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Fault Tolerant Guidance of Under-Actuated Satellite Formation Flying Using Inter-Vehicle Coulomb Force

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Abstract

In this study, satellite formation flying guidance in the presence of under actuation using inter-vehicle Coulomb force is investigated. The Coulomb forces are used to stabilize the formation flying mission. For this purpose, the charge of satellites is determined to create appropriate attraction and repulsion and also, to maintain the distance between satellites. Static Coulomb formation of satellites equations including three satellites in triangular form was developed. Furthermore, the charge value of the Coulomb propulsion system required for such formation was obtained. Considering Under actuation of one of the formation satellites, the fault-tolerance approach is proposed for achieving mission goals. Following this approach, in the first step fault-tolerant guidance law is designed. Accordingly, the obtained results show stationary formation. In the next step, to maintain the formation shape and dimension, a fault-tolerant control law is designed.

Keywords: Formation flying, Guidance law, Under actuation, Coulomb force.

Introduction

Satellites formation flying refers to satellites that can work together in a group and couple through a common control law. Since 1970s, studies on the application of several spacecraft in space missions or spacecraft formation flying, have been carried out. Recently, formation flights have become a significant challenging technology for future missions of NASA, ESA, and other space agencies. Guidance of a formation flying means producing any reference path, as an input for the relative state of a formation member, which follows the control law. Hill equations are often used for formation flight modeling [1-4].

Studies carried out on formation flying, Wolf and De Ridder (1993) identified the worst configuration of faulty satellites in a system and measured the disruption of the resulting coverage. In this research, it was found that the worst faulty layout occurs when the faulty satellites are adjacent [5]. Mishne and Ltd (2006) discussed the reconfiguration of a satellite formation maneuver in the presence of measurement errors. In this research, an optimal guidance law was presented for satellite maneuvering with specific final conditions [6]. DeWecka and Seddiqi (2007) investigated the optimization of a satellite system reconfiguration. This study reveals how a low-capacity system, in an optimal route, turns into a high-capacity system. The optimization criterion is ΔV minimization of the total transfer for reconfiguration [7]. Nunes (2012) proposed a method based on a genetic algorithm to optimize satellite complex and used another method for revealing different designs of the ground measurement system and its optimization [8]. Mushet et al. (2015) presented a new solution to the problem of allocating autonomous work for a self-organizing satellite system in the Earth's orbit. This method allows satellites to accumulate themselves above targets on the surface of the earth [9]. In the study conducted by Fakour et al. (2016), two major issues related to the relative motion of satellites in system for inter-satellite linking (ISL)were investigated; i.e., dynamic and control issues [10].

The Coulomb law was first introduced in 1979 at the Scatha Project, an American telecommunication satellite. In that project, active control of spacecraft potential was tested by emitting the charge using an electron beam. Afterward, NASA released a report in 2002 on the use of Coulomb inter-satellite forces in the formation flights. The idea behind this report is to control the spacecraft potential in the orbit by interacting with the surrounding space plasma environment. The comparison between Coulomb formations and achievements of traditional electric propulsion thrusters shows that significant savings (up to 90%) are possible on the mass of the propulsion system by the means of Coulomb control for

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all the studied formations [11]. Schaub et al. (2013) explored the challenges and prospects of the development of spacecraft formations using Coulomb forces. Coulomb forces allow the relative movement of satellites to be controlled without contamination. In this control, high fuel efficiency allows setting out high power missions [12]. Li et al. (2004) conducted a study on the formation flying of small satellites using the Coulomb forces. They examined the feasibility of achieving an optimum formation smaller than 100 m using Coulomb forces [13]. Berryman and Schaub (2007) conducted a study on charge analysis for Coulomb formations consisting of two or three satellites. This paper provides an analytical study of static Coulomb formations and summarizes a general method for calculating the product of the spacecraft required a charge for the formation of N spacecraft [14]. Ting WANG et al. (2016) analyze the co-linear formation of one-dimension, the planar formation of two-dimension and hexahedral formation of three-dimension for equal mass five-craft Coulomb formation flying. The authors discuss the conditions of an analytical solution to determine the net charges of craft in each case of static formation [15]. Rui Qi et al. (2018) proposes the Coulomb tether double-pyramid satellite formation, which includes a cluster of charged satellites repelling each other by Debye-shielded Coulomb forces and two counterweights connecting to each satellite via elastic massless tethers [16]. Emitting a negative charge in Coulomb formation satellites is accomplished with an electron gun, which usually provides up to 50 keV of emission energy corresponding to an acceleration voltage of up to 50 kV [17].

Modern systems require sophisticated controllers to meet the increasing functionality and safety demands. The presence of a fault in the actuators, sensors, or other components of the system, the conventional feedback controller may lead to undesirable performance or even instability in the system. To overcome such weaknesses and maintain functionality and stability characteristics, simultaneously, new ways for control systems design have been developed. It is crucial for sensitive systems in which safety is important, such as aircraft, spacecraft, and nuclear power plants. In these systems, the consequences due to minor errors in one part of the system are costly and, as a result, sensitivity to the reliability, safety, and fault/failure tolerance is generally high [18]. Many factors contribute to the collapse of the spacecraft attitude control system, with actuation systems and thruster systems having a large share [19]. The control of the under-actuated mechanical systems leads to a feedback control law that stabilizes the system in the presence of different uncertainties and external disturbances [20].

