x_1,x_2\right)=r_1r_2\left(1-r\right)\exp\left(-\alpha_1x_1-\alpha_2x_2\right)I_0\left(2\sqrt{\alpha_1\alpha_2rx_1x_2}\right)$$
   with the consideration that variables, $x_1,x_2,r_1,r_2>0$
   $0<r<1$
10. The distributions of inspection times of both the units are taken to be general with different c.d.f's.

State Description

The state description of the model highlights that S_0 is a state where the system is in a perfect state in which both the units are in good working condition. S_1, S_2, S_3, and S_4 are the states where one of the units is failed and is under inspection and repair, state S_5, S_6, S_7 and S_8 are the states where the system is in the utterly failed state.

Table 1. State Description

State	Description
S_0	It is a perfect state, and both the units are in good working condition.
S_1 S_2	The indicated states represent that any one unit of the system is in a failed state and under inspection. The other unit is in good working condition.
S_3 S_4	The indicated states represent that one failed unit is under repair after inspection. The other unit is in good working condition.
S_5 S_6	The states represent that the system is in totally failed mode and one unit is under inspection while the other unit is waiting for inspection.
S_7 S_8	The states represent that the system is in totally failed mode and one unit is under repair while the other unit is waiting for inspection.

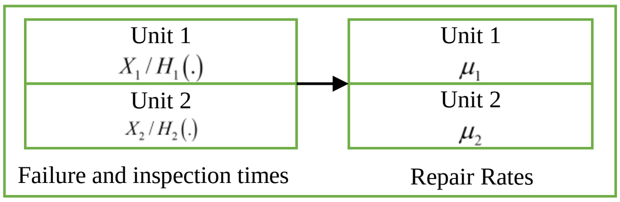
Up States

$S_0= (N_{1O}, N_{2O})$
$S_1= (F_{1i}, N_{2O})$
$S_2= (N_{1O}, F_{2i})$
$S_3= (F_{1r}, N_{2O})$
$S_4= (N_{1O}, F_{2r})$

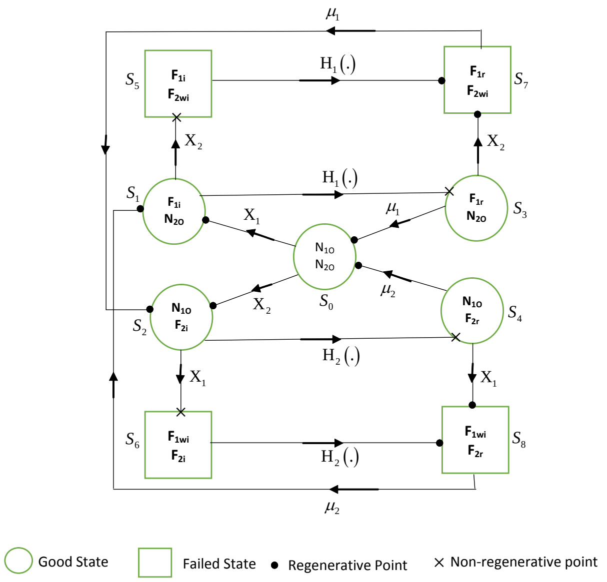
Down States

$S_5= (F_{1i}, F_{2wi})$
$S_6= (F_{1wi}, F_{2i})$
$S_7= (F_{1r}, F_{2wi})$
$S_8= (F_{1wi}, F_{2r})$

The transition diagram of the system model along with failure time variables, inspection time c.d.f's and repair rates are shown in fig. 1 (a, b). In the figure we observe that the epochs of transition from state S_1 to S_5 and S_3 and S_2 to S_4 and S_6 are non-regenerative as the future probabilistic behavior on these epochs depends upon the previous states. The all other entrance epochs are regenerative.



(a)



(b)

Fig. 1. (a) System configuration, **(b)** State transition diagram of the model: By the probabilistic assumptions, the following state transition diagram of system operation is possible. The operational state, regenerative states, non-regenerative states, and failed state are represented in the diagram **(b).**

Formulation of Mathematical Model: Transition Probabilities and Mean Sojourn Times

