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Original Research Article

Age Replacement Model of a Used System

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Abstract

As there are various replacement policies for maintaining and improving operating and non-operating systems, this paper investigated the properties of an age replacement model of a used series system of a certain age. The system is subjected to two types of failures: Type I and Type II. It is assumed that, Type I failure is a repairable one, while Type II failure is a non-repairable one. An analytical expression of the expected cost rate for a used series system with n components was obtained. A simple illustrative numerical example was made available to analyze the effectiveness and properties of the constructed replacement model.

Keyword: Age; Component; Failure; Replacement; System.

Notations

$C(T,n)$	Average cost rate of the system
$r(t)$	Rate of Type I failure of component B_i, for $i = 1, 2, 3, \ldots, n$
$F_i(t)$	Type II failure distribution function of component B_i, for $i = 1, 2, 3, .., n$
$R_i(t)$	Reliability function of component B_i, for $i = 1, 2, 3, .., n$, due to Type II failure
$F_S(t)$	Type II failure distribution function of the system.
$R_S(t)$	Reliability function of the system due to Type II failure
C_i	Cost of minimal repair of B_i due to Type I failure, for $i = 1, 2, 3, .., n$
C_P	Cost of scheduled replacement of the system
C_F	Cost of unscheduled replacement of the system due to Type II failure
T^*	The system's optimum replacement time

1. Introduction

The reliability of a complex system can be achieved and maintained through the application of redundancy and maintenance action. These techniques have been used in many natural complex systems, such as generators, radars, and airplanes, where failures during actual operation are costly or dangerous. Moreover, consecutive failures are dangerous for larger systems, so it is good to know when to replace or preventively maintain the systems before failure periodically.

There is extensive literature on replacement models involving minimal repair. Alamir and Mo [1] mentioned that maintaining a complex system output if the proper preventive maintenance schedule is not determined is why they presented an integrated preventive maintenance scheduling methodology for complex systems. Their method was developed to improve the system's reliability and minimize costs. Briš *et al.* [2] introduced a new approach for optimizing a complex system's maintenance strategy, which respects a given reliability constraint. Bris and Jahoda [3] introduced Weibull-based aging systems that underwent discrete maintenance optimization and devised a model to realize the optimization in a context with minimal system costs and a prescribed unavailability restriction. In trying to extend the classical *k*-out-of-*n* systems, Cerqueti [4] assigned different roles to the components of a coherent system in terms of reliability, such that the components were grouped into important and standard ones, and the failure of the system depends on how many components of the two sets are failed. Coria *et al.* [5] proposed a new method of analytical optimization of preventive maintenance policy with historical failure time data. Enogwe *et al.* [6] used a distribution of the probability of failure times and developed a replacement model for items that fail un-notice. In trying to extend from single unit and parallel systems to an arbitrary coherent system, Erliymaz [7] constructed an age-based preventive replacement policy for an arbitrary coherent system consisting of independent components with a typical discrete lifetime distribution. Fallahnezhad and Najafian [8] investigated the number of spare parts and installations for a unit and parallel systems to cut down the average cost per unit

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time. Lim *et al.* [9] studied the characteristics of some age substitution policies. Liu *et al.* [10] developed mathematical models of uncertain reliability of some multi-component systems.

In trying to increase the reliability and life of some targeted components with an increasing hazard rate of a coherent system consisting of independent and repairable components, Mirjalili and Kazempoor [11] introduced two policies. Mizutani et al. [12] applied replacement first and last to obtain optimal regular and random replacement policies for general age and periodic replacement models. Furthermore, they showed that replacement first includes age-based random replacement, periodic-based random replacement, and standard age replacement. Nakagawa [13] proposed a discrete age replacement model of a single unit. Nakagawa *et al.* [14] explored the advantages of some replacement policies. Safaei *et al.* [15] investigated a system's optimal preventive maintenance action based on some conditions. Safaei *et al.* [16] considered an age replacement policy for repairable series and parallel systems with n-dependent components. They used copula and formulated two optimal age replacement policies based on the minimum expected cost function and maximum availability function. Hence an offshore wind turbine was considered as a case study. Sanoubar *et al.* [17] constructed a long-run expected cost-rate minimization model with instantaneous replacements for a stochastically deteriorating system with self-announcing failures, such that the system is replaced at failure or at a prescribed replacement time, whichever occurs first. Usman et al. [18] used the (m,T) group replacement method and carried out an analysis on a replacement model developed to establish the optimum time for the replacement of burnt-out bulbs in street lighting systems. Sudheesh *et al.* [19] studied the age replacement policy discretely. Waziri and Yusuf [20] presented some age replacement models for a parallel-series system based on proposed policies. Waziri *et al.* [21] analyzed an age replacement model with minimal repair of a series-parallel system, such that the system contained three series subsystems, which are subsystem A, subsystem B, and subsystem C. Xie *et al.* [22] assessed the effects of safety barriers on the prevention of cascading failures. Yin and Cui [23] developed two types of shock models. They discussed the asymptotic behavior of each shock's damage evolution process and the use of aggregated stochastic processes. They derived the probabilities of each shock eventually disappearing and destroying the system. Yusuf et al. [24] developed a discrete fix-up limit time function for series and parallel formations based on some assumption to provide a chance of completing a fix-up action within a discrete-time, which can reduce the unplanned downtime. Zhang *et al.* [25] addressed the age replacement problem for parallel systems with mission durations, where they finally suggested that replacement policies should be applied in maintaining the pumps of a cooling water system of a nuclear power plant. Zhao *et al.* [26] surveyed some periodic replacement policies by considering the shortage and excess costs for a periodic. They discovered that the shortage and excess costs depend on future events outside the cycle.

