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Reliability Model of Fuzzy Consecutive k-out-of-n: F System

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Abstract

Redundancy is used to increase the reliability of different systems. The most common technique of redundancy is k-out-of-n structure which finds wide applications in industrial and military systems. consecutive k-out-of-n: F system C (k,n:F) is a special kind of k-out-of-n: F systems proposed in the design of integrated circuits, microwave relay stations in telecommunication systems, oil pipelines systems, vacuum systems in accelerators, computer ring networks, and spacecraft relay stations.

In this paper, we discuss a new algorithm for evaluating the fuzzy reliability of a fuzzy linear consecutive k-out-of-n: F system (Lin/C (k,n:F)) with independent, un-repairable, and non-identical components. Later, we introduce a model of un-repairable system consists of number of parallel subsystems when each subsystem is represented by Lin/C (k,n: F).we assume the failure time of any operating component in this system follows Rayleigh distribution with one parameter. Due to uncertainty and insufficient data, this parameter is fuzzy number with triangular shaped membership function estimated by using statistical data taken from random sample of each component. Furthermore, a numerical example is given for a fuzzy un-repairable parallel system with three subsystems when the failure time of any component in the system follows fuzzy Rayleigh distribution to show how to obtain analytically and represent graphically the fuzzy reliability function of the system.

Keywords: Consecutive k-out-of-n: F system, Rayleigh distribution, Fuzzy parameters, Reliability, Linear system

Nomenclature

$R_l^{(P)}(t)$	is the reliability of component l *in subsystem P at time* t,
$S_{b,j}^{i}$	is the b^{th} failed component that belonging to state j with i failed components and $(n-i)$ working components,
$S_{g,j}^{i}$	is the g^{th} working component that belonging to state j with i failed components and $(n-i)$ working components,
M	is the maximum number of failures that a Lin/C (k_P, n_P: F) subsystem may experience without failure,
$N(z, k_P, n_P)$	is the number of ways allocating z failed components such that at most $(k_P - 1)$ failed components are consecutive,
$\binom{n_P}{z}$	is the number of combinations of z components out of possible n_P,
$\left\lfloor \frac{z}{k_P} \right\rfloor$	is the smallest integer less than or equal to $\frac{z}{k_P}$.

Introduction

In the past decades, many articles concerning the modeling of reliability/availability of k-out-of-n systems have been published. Most of them introduced different algorithms to evaluate reliability/availability of the system when the system units have identical and independent life times with arbitrary distribution as in Arulmozhi [1]. While Parallel and series systems are special kinds of k-out-of-n: F system. Series system is equivalent to 1-out-of-n: F system, the parallel system is equivalent to n-out-of-n: F system.

There is a special type of k-out-of-n systems called consecutive k-out-of-n: F system $\mathrm{C(k, n{:}F)}$ which have been proposed for modeling many important and critical systems as integrated circuits, microwave relay stations in telecommunication systems, oil pipelines, vacuum systems in accelerators, computer ring networks, and spacecraft relay stations. Consecutive k-out-of-n: F system $\mathrm{C(k, n{:}F)}$ is an ordered sequence of n components that fails if and only if at least k consecutive components fail. Some researchers studied this system and presented different methods for computing its reliability in the cases of identical or non-identical components as Chao [2], Cluzeau [3], and Lin [4]. Components in this system can be arranged logically or physically in an open line $(\mathrm{Lin/C(k, n{:}F)})$ or in a circle $(\mathrm{Cir/C(k, n{:}F)})$ to perform a specific function. Different formulas were proposed to determine their reliability functions, see [5, 6, 7, 8]. Recently due to uncertainty and insufficient failure data in most systems, it was found that it is inadequate to use the conventional probability methods for analyzing the different reliability models. For this reason, fuzzy set, introduced firstly by Zadeh [9], was used to represent uncertain or unknown parameters in these models. Researchers as [10, 11, 12] introduced the fuzzy consecutive k-out-of-n: F system and obtained its fuzzy reliability. Also, Mon [13] analyzed this system by using the concept of confidence interval but Chiang [14] used fuzzy number arithmetic operations.