Using one-step unconditional transition probability, which can be obtained by simple probabilistic arguments, the non-zero elements of the transition probability $P = p_{ij}$ for the model are as follows-

$$p_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2},\; p_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2},\; p_{72} = p_{81} = 1$$

$$p_{15|x} = \int \bar{H}_1(u).dK_2(u|x) = p^{(5)}_{17|x},$$

$$p_{26|x} = \int \bar{H}_2(u).dK_1(u|x) = p^{(6)}_{28|x}$$

$$p^{(3)}_{10|x} = 1 - \int \mu_1 e^{-\mu_1 v} K_2(v|x) dv \int_0^v e^{\mu_1 u} dH_1(u),$$

$$p^{(3)}_{17|x} = \int e^{-\mu_1 v} dK_2(v|x) \int_0^v e^{\mu_1 u} dH_1(u)$$

$$p^{(4)}_{20|x} = 1 - \int \mu_2 e^{-\mu_2 v} K_1(v|x) dv \int_0^v e^{\mu_2 u} dH_2(u),$$

$$p^{(4)}_{28|x} = \int e^{-\mu_2 v} dK_1(v|x) \int_0^v e^{\mu_2 u} dH_2(u)$$

We observe that

$$p_{01} + p_{02} = 1,\; p_{15|x} + p^{(3)}_{17|x} + p^{(3)}_{10|x} = 1,$$

$$p^{(5)}_{17|x} + p^{(3)}_{17|x} + p^{(3)}_{10|x} = 1,\; p_{26|x} + p^{(4)}_{28|x} + p^{(4)}_{20|x} = 1, \text{ and } p^{(6)}_{28|x} + p^{(4)}_{28|x} + p^{(4)}_{20|x} = 1$$

From the conditional steady-state transition probabilities, the unconditional steady-state transition probability can be obtained by using the result

$$p_{ij} = \int p_{ij|x}.g(x)dx$$

as follows- $p^{(5)}_{17} = \int p^{(5)}_{17|x} \alpha_1 (1-r) e^{-\alpha_1(1-r)x} dx$ =p15

$$p^{(3)}_{17} = \int p^{(3)}_{17|x} \alpha_1 (1-r) e^{-\alpha_1(1-r)x} dx$$

$$p^{(3)}_{10} = \int p^{(3)}_{10|x} \alpha_1 (1-r) e^{-\alpha_1(1-r)x} dx$$

$$p^{(6)}_{28} = \int p^{(6)}_{28|x} \alpha_2 (1-r) e^{-\alpha_2(1-r)x} dx = p_{26}$$

$$p^{(4)}_{28} = \int p^{(4)}_{28|x} \alpha_2 (1-r) e^{-\alpha_2(1-r)x} dx$$

$$p^{(4)}_{20} = \int p^{(4)}_{20|x} \alpha_2 (1-r) e^{-\alpha_2(1-r)x} dx$$

Let X_i denotes the sojourn time in state S_i, then the mean sojourn time in state S is given as - $\psi_i = \int P(X_i > t)dt$

The mean sojourn times in various states are as follows:

$$\psi_0 = \frac{1}{(\alpha_1+\alpha_2)(1-r)},\; \psi_1 = \int e^{-\alpha_2(1-r)t} \bar{H}_1(t)dt$$

$$\psi_2 = \int e^{-\alpha_1(1-r)t} \bar{H}_2(t)dt \quad \psi_3 = \frac{1}{\mu_1 + \alpha_2(1-r)}$$

$$\psi_4 = \frac{1}{\mu_2 + \alpha_1(1-r)},\; \psi_5 = \int \bar{H}_1(t)dt \quad \psi_6 = \int \bar{H}_2(t)dt,,$$

$$\psi_7 = \frac{1}{\mu_1},\; \psi_8 = \frac{1}{\mu_2}$$

Analytical Study

Reliability and MTSF

Let the random variable T_i be the time to system failure when the system starts its operation from the state $S_i \in E$, then the reliability of the system is given as: $R_i(t) = P(T_i > t)$

To give $R_i(t)$, we assume that failed state S_5 to S_8 as an absorbing state of the system. Using the simple probabilistic arguments, one can quickly develop the recurrence relations among R_i (t); i=0, 1, 2, 3, 4. Taking the Laplace transform of the relationships and simplifying the resulting set of algebraic equations for $R_0^*(s)$, we get after omitting the argument 's' for brevity-