This research aims at maintaining the static formation of flight and the formation form. For this reason, after modeling, the motion of satellites is controlled by a proportional-derivative controller so that the relative distance between the satellites regulated and remains constant over the desired value. The important challenge studied in this research is the under actuation of a formation flight member. The approach adopted to face this challenge are to create fault-tolerance in the system. In the first step, a fault-tolerant guidance law is designed. After applying the proposed guidance law, most of the objectives are met; however, due to limitations, some part of the mission objectives is not fulfilled. For this reason, in the next step, the design of passive fault-tolerant control law is carried out. The results of concurrent application of the fault-tolerant control and guidance indicate that all mission objectives are met within acceptable limits of error.

Coulomb Control Concept

This concept follows the principle of Coulomb attraction/repulsion between charged bodies to control the spacing between nodes of a microsatellite cluster. The Coulomb control principle is easily conveyed by examining the interaction between two neighboring bodies capable of transferring electric charge.

For instance, two vehicles separated a distance d in space. Initially, both spacecraft are electrically neutral, i.e., the amount of negative charge (electrons) is equal to the amount of positive charge producing a net vehicle charge of zero and no interaction between the craft. Now, allow one craft to change its charge state through the emission of electrons. This is a trivial process utilizing an electron-gun or similar cathode device. If the electron beam is used to transfer an amount of negative charge, q_{sc} from spacecraft 1 (SC1) to spacecraft 2 (SC2), the net negative charge of SC2 willequal the net positive charge remaining on SC1, producing an attractive force between the spacecraft given by [11]:

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_{SC}^2}{d^2} \tag{1}$$

The Relations Describing the Coulomb Formations

In Coulomb formulations, $r_i$shows the initial position of $i$thspacecraft in a formation and vector $r_c$represents the initial position of the mass center of a spacecraft formation. Therefore, the position of the spacecraft relative to the center of mass is defined as $\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{r}_c$and, similarly, the position of the spacecraft i relative to spacecraft j defined as $\boldsymbol{\rho}_{ji} = \boldsymbol{\rho}_i - \boldsymbol{\rho}_j$. The hill frame is defined as follows: $H{:}\{O, \hat{\boldsymbol{o}}_r, \hat{\boldsymbol{o}}_\theta, \hat{\boldsymbol{o}}_h\}$.

Where $\hat{\boldsymbol{o}}_r$denotes the radial direction of the center of the earth to the outside, $\hat{\boldsymbol{o}}_h$the perpendicular direction to the orbit and $\hat{\boldsymbol{o}}_\theta$complete trinary, so that we have$\hat{\boldsymbol{o}}_\theta = \hat{\boldsymbol{o}}_h \times \hat{\boldsymbol{o}}_r$. For the unique circular motion of the center of mass that is considered in this research, $\hat{\boldsymbol{o}}_h$ is always the speed direction of the formation mass center.

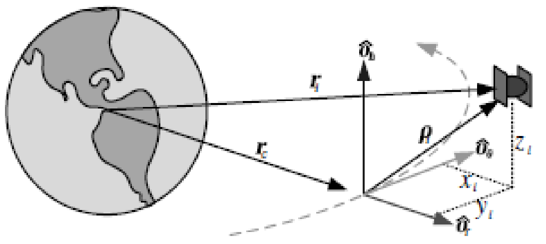

Fig. 1. An image of the rotating Hill coordinate frame [14]

As shown in Fig. 1, the ith relative position vector in the Hill-frame vector components is expressed as follows:

$$\boldsymbol{\rho}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \quad (1)$$

The formation spacecraft is assumed to be a charge point. Now, if each spacecraft has a q charge, the Coulomb interactions between the ith and the jth spacecraft are proportional to the product of their charge and the inverse of their squared separation distance. If the charges have the same sign, interactions are repulsive; otherwise, they are attractive. The motion equation for a Coulomb formation can be obtained by placing the Coulomb acceleration to the right of the Hill equations [14].

$$\ddot{x}_i - 2n\dot{y}_i - 3n^2 x_i = \frac{k_c}{m_i}\sum_{j=1}^{N} \frac{x_i - x_j}{\rho_{ji}^3} q_i q_j e^{-\frac{\rho_{ij}}{\lambda_d}} \quad (2)$$

$$\ddot{y}_i + 2n\dot{x}_i = \frac{k_c}{m_i}\sum_{j=1}^{N} \frac{y_i - y_j}{\rho_{ji}^3} q_i q_j e^{-\frac{\rho_{ij}}{\lambda_d}} \quad (3)$$

$$\ddot{z}_i + n^2 z_i = \frac{k_c}{m_i}\sum_{j=1}^{N} \frac{z_i - z_j}{\rho_{ji}^3} q_i q_j e^{-\frac{\rho_{ij}}{\lambda_d}} \quad (4)$$

where n refers to the mean orbit rate of motion of center of mass, kc denotes the Coulomb's constant (8.99×10^9 Nm^2/c^2), m_i is i satellite mass, and q_i is i satellite charge. It has to be noted that the orbital motion is linearized, while the Coulomb force remains in the nonlinear form. The exponential sentence on the right hand of Eqs. (2) to (4) dictates the rate in which the effect of the Coulomb decreases with increasing distance in a plasma environment. This decrease is a function of the Debye length of $\lambda_d$and is due to the protective effect created around a charged object at the plasma environment in the floating time. Therefore, in the analysis presented here, the formations for the GEO orbit, consisting of spacecraft with a separation distance of 10mare considered. In such formations, the exponential expression of the equation can be ignored.

To maintain the spacecraft stationary relative to the Hill frame, all derivatives in Eqs. (2) to (4) must be always zero. This relative equilibrium configuration is achieved by positioning and charging the spacecraft accurately in a way that the Hill frame accelerations are neutralized by the accelerations of the Coulomb force. These equilibriums are unstable without feedback. The determination of the relative equilibrium responses of Eqs. (2) to (4) with constant charge responses is not obvious and then can be neglected. A static Coulomb formation consisting of N spacecraft should satisfy the relative equilibrium constraints of 3N in Eqs. (2) to (4). Each spacecraft can select three degrees of freedom of (x_i, y_i, z_i) position, and q_i charge to neutralize the Hill frame accelerations.