In this paper, we will propose a model of un-repairable system with number of parallel branches. Each branch is represented by $\mathrm{Lin/C(k, n{:}F)}$ with independent and non-identical components. In **section 2**, we will propose the generalized expression used to evaluate the

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reliability of consecutive k-out-of-n:F system which was provided before by Barlow [15]. We will assume the failure time of any component follows the Rayleigh distribution with unknown parameter. Method for estimating this unknown parameter, by using the concept of $(1-\gamma)100\%$ confidence interval and the statistical data taken from random samples of this component, will be given in **Section 3**. Next, in **Section 4**, we will improve the expression mentioned in section 2 to evaluate the fuzzy reliability of fuzzy consecutive k-out-of-n:F system. Finally, in **Section 5** a numerical example will be given to illustrate the methodology of computing the fuzzy reliability of fuzzy consecutive k-out-of-n:F system.

The Model Description and Reliability

We will analyze a model for a system consisting of three parallel subsystems. Each subsystem, denoted by P; $\mathrm{P}=1,2,3$, has $\mathrm{n_P}$ components. Subsystem P works if and only if there are at least k_p consecutive components from n_P components work so it can be represented by Lin/C(k_P,n_P:F) as shown in Fig.1. First, we will give some assumptions for our model as follow:

1. Components are not identical but their failure times follow the same distribution.
2. Each Component takes only two possible states (the failure state or the working state).
3. Initially (at time $t=0$), all components are working and all are new.
4. There is no repair for failed components.

Depending on the previous assumptions, the system reliability can be obtained by:

$$R_{system}(t)=1-\prod_{P=1}^{3}\left[1-R_{line}^{P}(t;k_P,n_P)\right] \tag{1}$$

Where, $R_{line}^{P}(t;k_P,n_P)$ is the reliability of subsystem P.

There are various methods for evaluating $R_{line}^{P}(t;k_P,n_P)$ or the reliability of Lin/C(k_P,n_P:F). We will use the general method provided by Barlow [15] as it did not depend on the failure distribution of its components and is given by

$$R_{line}^{P}(t;k_P,n_P)=\prod_{l=1}^{n_P}R_l^{(P)}(t)+\sum_{i=0}^{k_P-1}\sum_{j=r}^{m}\left[\prod_{l\in S_{b,j}^{i}}^{i}\left(1-R_l^{(P)}(t)\right)\right]\left[\prod_{l\in S_{g,j}^{i}}^{n_P-i}R_l^{(P)}(t)\right]+\sum_{i=k_P}^{M}\sum_{j=\grave{r}}^{\grave{m}}\left[\prod_{l\in S_{b,j}^{i}}^{i}\left(1-R_l^{(P)}(t)\right)\right]\left[\prod_{l\in S_{g,j}^{i}}^{n_P-i}R_l^{(P)}(t)\right] \tag{2}$$

Where,

$$r=\sum_{z=0}^{i-1}\binom{n_P}{z},\ m=\sum_{z=1}^{i}\binom{n_P}{z},\ \grave{r}=\sum_{z=0}^{i-1}N(z,k_P,n_P),\ \grave{m}=\sum_{z=1}^{i}N(z,k_P,n_P)$$

$$\mathrm{N}(\mathrm{z},\mathrm{k_P},\mathrm{n_P})=\begin{cases}0\ , & \mathrm{z}>M\\ \binom{\mathrm{n_P}}{\mathrm{z}}\ , & 0<z<k_P-1\\ \sum_{w=0}^{\left\lfloor\frac{z}{k_P}\right\rfloor}(-1)^{w}\binom{n_P-z+1}{w}\binom{n_P-wk_P}{n_P-z}\ , & k_P<z<n_P\end{cases} \tag{3}$$

and $M=\begin{cases}\mathrm{n_P}-\left[\frac{\mathrm{n_P}+1}{\mathrm{k_P}}\right]+1 & \text{if } (\mathrm{n_P}+1) \text{ is integer}\\ \mathrm{n_P}-\left[\frac{\mathrm{n_P}+1}{\mathrm{k_P}}\right] & otherwise\end{cases}$