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Where
$$N_1(s) = \begin{vmatrix} z_0^* & -q_{01}^* & -q_{02}^* \\ z_1^* + q_{13}^* z_3^* & 1 & 0 \\ z_2^* + q_{24}^* z_4^* & 0 & 1 \end{vmatrix} =$$

$$z_0^* + q_{01}^*(z_1^* + q_{13}^* z_3^*) + q_{02}^*(z_2^* + q_{24}^* z_4^*)$$

and

$$D_1(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* \\ -q_{10}^{(3)*} & 1 & 0 \\ -q_{20}^{(4)*} & 0 & 1 \end{vmatrix}$$

$$= 1 - q_{01}^* q_{10}^{(3)*} - q_{02}^* q_{20}^{(4)*}$$

The mean time to system failure (MTSF) is given by $E(T_0)=\lim_{s\to 0} R_0^*(s)$

Using the results $z_i^*(0)=\psi_i$ and $q_{ij}^*(0)=p_{ij}$, we get

$$E(T_0)=\frac{N_1}{D_1}$$

Where $N_1=\psi_0+P_{01}(\psi_1+P_{13}\psi_3)+P_{02}(\psi_2+P_{24}\psi_4)$

and $D_1=1-P_{01}P_{10}^{(3)}-P_{02}P_{20}^{(4)}$

Table 2. Variation in the values of the MTSF of the system

α_1	$r=0.25$			$r=0.50$		
	$\mu_1=0.2$	$\mu_1=0.3$	$\mu_1=0.4$	$\mu_1=0.2$	$\mu_1=0.3$	$\mu_1=0.4$
0.01	2103.	2324.	2828.0	4524.5	5013.6	6106.2
0.02	1122.	1234.	1505.3	2368.5	2613.4	3187.9
0.03	797.3	872.4	1067.5	1651.7	1815.3	2218.2
0.04	636.1	693.1	851.11	1294.6	1417.7	1735.7
0.05	540.5	586.7	723.29	1081.6	1180.4	1448.2
0.06	477.7	516.8	639.87	940.47	1023.2	1258.2
0.07	433.6	467.8	581.91	840.48	911.85	1124.0
0.08	401.3	431.9	539.91	766.2	829.08	1024.8
0.09	376.8	404.7	508.64	709.08	765.43	948.82
0.10	357.8	383.6	484.93	663.96	715.15	889.19

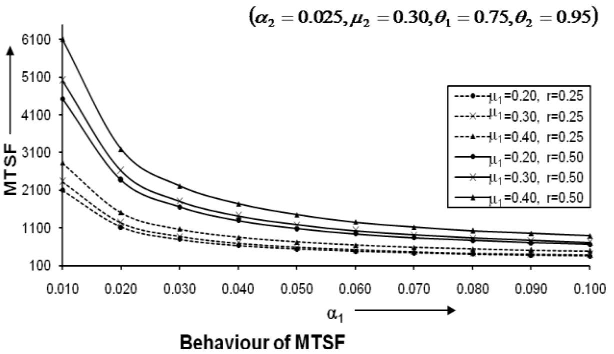


Fig. 2. Behavior of MTSF w.r.t α_1 for different values of μ_1 and r

Availability

Let $A_i(t)$ be the probability that the system is up at epoch t when initially it starts functioning from the state $S_i\ (i=0,1,2,7,8)$. by using probabilistic augments we get the following set of recurrence relations among $A_i(t)$ –

$$A_0(t)=z_0(t)+q_0(t)\ ©\ A_1(t)+q_{02}(t)\ ©\ A_2(t)$$

$$A_1(t)=z_1(t)+q_{13}(t)\ ©\ z_3(t)+q_{10}^{(3)}(t)\ ©\ A_0(t)+q_{17}^{(3)}(t)\ ©\ A_7(t)+q_{17}^{(5)}(t)\ ©\ A_7(t)$$

$$A_2(t)=z_2(t)+q_{24}(t)\ ©\ z_4(t)+q_{20}^{(4)}(t)\ ©\ A_0(t)+q_{28}^{(4)}(t)\ ©\ A_8(t)+q_{28}^{(6)}(t)\ ©\ A_8(t)$$

$$A_7(t)=q_{72}(t)\ ©A_2(t)$$

$$A_8(t)=q_{81}(t)\ ©\ A_1(t)$$

Taking Laplace transformation of the above equations, the solution for $A_i^*(s)$ can be put in the following form

$$\begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_7^* \\ A_8^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 \\ -q_{10}^{(3)*} & 1 & -q_{12}^* & -q_{17}^* & 0 \\ -q_{20}^{(4)*} & 0 & 1 & 0 & -q_{28}^* \\ 0 & 0 & -q_{72}^* & 1 & 0 \\ 0 & -q_{81}^* & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} z_0^* \\ z_1^*+q_{13}^*z_3^* \\ z_2^*+q_{24}^*z_4^* \\ 0 \\ 0 \end{bmatrix}$$