To reduce the complexity of the analysis, the equations of motion are scaled according to the orbital rates of n and k_c. This scaling is done by introducing the scaled-charge of$\tilde{q} \equiv q\sqrt{k_c}/n$. By substituting this new variable, excluding the exponential expression, and zeroing all the derivatives, the static equations for a Coulomb formation are written as follows:

$$m_i \frac{\ddot{x}_i}{n^2} = 3x_i m_i + \sum_{j=1}^{N} \frac{x_i - x_j}{\rho_{ji}^3} \tilde{q}_i \tilde{q}_j = 0 \quad (5)$$

$$m_i \frac{\ddot{y}_i}{n^2} = \sum_{j=1}^{N} \frac{y_i - y_j}{\rho_{ji}^3} \tilde{q}_i \tilde{q}_j = 0 \quad (6)$$

$$m_i \frac{\ddot{z}_i}{n^2} = -z_i m_i + \sum_{j=1}^{N} \frac{z_i - z_j}{\rho_{ji}^3} \tilde{q}_i \tilde{q}_j = 0 \quad (7)$$

If these conditions are met for a Coulomb formation, the Hill frame accelerations will exactly match the Coulomb accelerations and the spacecraft will remain constant relative to the Hill frame. With regard to Eqs. (5) to (7), one can find that the equations are linear relative to the product of charges ($\tilde{q}_i\tilde{q}_j$). This fact is important in analytical point of view. So, the product of charges (Q_{ij}) is given as follows:

$$Q_{ij} \equiv \tilde{q}_i \tilde{q}_j \quad (8)$$

A static formation of the Hill frame must meet two necessary conditions. The first condition is that the center of the mass of the formation should be located at the origin of the Hill frame and the second condition is that the main inertial axes of the formation should be aligned with the axes of the Hill frame [14].

For the relative distances to remain constant, the controller is applied to the above formation. Regarding the behavior of the system, a proportional-derivative controller is considered to converge answers to the optimum range. Due to the oscillations observed, the derivative part is also added to the controller. Since the behavior of the system does not have a permanent error, there is no need for an integrator controller. So, here, a proportional controller regulates the relative distance between satellites to reach the optimum value. This controller is designed according to the relationship between relative distances and the product of charges. An overview of the relationship between the system and the controller is presented in Fig. 2. Since, in such formations, the goal is to maintain the distance between the satellites such that the optimum distance is the same as the initial distance. The applied control law is as follows:

$$u = k_0[k_1(\rho_d - \rho) + k_2(\dot{\rho}_d - \dot{\rho})] \quad (9)$$

Where the coefficient $k_0$is the ratio of relative distances to the product of charges and k_1 and k_2 are proportional and derivative gains of the controller, respectively.

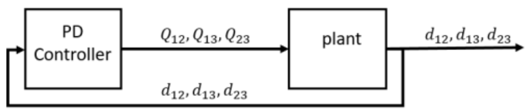

Fig. 2. Block diagram of the system and the controller in the aligned formation

Equations of Formations Consisting of Three Spacecraft

An arbitrary triangular formation consisting of three spacecraft should satisfy the conditions of the equations. According to this formation, each spacecraft should stay motionless in$\hat{\boldsymbol{o}}_r$,$\hat{\boldsymbol{o}}_\theta$, and$\hat{\boldsymbol{o}}_h$ directions (Fig. 1). These equations are reduced to six equations by verifying that a formation consisting of three spacecraft are essentially planar and one main axis of each planar formation is perpendicular to the formation plane. So, to satisfy the main axis condition, three spacecraft should be placed on one of the three planes that are perpendicular to the axes of the Hill frame. The spacecraft of the formation should be placed on a plane that is formed by two axes of the Hill frame. So, in equations, it is not necessary to consider the movement perpendicular to this plane. So, the position of each spacecraft is determined with two values: the distance $x_i$in the direction $\hat{o}_1$, and the distance $z_i$in the direction $\hat{o}_2$, where $\hat{o}_1$ and $\hat{o}_2$ can be defined as any combination of directions$\hat{\boldsymbol{o}}_r$, $\hat{\boldsymbol{o}}_\theta$, and $\hat{\boldsymbol{o}}_h$. The rest of the acceleration equations that must be met for static formations are as follows:

$$m_i \frac{\ddot{x}_1}{n^2} = -3m_1x_1 + \frac{x_1-x_2}{\rho_{12}^3}Q_{12} + \frac{x_1-x_3}{\rho_{13}^3}Q_{13} = 0 \quad (10)$$

$$m_i \frac{\ddot{x}_2}{n^2} = -3m_2x_2 + \frac{x_2-x_1}{\rho_{12}^3}Q_{12} + \frac{x_2-x_3}{\rho_{23}^3}Q_{23} = 0 \quad (11)$$

$$m_i \frac{\ddot{x}_3}{n^2} = -3m_3x_3 + \frac{x_3-x_1}{\rho_{13}^3}Q_{13} + \frac{x_3-x_2}{\rho_{23}^3}Q_{23} = 0 \quad (12)$$

$$m_i \frac{\ddot{z}_1}{n^2} = m_1z_1 + \frac{z_1-z_2}{\rho_{12}^3}Q_{12} + \frac{z_1-z_3}{\rho_{13}^3}Q_{13} = 0 \quad (13)$$

$$m_i \frac{\ddot{z}_2}{n^2} = m_2z_2 + \frac{z_2-z_1}{\rho_{12}^3}Q_{12} + \frac{z_2-z_3}{\rho_{23}^3}Q_{23} = 0 \quad (14)$$

$$m_i \frac{\ddot{z}_3}{n^2} = m_3z_3 + \frac{z_3-z_1}{\rho_{13}^3}Q_{13} + \frac{z_3-z_2}{\rho_{23}^3}Q_{23} = 0 \quad (15)$$

These equations do not include first derivatives because the first derivatives for a static formation are always set to zero.