Several authors have published many papers on age replacement models for single and multi-component systems. However, there are limited papers on replacement models for used units and systems. The aim of this paper is to develop a replacement model with minimal repair for a used system, with the hope that the proposed model provides a useful quantitative tool for managers to evaluate the system performance and design an optimal maintenance policy.

2. Description of the system

Consider a used system of age x $(0 < x < \infty)$ with n identical components. It is assumed that each of the n components is subjected to two types of failures, which are Type I and Type II failures. Type I failure is repairable, which is rectified by minimal repair. In contrast, Type II failure is a non-repairable failure, which, if it occurs, the failed component is replaced entirely with used one of age x, where x is previously specified. Since all the n components are subjected to Type I and Type II failures, the whole system is also subjected to Type I and Type II failures. If the system fails due to Type I failure, it is minimally repaired and allowed to continue operating from where it stopped. If the system fails due to Type II failure, the system is replaced with a used one of age x. Let C_i be the cost of minimal repair of the failed component B_i, C_P be the cost of the constant scheduled replacement of the system due to Type II failure and C_F be the cost of unscheduled replacement due to Type II failure within every replacement cycle. The mathematical expressions of the reliability function and failure rates based on some assumptions were used for the construction of the average cost rate of the system.

3. Assumptions

1. Suppose that x $(0 < x < \infty)$ was the age of the used system.
2. If the system fails due to Type I failure, the system is minimally repaired.
3. If the system fails due to Type II failure, the failed system is replaced completely with a new one of age x.
4. Both Type I and Type II failures arrives according to non-homogeneous Possion process.
5. The system is replaced at scheduled time $(T + x)$ after its exchanged or at the first instance of Type II failure, whichever occurs first.
6. The cost of scheduled replacement of the system is less than the cost of unscheduled replacement due to Type II failure, that is, $C_p < C_F$.
7. All cost are positive numbers.
8. All the required resources are available for replacement and minimal repair action.
9. The time of repair or replacement is negligible.

4. Proposed model

Based on the assumption, the failure time distribution of the system based on Type II failure is

$$F_s(t|x) = \frac{F_s(t+x)-F_s(x)}{R_S(x)}, \text{ for } F_s(x) < 1, t \geq 0, \quad (1)$$

where $R_S(t) = 1 - F_s(t)$ and $R_S(t,x) = 1 - F_s(t,x)$.

The reliability function of the system, due to Type II failure, is

$$R_S(T,x) = \prod_{i=1}^{n} R_i(T,x), \quad (2)$$

where $R_i(T,x)$ is the reliability function of the component B_i based on Type II failure T.

The cost of the scheduled replacement time of the entire system within one replacement cycle is

$$Cost\ of\ scheduled\ replacement = C_p \quad (3)$$

The cost of unscheduled replacement time of the entire system within one replacement cycle, is

$$Cost\ of\ unscheduled\ replacement = C_F F_S(T|x) \quad (4)$$

The mean replacement time of the entire system, within the planned time of one replacement cycle, is

$$Mean\ time\ of\ one\ replacement\ cycle = \int_0^T R_S(t+x)dt / R_S(x) \quad (5)$$

The cost of minimal repair of component B_i due to Type I failure before the scheduled time of one replacement cycle for $i = 1,2,\ldots n$, is

$$Cost\ of\ minimal\ repair = \int_x^{T+x} \sum_{i=1}^{n} C_i r_i(t)\, dt \quad (6)$$