Where $q_{17}^*=q_{17}^{(3)*}+q_{17}^{(5)*}$ and $q_{28}^*=q_{28}^{(4)*}+q_{28}^{(6)*}$

Simplifying the above matrix equation for $A_0^*(s)$, we get

$$A_0^*(s)=\frac{N_2(s)}{D_2(s)},$$

where

$$N_2(s)=\left(1-q_{17}^*q_{72}^*q_{28}^*q_{81}^*\right)z_0^*+\left(q_{01}^*+q_{02}^*q_{28}^*q_{81}^*\right)\left(z_1^*+q_{13}^*z_3^*\right)+\left(q_{02}^*+q_{01}^*q_{17}^*q_{72}^*\right)\left(z_2^*+q_{24}^*z_4^*\right)$$

and

$$D_2(s)=\left(1-q_{17}^*q_{72}^*q_{28}^*q_{81}^*\right)-\left(q_{01}^*+q_{02}^*q_{28}^*q_{81}^*\right)q_{10}^{(3)*}-\left(q_{02}^*+q_{01}^*q_{17}^*q_{72}^*\right)q_{20}^{(4)*}$$

In the long run, the probability that the system will be UP state is given by

$$A_0=\lim_{s\to 0}A_0(t)=\lim_{s\to 0}sA_0^*(s)=\lim_{s\to 0}\frac{sN_2(s)}{D_2(s)}$$

As $D_2(s)\to 0$, as $s\to 0$, therefore the steady state availability, by using L-Hospital rule, we have

$$A_0=\lim_{s\to 0}\frac{N_2(0)}{D_2'(0)}=\frac{N_2}{D_2}$$

Where

$$N_2=(1-p_{17}p_{28})\psi_0+(p_{01}+p_{02}p_{28})(\psi_1+p_{13}\psi_3)+(p_{02}+p_{01}p_{17})(\psi_2+p_{24}\psi_4)$$

and

$$D_2=\left[1-\left(p_{17}^{(3)}+p_{17}^{(5)}\right)\left(p_{28}^{(4)}+p_{28}^{(6)}\right)\right]\psi_0+\left(1-p_{02}p_{20}^{(4)}\right)\left[\psi_1+p_{13}\psi_3+p_{15}\psi_5+\left(p_{17}^{(3)}+p_{17}^{(5)}\right)\psi_7\right]+\left(1-p_{01}p_{10}^{(3)}\right)\left[\psi_2+p_{24}\psi_4+p_{26}\psi_6+\left(p_{28}^{(4)}+p_{28}^{(6)}\right)\psi_8\right]$$

Table 3. Variation in the values of the Availability of the system

α_1	$r=0.25$			$r=0.50$		
	$\mu_1=0.2$	$\mu_1=0.3$	$\mu_1=0.4$	$\mu_1=0.2$	$\mu_1=0.3$	$\mu_1=0.4$
0.010	0.9885	0.9921	0.9935	0.9888	0.9923	0.9937
0.020	0.9779	0.9847	0.9874	0.9786	0.9853	0.9879
0.030	0.9678	0.9777	0.9817	0.9691	0.9787	0.9824
0.040	0.9584	0.9711	0.9762	0.9604	0.9726	0.9774
0.050	0.9495	0.9648	0.971	0.9523	0.9668	0.9726
0.060	0.9411	0.9588	0.966	0.9447	0.9614	0.9681
0.070	0.9332	0.9531	0.9613	0.9377	0.9563	0.9639
0.080	0.9258	0.9477	0.9569	0.9311	0.9516	0.9601
0.090	0.9187	0.9425	0.9526	0.9249	0.9471	0.9562
0.100	0.912	0.9376	0.9485	0.9191	0.9428	0.9527