The Simulation and Verification of Three Satellites in the Triangular Mode

In this section it is assumed that the satellites with the equal mass of 1 kg are located in arbitrary positions according to table 1.

Table 1. Initial Position of the satellites

satellite	x(m)	y(m)	z(m)
1	-7	0	-7
2	2	0	10
3	5	0	-3

Based on obtained results, there are three satellites that are located in the plane composed of x-axis and z-axis. The aim of this section is to apply guidance and control algorithms for satellites to reach desired positions and to form the desired isosceles triangle formation by adjusting their relative distances. Fig. 3 shows the desired satellite positions.

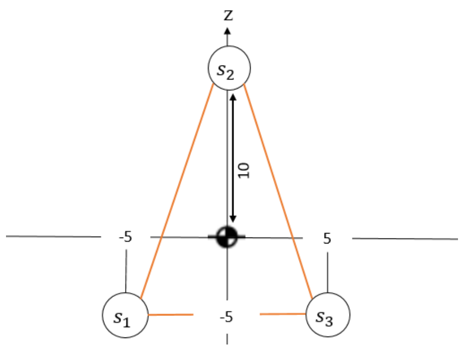

Fig. 3. Position of satellites in the desired isosceles triangle formation

Desired distances between satellites are equal to$\rho_{12} = 10$, $\rho_{13} = 15.81$, and $\rho_{12} = 10$. Using Eqs. (10), (11), and (13), the product of the charge required to form this formation is equal to Q_{12}=-1317.26, Q_{13}=1666.6, and Q_{23}=-1317.26. With the implementation of the equations and constraints expressed, the distance variations between the satellites can be illustrated as Fig. 4.

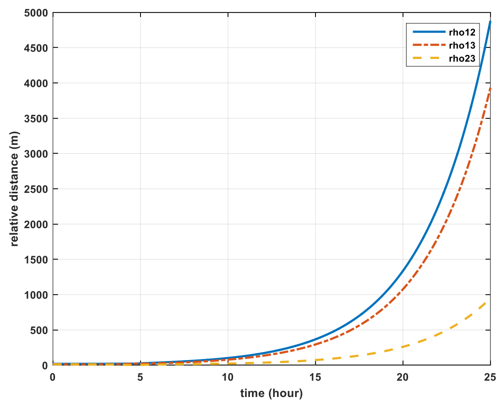

Fig. 4. Relative distances variations of satellites with no controller

As in the previous part, the proportional-derivative controller is used for the isosceles triangular formation. Fig. 5shows this result.

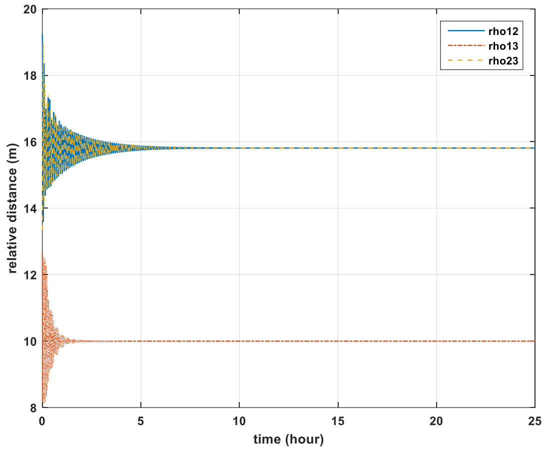

Fig. 5. Relative distances variations in the isosceles triangle formation with controller and correct gain

By using guidance and control algorithms, satellites reach to desired positions and formed the desired isosceles triangle formation by adjusting their relative distances. So it is verifying that the dynamic model of system and algorithms which used, are correctly applied.

Underactuation

An under-actuated mechanical system is a system in which the number of control inputs is less than the number of degree of freedom. Therefore, due to system Underactuation, failure of actuators that are responsible for producing independent force/torque for controlling output occurs. In this case, the conventional state feedback control law is not responsive, and the system is severely disrupted in terms of durability and performance.

In this article, the underactuation is considered in one of the satellites on the formation flying. The underactuation of this satellite occurs when the Coulomb actuator in the satellite – which generates the electrical charge required for interaction with other satellites – is faulty and unable to act properly. In fact, in this state, q is not produced correctly. q is one of the product parameters of $Q_{ij}=q_iq_j$ and Q_{ij} is the interaction factor between the satellites. Based on the equations of Coulomb formation flying motion (Eqs. 10-15), the motion of the satellites changes and the relative distance between the satellites increases. Referring to equation Qij=qiqj, zeroing the charge of one satellite causes zeroing Q_{ij} and, the interaction between the satellites is lost. Therefore, the underactuation of this system can be retrieved in a situation where the actuators charge is non-zero. Simulation of the above-mentioned conditions can be achieved by considering a formation consisting of 3 satellites in form of an isosceles triangle as studied in part 4 with the initial coordinates according to desired distances and a mass equal to 1 kg.