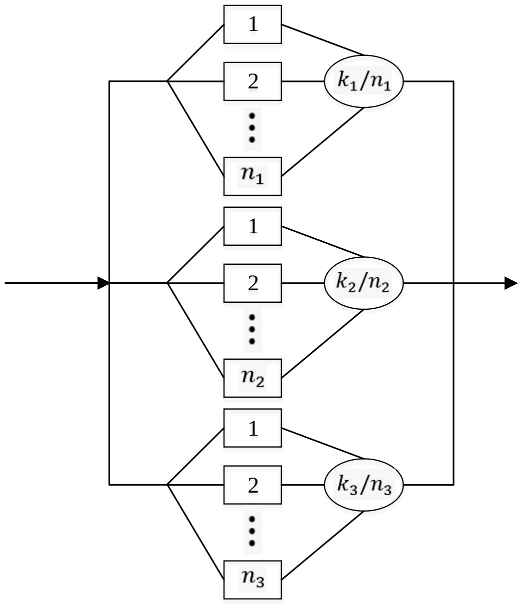

Fig.1: Parallel system consists of Lin/C(k_P,n_P:F) branches

The Fuzzy Reliability of Each Component

For the components wear rapidly with time, Rayleigh distribution provides lifetime model with linearly increasing failure rate. In the proposed model, we consider the failure time of any component l in subsystem P is a random variable following Rayleigh distribution with parameter θ_l^P so its reliability is given by

$$R_l^{(P)}(t)=\exp(-t^2/2(\theta_l^P)^2);\quad P=1,2,3 \text{ and } l=1,2,\dots,n_P \tag{4}$$

In real models, the value of the lifetime distribution's parameters is unknown due to imprecision of the lifetime data. For this reason, we can here estimate $(1-\gamma)100\%$ confidence interval of the Rayleigh distribution's parameter θ_l^P, by using random sample taken from component l with size q; $q\geq 30$ and observed lifetime data X_i, see [16], as follows:

$$\hat{\theta}_l^P-Z_{\frac{\gamma}{2}}\sqrt{var\left(\theta_l^{P^M}\right)}\leq\theta_l^P\leq\hat{\theta}_l^P+Z_{\frac{\gamma}{2}}\sqrt{var\left(\theta_l^{P^M}\right)}\ ;\ P=1,2,3,\ l=1,2,\dots,n_P \tag{5}$$

Where, $\hat{\theta}_l^P$ is the point estimation of the parameter θ_l^P which equals $\sqrt{\frac{\sum_{i=1}^{q}X_i^2}{2q}}$, $var\left(\theta_l^{P^M}\right)$ is the variance which equals $\frac{\left(\theta_l^{P^M}\right)^2}{4q}$, and γ is the level of significance; $0\leq\gamma\leq 1$

$$\tilde{\theta}_l^P[\alpha]=\left[\tilde{\theta}_l^{P^L}(\alpha),\tilde{\theta}_l^{P^U}(\alpha)\right]=\left[\theta_l^{P^L}+\alpha\left(\theta_l^{P^M}-\theta_l^{P^L}\right),\ \theta_l^{P^U}-\alpha\left(\theta_l^{P^U}-\theta_l^{P^M}\right)\right];\ \alpha\in[0,1] \tag{6}$$

The Fuzzy Reliability of the System with Fuzzy Parameters

We can obtain the fuzzy reliability function of our system by the following steps:

Step.1: construct the confidence intervals of the parameters $\theta_l^P;\ P = 1, 2, 3,\ l = 1, 2, \dots, n_P$ by relation (5) which can be converted to triangular fuzzy number $\tilde{\theta}_l^P$.

Step.2: determine the corresponding crisp intervals ($\alpha - cut$) for $\tilde{\theta}_l^P$ as given in relation (6) at certain values of $\alpha;\ \alpha \in [0, 1]$.

Step.3: get α-*cut* of the fuzzy reliability $\tilde{R}_l^{(P)}(t)$ of each component $l;\ l = 1, 2, \dots, n_P$ in subsystem $P;\ P = 1, 2, 3$ as follow:

$$\tilde{R}_l^{(P)}[t, \alpha] = \left[\tilde{R}_l^{(P)^L}(t, \alpha), \tilde{R}_l^{(P)^M}(t, \alpha)\right];$$
$$\tilde{R}_l^{(P)^L}(t, \alpha) = \exp\left[-t^2/2\left(\theta_l^{P^U} - \alpha\left(\theta_l^{P^U} - \theta_l^{P^M}\right)\right)^2\right] \tag{7-1}$$

$$\tilde{R}_l^{(P)^U}(t, \alpha) = \exp\left[-t^2/2\left(\theta_l^{P^L} + \alpha\left(\theta_l^{P^M} - \theta_l^{P^L}\right)\right)^2\right] \tag{7-2}$$