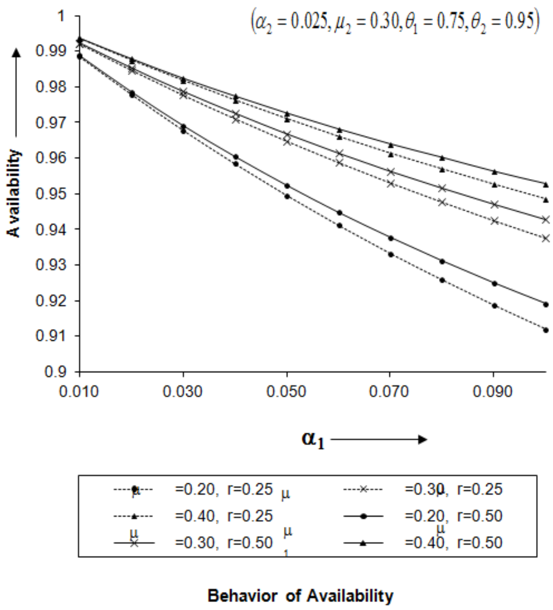


Fig. 3. Behavior of Availability A_0 w.r.t α_1 for different values of μ_1 and r

Busy Period of Repairman

Let $B_i(t)$ be the probability that the repairman is busy in the repair of a failed unit at time t when the system initially starts from the state $S_i \in E$. here the probabilistic arguments yield the following integral equations:

$$B_0(t) = \int_0^t q_{01}(u)du B_1(t-u) + \int_0^t q_{02}(u)du B_2(t-u)$$
$$= q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t)$$
$$B_1(t) = q_{13}(t)©z_3(t) + q_{10}^{(3)}(t)©B_0(t)$$
$$+q_{17}^{(3)}(t)©B_7(t) + q_{17}^{(5)}(t)©B_7(t)$$
$$B_2(t) = q_{24}(t)©z_4(t) + q_{20}^{(4)}(t)©B_0(t)$$
$$+q_{28}^{(4)}(t)©B_8(t) + q_{28}^{(6)}(t)©B_8(t)$$
$$B_7(t) = z_7(t) + q_{72}(t)©B_2(t)$$
$$B_8(t) = z_8(t) + q_{81}(t)©B_1(t)$$

Where, $z_7(t) = e^{-\mu_1 t}$ and $z_8(t) = e^{-\mu_2 t}$

Taking the Laplace transform of the above relations and simplifying the resulting set of algebraic equations for $B_0^*(s)$, we have

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)}$$

where $N_3(s) = \left(q_{01}^* + q_{02}^* q_{28}^* q_{81}^*\right)\left(q_{13}^* z_3^* + q_{17}^* z_7^*\right) + \left(q_{02}^* + q_{01}^* q_{17}^* q_{72}^*\right)\left(q_{24}^* z_4^* + q_{28}^* z_8^*\right)$

and $D_2(s)$ is given by as same as in availability.

In the long run, the probability that the repairman will be busy in the repair of a failed unit is given by

$$B_0 = \lim(t \to 0) B_0(t) = \lim(s \to 0) s B_0^*(s) = \frac{N_3}{D_2}$$

Similarly, let $I_i(t)$ be the probability that the repairman is busy in the installation of a failed unit at epoch t when the system initially starts from the state $S_i \in E$. here the probabilistic arguments yield the following integral equations:

$$I_0(t) = q_{01}(t)©I_1(t) + q_{02}(t)©I_2(t)$$
$$I_1(t) = Z_1(t) + q_{15}(t)©z_5(t) + q_{10}^{(3)}(t)©I_0(t)$$
$$+q_{17}^{(3)}(t)©I_7(t) + q_{17}^{(5)}(t)©I_7(t)$$
$$I_2(t) = Z_2(t) + q_{26}(t)©z_6(t) + q_{20}^{(4)}(t)©I_0(t)$$
$$+q_{28}^{(4)}(t)©I_8(t) + q_{28}^{(6)}(t)©I_8(t)$$
$$I_7(t) = q_{72}(t)©I_2(t)$$
$$I_8(t) = q_{81}(t)©I_1(t)$$

where $z_5(t) = \overline{H}_1(t)$ and $z_6(t) = \overline{H}_2(t)$

Taking the Laplace transform of the above relations and simplifying the resulting set of algebraic equations for $I_0^*(s)$, we have

$$I_0^*(s) = \frac{N_4(s)}{D_2(s)}$$

where

$$N_4(s) = \left(q_{01}^* + q_{02}^* q_{28}^* q_{81}^*\right)\left(z_1^* + q_{15}^* z_5^*\right) + \left(q_{02}^* + q_{01}^* q_{17}^* q_{72}^*\right)\left(z_2^* + q_{26}^* z_6^*\right)$$