Now, according to the description given, it is assumed that the Coulomb actuator on satellite 2 of this triangular formation is defective. Therefore, producing of q_2 is reduced by 40%. So:

$$q_{2_{new}} = 0.6q_2 \quad (16)$$

This change in the product of charge (Q12 and Q23) also affects and prevents the production of Coulomb acceleration as much as necessary. Therefore, the attraction and repulsive forces produced are less than the amount that can hold the satellites in the initial distance. Fig. 6 shows the results of this fault.

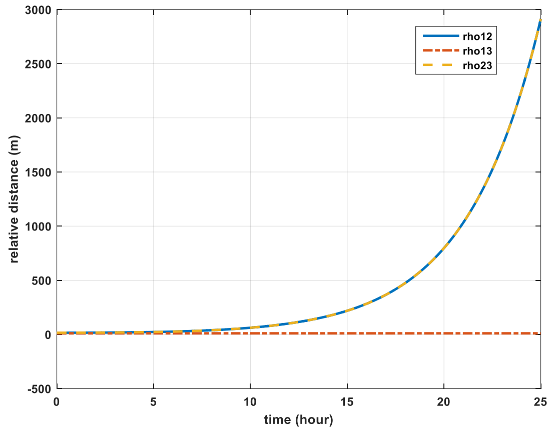

Fig. 6. relative distances of satellites in the underactuation system

As shown in the graph, because of the insufficient amount of force between the Coulomb satellites required for attraction, the distance between satellites 1 and 2 and the distance between satellites 2 and 3 increase with time and the formation is not in desired form anymore. In fact, satellite 2 is distanced from two other satellites and is separated from the formation.

The approach of creating fault tolerance in the underactuation system

In the face of the underactuation phenomenon and, in general, fault in one of the satellites of formation flying, the common solution is to remove the faulty satellite and replace a new satellite. So, in most formation flying, the number of satellites that are designed and manufactured is more than the number required for that formation; e.g., Iridium satellite constellation that has six spare satellites for 76 active satellites. Obviously, this approach involves a high cost. In this study, faulty satellites are not excluded, but the guidance and control laws are designed and applied to the formation members so that satellites can continue to fly and fulfill the mission despite the underactuation. The cost of providing a spare satellite is eliminated by using this approach.

In terms of fault tolerance creation, two basic functions can be distinguished for reconfiguration: Fault Tolerance Control (FTC) and Fault Tolerance Guidance (FTG).

FTC systems look for low-performance levels in faulty situations. In general, an FTC does not offer optimal performance for the system, but it can neutralize the effects of system failures. FTG can provide more flexibility for safe recovery in case of degraded flight conditions. In fact, processor planning capacities can be used to resume mission activities without ground intervention after a fault is detected and confirmed. The FTC and FTG provide strategies to prevent and suppress potentially dangerous, out-of-range or hazardous system behaviors [21].

In fact, introducing the FTC is an economic way for gaining reliability and safety in automatic systems. The FTC implementation strategy involves designing intelligent software that monitors the behavior of the components [22].

Designing fault tolerant guidance law

In this research, first, the fault tolerance guidance approach is proposed such that the guidance law is designed to meet the objectives of the mission. The two main objectives of the mission are to maintain the formation of flying stability and formation form. Maintaining the form actually means maintaining the geometric shape of the formation and the dimensions of the formation.

As stated above, to have a static formation, the following two constraints must be met:

1) The formation center of mass must be located at the origin of the Hill frame.
2) The principal axes of the formation must be aligned with the axes of the Hill frame.

In the triangular formation discussed in this study, Eqs. (17) to (19) ensure the fulfillment of constraints 1 and 2, respectively.

$$m_1x_1 + m_2x_2 + m_3x_3 = 0 \tag{17}$$

$$m_1z_1 + m_2z_2 + m_3z_3 = 0 \tag{18}$$

$$m_1x_1z_1 + m_2x_2z_2 + m_3x_3z_3 = 0 \tag{19}$$

So, to achieve the first goal of the mission, namely the formation flying stability, it is necessary to meet the statements proposed in Eqs. (17) to (19), which are a function of the mass and position of satellites.

In the next step, to achieve the second goal, i.e., maintaining the triangular form, a new statement is needed. As stated above, maintaining the form involves maintaining the geometric shape of the formation and the dimensions of the formation. Since in this simulation, the geometric shape of the formation is an isosceles triangle and the problem always tends to maintain this form, the side length of the triangles, in other words, the spacing between satellites 1 and 2, and 2 and 3 must always be equal. Accordingly, Eq. (20) is established as follows:

$$\sqrt{(d_2-d_1)^2+(e_2-e_1)^2} = \sqrt{(d_3-d_2)^2+(e_3-e_2)^2} \tag{20}$$

Maintaining the dimensions of the triangle means that the size of the sides of the triangle remains at a certain range. So Eq. (21) is written as:

$$\sqrt{(d_2-d_1)^2+(e_2-e_1)^2} = \sqrt{(d_3-d_2)^2+(e_3-e_2)^2} = r \tag{21}$$

Where is chosen according to the initial dimensions of the triangle and the desired range for the formation. In this simulation, the initial size of the sides of the isosceles triangles is 15.81 m and the optimum range for the spacing between satellites is 40 m. So, with this condition, the maximum value can be 40.

Now, by investigating the two main goals of the problem, four Eqs. (17) to (18) and (19) are the constraints that must always be established.