Step.4: get α-*cut* of the fuzzy reliability $\tilde{R}_{line}^P(t)$ of each subsystem $P;\ P = 1,2,3$ which is Lin/C (k_P, n_P:F) as follow:

$$\tilde{R}_{line}^P[t; k_P, n_P, \alpha] = \left[\tilde{R}_{line}^{P\ L}(t; k_P, n_P)(\alpha), \tilde{R}_{line}^{P\ U}(t; k_P, n_P)(\alpha)\right]$$
$$\tilde{R}_{line}^{P\ L}(t; k_P, n_P, \alpha) = \prod_{l=1}^{n_P} \tilde{R}_l^{(P)^L}(t, \alpha) + \sum_{i=0}^{k_P-1} \sum_{j=r}^{m} \left[\prod_{l \in S_{b,j}^i}^{i} \left(1 - \tilde{R}_l^{(P)^L}(t, \alpha)\right)\right]\left[\prod_{l \in S_{g,j}^i}^{n_P - i} \tilde{R}_l^{(P)^L}(t, \alpha)\right] + \sum_{i=k_P}^{M} \sum_{j=\grave{r}}^{\grave{m}} \left[\prod_{l \in S_{b,j}^i}^{i} \left(1 - \tilde{R}_l^{(P)^L}(t, \alpha)\right)\right]\left[\prod_{l \in S_{g,j}^i}^{n_P - i} \tilde{R}_l^{(P)^L}(t, \alpha)\right] \tag{8-1}$$

$$\tilde{R}_{line}^{P\ U}(t; k_P, n_P, \alpha) = \prod_{l=1}^{n_P} \tilde{R}_l^{(P)^U}(t, \alpha) + \sum_{i=0}^{k_P-1} \sum_{j=r}^{m} \left[\prod_{l \in S_{b,j}^i}^{i} \left(1 - \tilde{R}_l^{(P)^U}(t, \alpha)\right)\right]\left[\prod_{l \in S_{g,j}^i}^{n_P - i} \tilde{R}_l^{(P)^U}(t, \alpha)\right] + \sum_{i=k_P}^{M} \sum_{j=\grave{r}}^{\grave{m}} \left[\prod_{l \in S_{b,j}^i}^{i} \left(1 - \tilde{R}_l^{(P)^U}(t, \alpha)\right)\right]\left[\prod_{l \in S_{g,j}^i}^{n_P - i} \tilde{R}_l^{(P)^U}(t, \alpha)\right] \tag{8-2}$$

Step.5: get α-*cut* of the fuzzy reliability $\tilde{R}_{system}(t)$ of our parallel system as follow:

$$\tilde{R}_{system}[t, \alpha] = \left[\tilde{R}_{system}{}^L(t, \alpha), \tilde{R}_{system}{}^U(t, \alpha)\right];$$
$$\tilde{R}_{system}{}^L(t, \alpha) = 1 - \prod_{P=1}^{3}\left[1 - \tilde{R}_{line}^{P\ L}(t; k_P, n_P, \alpha)\right] \tag{9-1}$$

$$\tilde{R}_{system}{}^U(t, \alpha) = 1 - \prod_{P=1}^{3}\left[1 - \tilde{R}_{line}^{P\ U}(t; k_P, n_P, \alpha)\right] \tag{9-2}$$

Numerical Example

Take a model for a system consists of three parallel subsystems. The first subsystem is represented by $Lin/C(k_1, n_1{:}\,F)$, the second is $Lin/C\,(k_2, n_2{:}\,F)$, and the third is $Lin/C(k_3, n_3{:}\,F)$; $k_1 = 2, n_1 = 4, k_2 = 2, n_2 = 3, k_3 = 2, n_3 = 3$. The lifetime of each component $l; l = 1,2, \dots, n_P$ in a subsystem $P; P = 1, 2, 3$ follows Rayleigh distribution with fuzzy parameter $\tilde{\theta}_l^P$.