In the long run, the probability that the repairman will be busy in the installation of a failed unit is given by

$$I_0 = \lim(t \to 0) I_0(t) = \lim(s \to 0) s I_0^*(s) = \frac{N_4}{D_2}$$

where $N_4 = \left(1 - p_{02} p_{20}^{(4)}\right)(\psi_1 + P_{15}\psi_5) + \left(1 - p_{01} p_{10}^{(3)}\right)[\psi_2 + P_{26}\psi_6]$

Profit Analysis

Let K_0, K_1 and K_2 be the revenue generated, service cost of repair and installation charge per unit time in the interval [0, t) respectively. Then net expected profit incurred by the system operation in the interval $[0, t)$ is given by

$P(t)$ = Expected revenue in $[0, t)$ – Expected amount paid to a repairman in $[0, t) = K_0 \mu_{up}(t) - K_1 \mu_b(t) - K_2 \mu_1(T)$

where,

$\mu_{up}(t)$ = Expected up (operative) time of the system during $[0, t)$ $= \int_0^t A_0(u)du$

So that $\mu_{up}^*(s) = A_0^*(s)/s$

Similarly,

$\mu_b(t) =$ Expected busy period of the repairman in the repair of a failed unit during $[0, t) = \int_0^t B_0(u)du$

And $\mu_l(t) =$ The expected busy period of the repairman in the installation of a failed unit during $[0, t) = \int_0^t l_0(u)du$.

So that $\mu_b^*(s) = B_0^*(s)/s$ and $\mu_1^*(s) = l_0^*(s)/s$

The expected profit per unit time in a steady state is given by

$$P = \lim(t \to 0)P(t)/t = \lim(s \to 0)s^2 P^*(s)$$
$$= K_0 \lim(s\to 0)sA_0^*(s) - K_1 \lim(s\to 0)sB_0^*(s) - K_2 \lim(s\to 0)sl_0^*(s)$$
$$= K_0 A_0 - K_1 B_0 - K_2 l_0$$

Table 4. Variation in the values of the cost function of the system

α_1	$r = 0.25$			$r = 0.50$		
	$\mu_1 = 0.2$	$\mu_1 = 0.3$	$\mu_1 = 0.4$	$\mu_1 = 0.2$	$\mu_1 = 0.3$	$\mu_1 = 0.4$
0.010	435.59	436.51	437.13	440.68	441.22	441.56
0.020	430.14	434.19	436.64	437.18	439.94	441.52
0.030	424.95	431.83	435.97	433.78	438.62	441.37
0.040	419.99	429.44	435.17	430.46	437.24	441.12
0.050	415.26	427.03	434.25	427.22	435.83	440.78
0.060	410.74	424.62	433.23	424.07	434.38	440.36
0.070	406.41	422.22	432.12	421	432.91	439.87
0.080	402.25	419.82	430.94	418.01	431.41	439.32
0.090	398.27	417.44	429.7	415.09	429.89	438.7
0.100	394.44	415.08	428.41	412.24	428.37	438.03

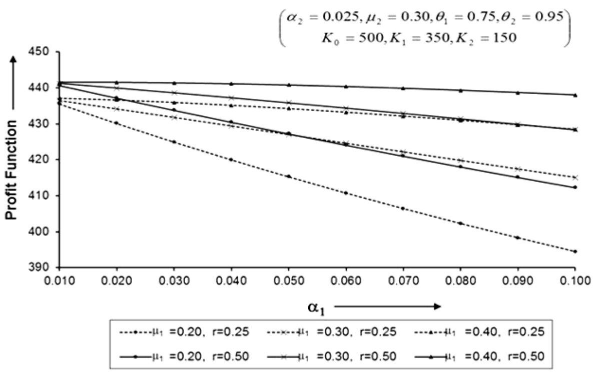


Fig. 4. Behavior of profit function P w.r.t α_1 for different values of μ_1 and r

Particular Cases

This model studies a general two parallel dissimilar units with inspection and some of the similar systems can be deduced as particular cases by taking inspection time distributions as negative exponential. Thus,

$H_1(t) = 1 - e^{-\theta_1 t}$ And $H_2(t) = 1 - e^{-\theta_2 t}$ then

$$P_{15|x} = \int e^{-\theta_1 u}\alpha_2 e^{-(\alpha_2 u+\alpha_1 rx)} l_0\left(2\sqrt{\alpha_1\alpha_2 rxu}\right)du$$
$$= \alpha_2 e^{-\alpha_1 rx}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\int e^{-(\alpha_2+\theta_1)u}u^j du$$
$$= \alpha_2 e^{-\alpha_2 rx}\sum_{j-0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\frac{\Gamma(j+1)}{(\alpha_2+\theta_1)^{j+1}}$$
$$= \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')} = P_{17|x}^{(5)}, \alpha_2' = \frac{\alpha_2}{\alpha_2+\theta_1}$$