Since the mass of all three satellites is considered equal to 1 kg, we have:

$$d_1 + d_2 + d_3 = 0 \tag{22}$$

$$e_1 + e_2 + e_3 = 0 \tag{23}$$

$$d_1e_1 + d_2e_2 + d_3e_3 = 0 \tag{24}$$

$$\sqrt{(d_2-d_1)^2+(e_2-e_1)^2} = \sqrt{(d_3-d_2)^2+(e_3-e_2)^2} = r \tag{25}$$

In addition, despite the underactuation in satellite 2 and the disturbance in flying formation, placing a constant value as a desired value for relative distances does not work anymore. In other words, in the current situation, the system needs to determine the correct position of the satellites and generate the reference path for the control block input. Considering the underactuation, satellite 2 is placed in a new position that cannot be changed or modified. Therefore, in this situation, the position of the other two satellites should be determined in such a way that the constraints proposed in Eqs. (22) to (25) are met, suggesting that the mission is preserved.

As a result, the equation system, including Eqs. (22) to (25), is proposed as a fault tolerance guidance law. So, the faulty satellite position is entered as an input to the guidance block, and the correct position is obtained for two other satellites using the equations in the guidance law. Then, by determining the correct position of all satellites, the desired relative distance is obtained and entered as reference values to the control block. A view of the block diagram of the system, the guidance law, and the controller is presented in Fig. 7.

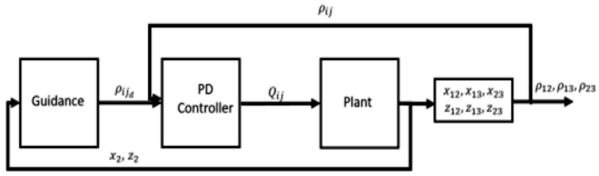

Fig. 7. Block diagram of the system

It is expected that by applying this guidance method, despite the underactuation of one of the satellites, the two constraints of the formation flying stability and maintaining the formation form are met. Figs. 8 to 11 present the obtained results.

Fig. 8 and 9 shows that the center of mass condition is met along x and z axes, with an accuracy of about 10^{-10}.

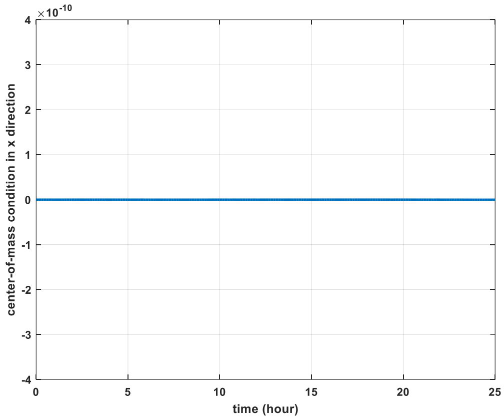

Fig. 8. The graph for establishing a condition for the mass center along the x-axis

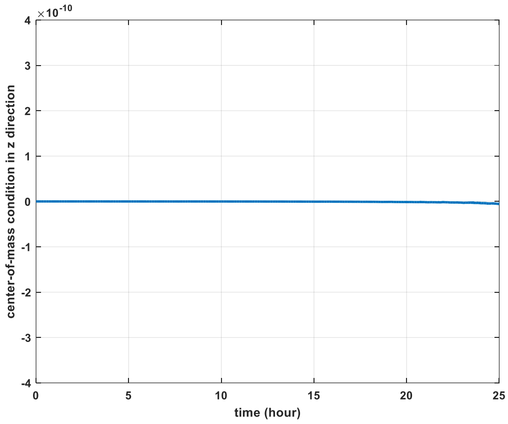

Fig. 9. The graph for establishing a condition for the mass center along the z-axis

Fig. 10 represents the condition of alignment of the main inertia axes of the formation on the main axis of the Hill frame. According to this fig., after 15 hours, the values diverge from zero and become fully divergent after 22 hours.

Fig. 11 provides two important points. First, the two curves in the chart, which represent the relative distances ρ_{12}andρ_{23} always fit with each other. This matching means establishing the condition to maintain the geometric shape of the formation and remaining the triangle in the isosceles form at all times. Second, relative distances, despite the matching, are rising over time exceeding 1,000 m. This increase reflects the distance between the formation satellites from each other and, in fact, the increase in the dimensions of the triangle. Thus, the results show that one of the goals of the problem has not been realized using fault tolerance guidance approach.

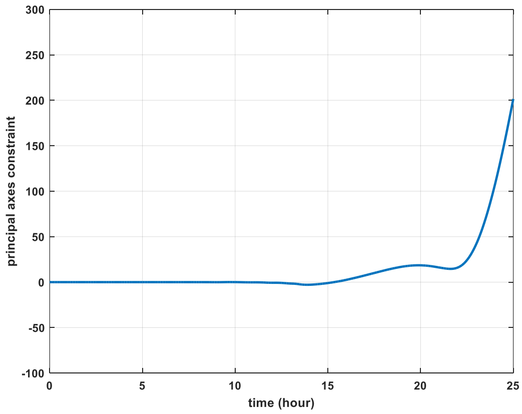

Fig. 10. The graph for establishing a condition for inertia axes

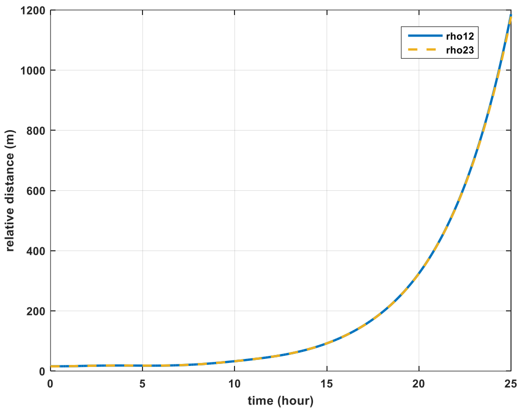

Fig. 11. The graph for establishing a condition for maintaining shape and dimensions of formation

Design of fault-tolerant control law

Considering the results of the applied fault-tolerant guidance approach, to offset its limitation, a fault-tolerant control approach is proposed. In this approach, a modification is applied in the expressed proportional-derivative controller. In other words, to meet the goal of maintaining dimensions, which has not been achieved in the guidance approach, a new design is developed according to the nature of the Coulomb formations issue.