To validate the performance of our model, we can get the fuzzy reliability function of this system by using the steps determined in the previous section with the aid of samples' statistical data, shown in Table 1, taken from random samples of each component.

Table 1: The statistical data used to evaluate the fuzzy parameter $\tilde{\theta}_l^P$ for each component l in subsystem P

Subsystem number "P"	Component number "l"	Parameter	γ	q	$\sum_{i=1}^{q} X_i^2$	$\theta_l^{P^M}$
1	1	θ_1^1	0.03	50	700	2.65
	2	θ_2^1	0.07	50	750	2.7386
	3	θ_3^1	0.045	55	880	2.83
	4	θ_4^1	0.03	50	700	2.65
2	1	θ_1^2	0.045	45	750	2.887
	2	θ_2^2	0.04	70	1250	2.988
	3	θ_3^2	0.07	70	1250	2.988
3	1	θ_1^3	0.075	48	750	2.795
	2	θ_2^3	0.05	50	800	2.83
	3	θ_3^3	0.06	80	1400	2.958

According to relation (5), we can calculate the crisp intervals ($\alpha - \text{cut}$) for fuzzy parameters in our model at distinct values of α's; $\alpha \in [0, 1]$. After that determine the system fuzzy reliability by substituting in equations (7) and (8) then (9).

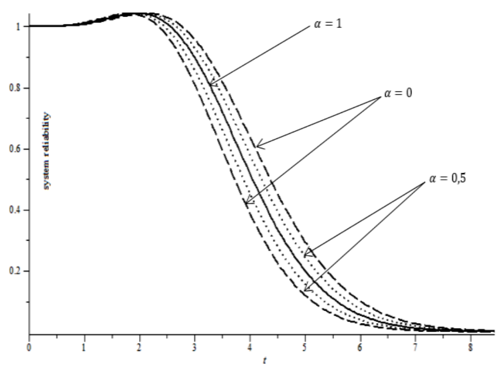

Fig. 2: The system fuzzy reliability versus the time

As shown in Fig.2, we can represent graphically the system fuzzy reliability function $\widetilde{\mathrm{R}}_{\mathrm{system}}[\mathrm{t}, \alpha]$ versus the time at $\alpha - \mathrm{cut} = 0, 0.5, \text{and } 1$ by using MAPLE program and represent the reliability at instant value of time $\mathrm{t} = 1.5$ as shown in Fig.3 which can be approximated to be a triangular shaped fuzzy number.

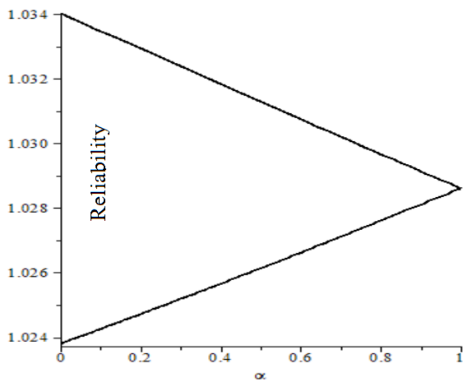

Fig. 3: The system fuzzy reliability at time $t = 1.5$

Conclusion

In this paper, we proposed a model of un-repairable parallel system consisting of three branches. Each branch is a linear consecutive k-out-of-n:F system with independent but not identical components. We assumed the lifetime of each component follows the Rayleigh distribution with one fuzzy parameter represented by triangular shaped membership function. The membership function can be estimated from statistical data obtained from random samples. We modify Barlow algorithm for evaluating the fuzzy reliability of each branch then we easily obtain the fuzzy reliability of the system by using $\alpha - \mathrm{cut}$ technique. We also draw the results by using MAPLE software program.

According to the results, we found that for each value of α, the fuzzy reliability function wasn't one curve as usual but each had infinite number of curves around a main curve in the crisp case evaluated at α=1 and all curves were bounded by the lower and upper limit curves at α= 0. In addition, the reliability can be approximated to be triangular shaped fuzzy number at specific value of time t.

This model can be applied in various industrial applications. As an extension to this work, we can develop other complex systems consisting of linear consecutive k-out-of-n:F systems with different lifetime distributions with fuzzy parameters.

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