Similarly,

$$P_{26|x} = \int e^{-\theta_2 u}\alpha_1 e^{-(\alpha_1 u+\alpha_2 rx)} l_0\left(2\sqrt{\alpha_1\alpha_2 rxu}\right)du$$
$$= \alpha_1 e^{-\alpha_2 rx(1-\alpha_1')} = P_{28|x}^{(6)}, \alpha_1 = \frac{\alpha_1}{\alpha_1+\theta_2}$$

$$P_{10|x}^{(3)} = 1 - \int \mu_1 e^{-\mu_1 v}K_2(v|x)dv\int_0^v e^{\mu_1 u}\theta_1 e^{-\theta_1 u}du$$
$$= 1 - \frac{\mu_1\theta_1}{\theta_1-\mu_1}\int e^{-\mu_1 u}K_2(v|x)dv + \frac{\mu_1\theta_1}{\theta_1-\mu_1}\int e^{-\theta_1 u}K_2(v|x)dv$$
$$= 1 - \frac{\theta_1}{\theta_1-\mu_1}\alpha_2'' e^{-\alpha_1 rx(1-\alpha_2'')} + \frac{\mu_1}{\theta_1-\mu_1}\alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}$$

, where $\alpha_2'' = \dfrac{\alpha_2}{\alpha_2+\mu_1}$

$$P_{17|x}^{(3)} = \int e^{-\mu_1 v}dK_2(v|x)\int_0^v \theta_1 e^{-(\theta_1-\mu_1)u}du$$
$$= \frac{\theta_1}{\theta_1-\mu_1}\left[\alpha_2'' e^{-\alpha_1 rx(1-\alpha_2'')} - \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}\right]$$

$$P_{20|x}^{(4)} = 1 - \int \mu_2 e^{-\mu_2 v}K_1(v|x)dv\int_0^v e^{\mu_2 u}\theta_2 e^{-\theta_2 u}du$$
$$= 1 - \frac{\theta_2}{\theta_2-\mu_2}\alpha_1'' e^{-\alpha_2 rx(1-\alpha_1'')} + \frac{\mu_2}{\theta_2-\mu_2}\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')},$$

where $\alpha_1'' = \dfrac{\alpha_1}{\alpha_1+\mu_2}$

$$P_{28|x}^{(4)} = \int e^{-\mu_2 v}dK_1(v|x)\int_0^v e^{\mu_2 u}\theta_2 e^{-(\theta_2\mu_1)u}du$$
$$= \frac{\theta_2}{\theta_2-\mu_2}\left[\alpha_2'' e^{-\alpha_1 rx(1-\alpha_2')}\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right]$$

$$P_{15} = P_{17}^{(5)} = \int \alpha_2'' e^{-\alpha_1 rx(1-\alpha_2')}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx$$
$$= \frac{\alpha_2'(1-r)}{1-\alpha_2' r}$$

$$P_{26} = P_{28}^{(6)} = \int \alpha_1'' e^{-\alpha_2 rx(1-\alpha_1')}\alpha_2(1-r)e^{-\alpha_2(1-r)x}dx$$
$$= \frac{\alpha_1'(1-r)}{1-\alpha_1' r}$$

$$P_0^{(3)} = \int \alpha_1(1-r)e^{-\alpha_1(1-r)x}dx - \frac{\theta_1}{\theta_1-\mu_1}\int \alpha_2'' e^{-\alpha_1 rx(1-\alpha_2'')}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx + \frac{\mu_1}{\theta_1-\mu_1}$$
$$\int \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx$$
$$= 1 - \frac{\theta_1}{\theta_1-\mu_1}\frac{\alpha_2''(1-r)}{1-\alpha_2''r} + \frac{\mu_1}{\theta_1-\mu_1}\frac{\alpha_2'(1-r)}{1-\alpha_2'r}$$

$$P_{17}^{(3)} = \frac{\theta_1}{\theta_1-\mu_1}\int\left[\alpha_2'' e^{-\alpha_1 rx(1-\alpha_2'')} - \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}\right]$$
$$\times \alpha_1(1-r)e^{-\alpha_1(1-r)x}dx = \frac{\theta_1}{\theta_1-\mu_1}\left[\frac{\alpha_2''(1-r)}{1-\alpha_2''r} - \frac{\alpha_2'(1-r)}{1-\alpha_2'r}\right]$$