Since the Coulomb force between two satellites is a function of their charge product, by reducing the charge of one satellite, it can be offset by increasing the charge of another satellite and, consequently, the force can be preserved without change.

From this perspective, the controller is designed so that where the final charge shares of satellite 1 and 3, (q_1 and q_3) that are determined and commanded to the actuators, their gains have two varying modes. First, when q_2 is produced in the proper range and the system is in full-actuated mode, the k_4 gain is always equal to 1, and the actuator of each satellite generates only its designated charge share. But when the actuator of satellite 2 is faulty and the quantity of q_2 is not produced to the optimum value, the gain is equal to k_5, and the actuator of satellites

1 and 3 produces a coefficient of the specified values of q_1 and q_3. In this way, the required amounts of Q_{12} and Q_{23} are still provided.

The numerical value of K_5 is based on the amount of the selected satellite charge lost. Since satellite 2 charge reduced by 40%, the other satellite charge should increase 1.67 times. Thus, the result of their multiplication will be constant. As a result, the k_5 gain for this simulation should be at least 1.67.

$$q_1 = \begin{cases} k_4 q_1 & if \quad q_2 \geq 0.6 q_{2_{desired}} \\ k_5 q_1 & if \quad q_2 < 0.6 q_{2_{desired}} \end{cases} \quad (26)$$

$$q_3 = \begin{cases} k_4 q_3 & if \quad q_2 \geq 0.6 q_{2_{desired}} \\ k_5 q_3 & if \quad q_2 < 0.6 q_{2_{desired}} \end{cases} \quad (27)$$

The results of implementing the fault-tolerant control approach for this system are shown in Figs. 12 to 15.

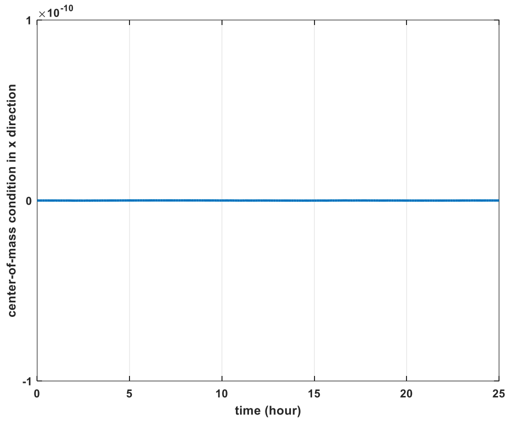

Fig. 12. The graph for establishing a condition for the center of mass along axis x

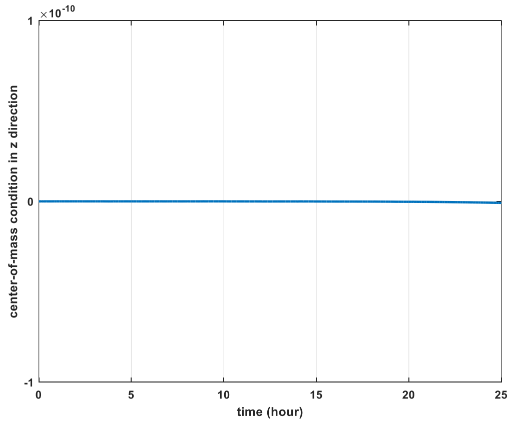

Fig. 13. The graph for establishing a condition for the center of mass along the z-axis

Fig. 12 and 13 shows that the center of mass condition is met along x and z axes, with an accuracy of about 10^{-10}.

Fig. 14 illustrates the condition of alignment the main inertia axes with the main axis of the Hill frame. In Fig. 10, this condition is established only for up to 12 hours after the simulation and then started to deviate from zero. But, Fig. 14 shows that this condition is established with an accuracy of about 10^{-3}.

From Fig. 15, two points can be inferred: First, the two curves in the diagram representing the relative distances of ρ_{12} and ρ_{23} always fit with each other. This matching means the establishment of a condition for maintaining the geometric shape of the formation and remaining triangle in the form of an isosceles triangular in all times. The second point is that relative distances, are limited over time in a range of 15 to 25 meter.

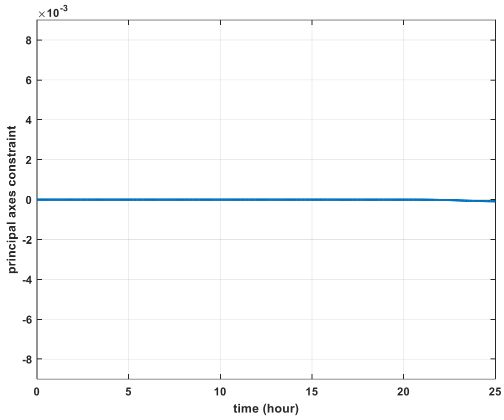

Fig. 14. The graph for establishing a condition for Inertia Axes

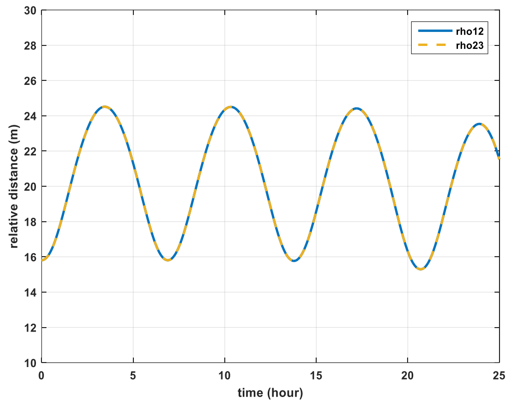

Fig. 15. The graph for establishing a condition for maintaining shape and dimensions of formation