Using equations (1) to (6), the average replacement cost rate is

$$C(T,x) = \frac{C_p + C_F F_S(T|x) + \int_x^{T+x} \sum_{i=1}^{n} C_i r_i(t)dt}{\int_0^{t_0} R_S(t+x)dt / R_S(x)} \quad (7)$$

Equation (7) can be further written as

$$C(T,x) = \frac{C_p R_S(x) + C_F\left(F_S(T+x) - F_S(x)\right) + R_S(x)\int_x^{T+x} \sum_{i=1}^{n} C_i r_i(t)dt}{\int_0^T R_S(t+x)dt} \quad (8)$$

where

- C_P: Cost of scheduled periodic replacement of the system at constant periodic time,
- C_F: Cost of unscheduled replacement of the system due to Type II failure,
- C_i: Cost of minimal repair of failed component B_i.

Noting that $C(T,x)$ is assumed to be an objective function of an optimization problem, and the aim is to determine T^* that minimizes $C(T,x)$.

5. Numerical example

This section will present a numerical example to analyze and study the properties of the replacement model constructed. For a simple illustration of the model constructed, a system with ten components was considered, and it was assumed that all ten components follow the Weibull distribution. Let the Type I failure of the component B_i follows exponential distribution:

$$r_i(t) = \beta_i, \text{ for } i = 1,2,\ldots n \quad (9)$$

While, let the Type II failure of component B_i follows Weibull distribution :

$$F_i(t) = 1 - e^{(-\lambda_i t^{\alpha_i})}, i = 1,2,3,\ldots \text{n} \quad (10)$$

For simple illustration of the model, five components were considered. Let the set of parameters, cost of repair and replacement were used throughout this particular example:

1. $\lambda_1 = 0.4$, $\lambda_2 = 0.4$, $\lambda_3 = 0.3$, $\lambda_4 = 0.3$ and $\lambda_5 = 0.2$.
2. $\alpha_1 = 4, \alpha_2 = 4, \alpha_3 = 3, \alpha_4 = 2$ and $\alpha_5 = 2$.
3. $\beta_i = 0.5$, for $i = 1,2,3,4,5$.
4. $C_P = 15$ and $C_F = 25$.
5. $C_i = 0.75$, for $i = 1,2,3,4,5$.

By substituting the parameters of the Type I and Type II failures in equations (9) and (10), the equations below are obtained as follows:

$$r_i(t) = 0.5, \text{ for } i = 1,2,3,4,5. \quad (11)$$
$$F_1(t) = 1 - Exp(-0.4t^4) \quad (12)$$
$$F_2(t) = 1 - Exp(-0.4t^4) \quad (13)$$
$$F_3(t) = 1 - Exp(-0.3t^3) \quad (14)$$
$$F_4(t) = 1 - Exp(-0.3t^2) \quad (15)$$
$$F_5(t) = 1 - Exp(-0.2t^2) \quad (16)$$

Table 1. The Cost $C(T,x)$ for Some Values of x

T	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1.$
0.1	3687.05	3892.07	4151.61	4471.70	4859.99
0.2	1989.47	2220.20	2521.27	2907.90	3400.67
0.3	1467.44	1731.38	2088.021	2566.91	3210.71
0.4	1248.60	1556.17	1988.43	2597.84	3465.49
0.5	1161.66	1527.53	2064.64	2862.96	4071.40
0.6	1153.52	1598.69	2284.27	3362.86	5105.66
0.7	1206.70	1761.86	2662.53	4168.45	6775.98
0.8	1319.68	2030.70	3250.91	5427.83	9482.94
0.9	1501.90	2438.96	4147.06	7411.56	13979.03
1	1774.20	3047.53	5522.89	10610.95	21708.94
1.1	2172.95	3960.48	7681.43	15939.59	35544.68
1.2	2758.96	5356.42	11169.33	25153.68	61428.08
1.3	3634.03	7548.53	17004.26	41759.02	112190.40
1.4	4971.97	11102.22	27149.16	73043.88	216822.46
1.5	7077.59	17072.93	45538.40	134824.49	444004.17

Table 2. The Optimal Replacement Time of the System Obtained from Different Values of x (Extracted from Table 1)

$\boldsymbol{x}$	$x = 0.2$	$x = 0.4$	$x = 0.6$	$x = 0.8$	$x = 1.$
$\boldsymbol{T^*}$	$T^* = 0.6$	$T^* = 0.5$	$T^* = 0.4$	$T^* = 0.3$	$T^* = 0.3$