$$P_{20}^{(4)} = \int \alpha_2(1-r)e^{-\alpha_2(1-r)x}dx - \frac{\theta_2}{\theta_2-\mu_2}\int \alpha_1'' e^{-\alpha_2 rx(1-\alpha_1'')}\alpha_2(1-r)e^{-\alpha_2(1-r)x}dx +$$
$$\frac{\mu_2}{\theta_2-\mu_2}\int \alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\alpha_2(1-r)e^{-\alpha_2(1-r)x}dx$$
$$= 1 - \frac{\theta_2}{\theta_2-\mu_2}\frac{\alpha_1''(1-r)}{1-\alpha_1''r} + \frac{\mu_2}{\theta_2-\mu_2}\frac{\alpha_1'(1-r)}{1-\alpha_1'r}$$

$$P_{28}^{(4)} = \frac{\theta_2}{\theta_2-\mu_2}\int\left[\alpha_1'' e^{-\alpha_2 rx(1-\alpha_1'')} - \alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right]\alpha_2(1-r)e^{-\alpha_2(1-r)x}dx$$
$$= \frac{\theta_2}{\theta_2-\mu_2}\left[\frac{\alpha_1''(1-r)}{1-\alpha_1''r} - \frac{\alpha_1'(1-r)}{1-\alpha_1'r}\right]$$

The changes in ψ_1, ψ_2, ψ_5 and ψ_6 are as follows $\rightarrow$

$$\psi_1 = \int e^{-\alpha_2(1-r)t}e^{-\theta_1 t}dt = \frac{1}{\left\{\theta_1 + \alpha_2(1-r)\right\}},$$
$$\psi_2 = \int e^{-\alpha_1(1-r)t}e^{-\theta_2 t}dt = \frac{1}{\left\{\theta_2 + \alpha_1(1-r)\right\}}$$
$$\psi_5 = \int e^{-\theta_1 t}dt = \frac{1}{\theta_1} \text{ And } \psi_6 = \int e^{-\theta_2 t}dt = \frac{1}{\theta_2}$$

Conclusion and Scope

In the model for two parallel distinguishable units with correlated lifetimes, inspected from time to time have analyzed, using Markov regenerative point technique and the various system parameters MTSF, steady-state availability, a busy period of repairman and profit function have computed. Observe the effect of correlation on the system performance in steady state; failure parameter α_1 and repair parameter μ_1 have investigated through variations in the parameters. The computations have done by fixing the parameters as $\alpha_2 = 0.025, \mu_2 = 0.30, \theta_1 = 0.75$ and $\theta_2 = 0.95$.

Table 2 and the fig. 2 yield the MTSF for correlation coefficient $r = 0.25$ and 0.5 and one can observe that MTSF decreases uniformly α_1 increases from 0.01 to 0.10 and μ_1 from 0.20 to 0.40. It concludes that MTSF is higher for higher values of correlation coefficient and repair parameter μ_1. Also, MTSF decreases significantly in the beginning, and after that, it decreases approximately in a constant manner. Table-3 and the fig. 3 provide clear guidance, how the availability decreases gradually as the failure rate α_1 and repair rate μ_1 increases for both the correlation coefficient values. Increasing parameters (α_1, μ_1), MTSF and availability decrease. Table-4 and the corresponding fig. 4 deals with variation of expected profit $E_p(t)$ function for different values of parameter K_0 (Revenue generation), K_1(Service cost), and K_2 (installation charge), per unit time in the [0, t]. These costs have been fixed at $K_0 = 500, K_1 = 350$ and $K_2 = 150$ .It reveals that expected profit decreases linearly α_1 increase and it increases with the increase in the values of r and μ_1.

Maintaining high or required level of reliability is often an essential requisite for improving system reliability. The model developed in this paper may be highly beneficial to engineers, maintenance managers, system designers etc., for the proper maintenance, optimal maintenance policy, performance evaluation and safety aspect of the system. The approaches used in the paper provide a useful tool for practical reliability analysis of two non-identical units' parallel system under prior inspection before maintenance.

A fast way to develop a more realistic model would be to assume three modes like a normal, partial failure and total failure. Another improvement would be to include the warranty in repair that will decrease the maintenance cost of the system. The introduction of the controller in both the units would also make the model more realistic. Finally, the introduction of uncertainty in the model parameters would also give more accurate results.

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