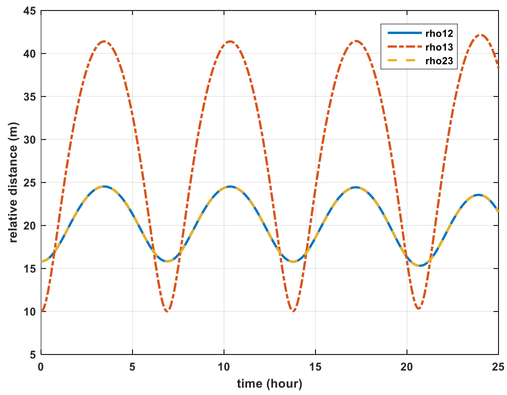

Fig. 16. Relative distance variations of satellites in under-actuated mode using the guidance and control approach

Fig. 16 shows that, in general, all the relative distances of a triangular formation are limited within the range of 10 to 43 meter. Indeed, with the simultaneous use of fault-tolerant guidance and fault-tolerant control approaches, both goals of the study were achieved; i.e., 1) maintaining formation flying stationary and 2) maintaining the form of formation in the underactuation mode of one member of the formation.

For checking the robustness of the designed fault tolerant algorithm in presence of the disturbances, a random disturbance applies on actuator of satellites 1and 3 by variance of 5%. Figs.17 to 19 show that formation flying performance is acceptable and mission's goals are fulfilled.

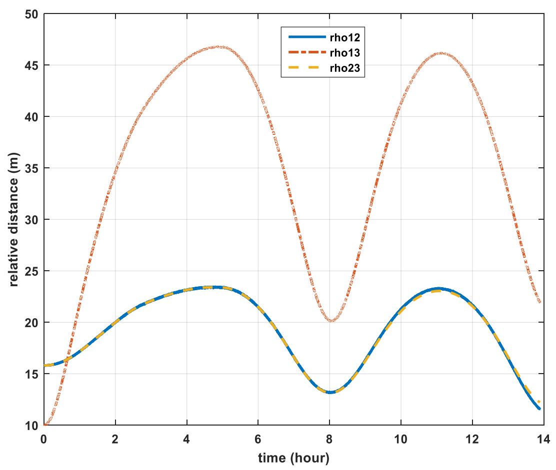

Fig. 17. Relative distance variations of satellites in presence of noise

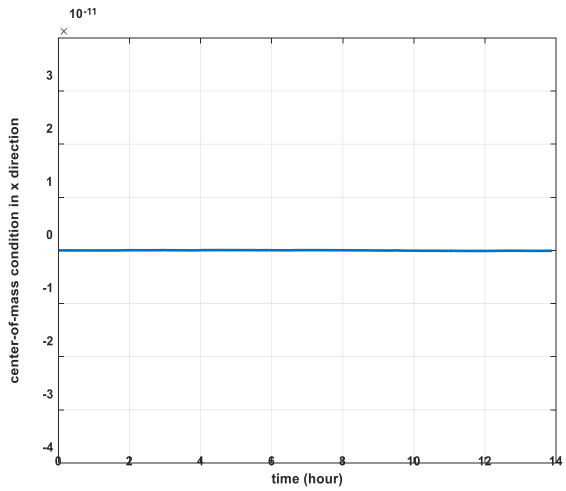

Fig. 18. The graph for establishing a condition for the center of mass along axis x

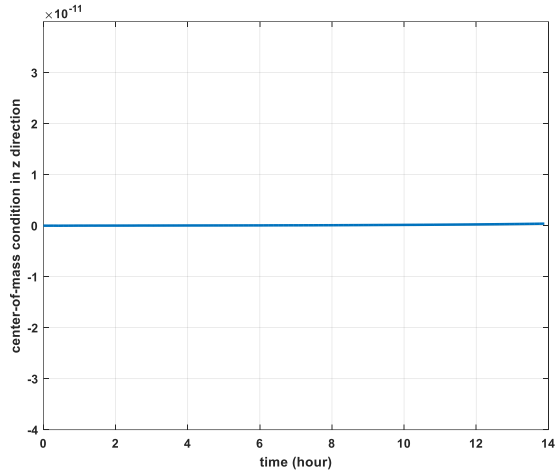

Fig. 19. The graph for establishing a condition for the center of mass along axis z

Summary and Conclusion

In this study, satellite formation flying guidance is studied in the presence of underactuation phenomenon. The Hill equations are used for describing the relative motion of satellites. The modeling was done using the concept of Coulomb forces as inter-satellite forces. Therefore, by controlling the value of electrical charge of satellites, their distance relative to each other can be controlled. First, the triangular formations including three satellites were modeled and controlled using a proportional-derivative controller. The results show that in this state the correct charge of satellites and control gains can be determined. Accordingly, it is possible to maintain their relative distance and the overall form of the formation. Then, the underactuation is considered in one of the formation satellites. The most frequently adopted approach is to remove faulty satellite and replace another satellite. The novelty of this research is in proposing a method that does not require removal of underactuated satellite and the construction of an additional satellite for replacement. In this method, Despite the underactuation of the satellite, guidance law and control law is designed, and the formation flight mission is not disturbed. Therefore, at this stage, a guidance law must be designed to maintain the relative distances and the formation form, despite the underactuation phenomenon. Robustness of the designed fault tolerant algorithm in the presences of disturbance was checked and showed that until 5% it can be work properly.

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