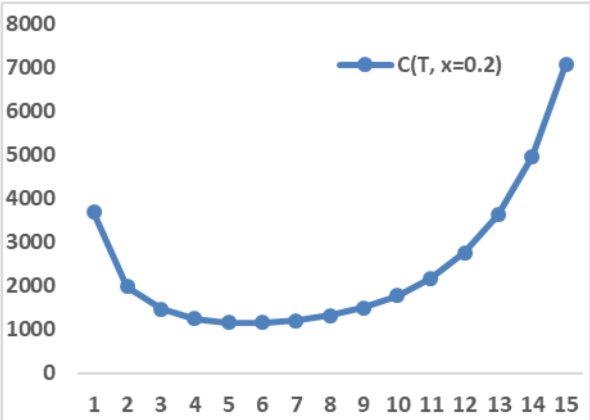

Figure 1. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for x Equal to 0.2

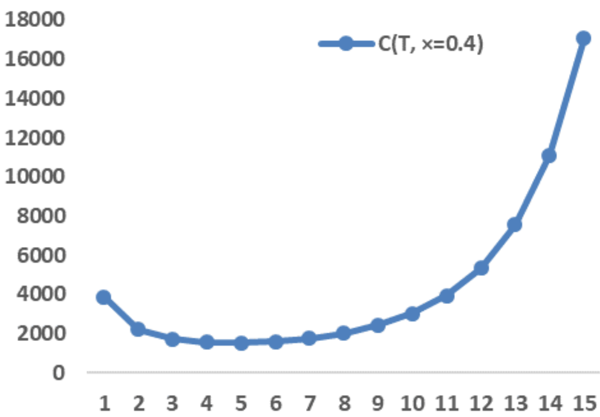

Figure 2. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for x Equal to 0.4

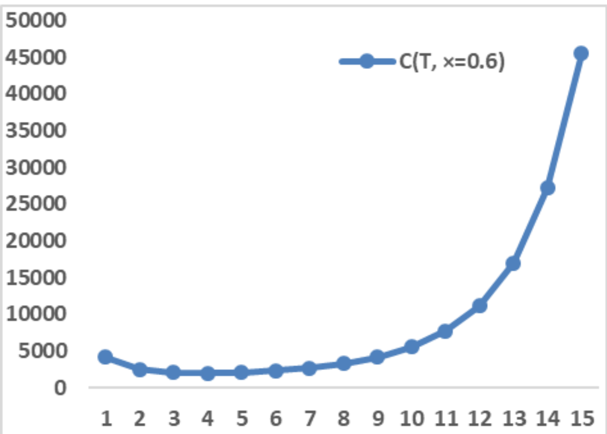

Figure 3. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for x Equal to 0.6

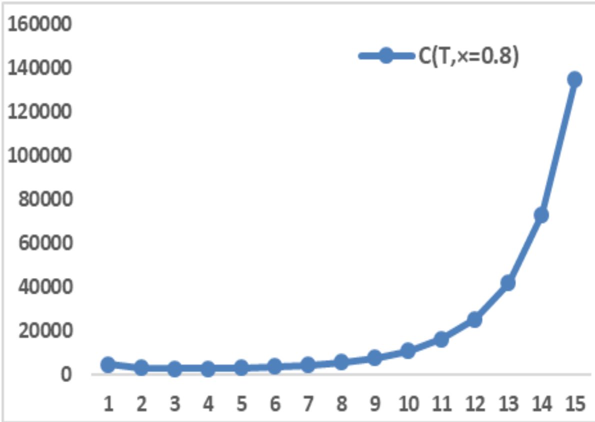

Figure 4. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for x Equal to 0.8

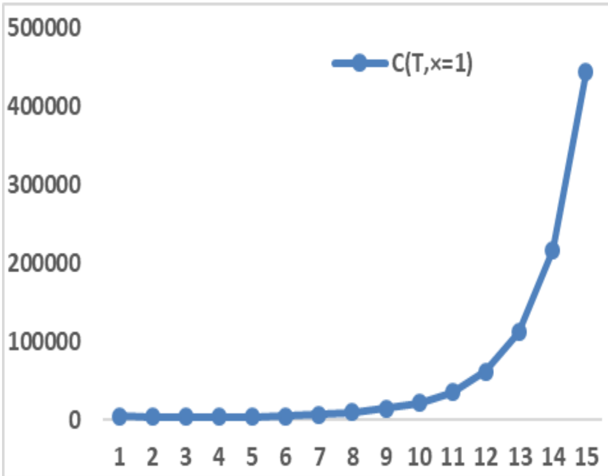

Figure 5. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for x Equal to 1

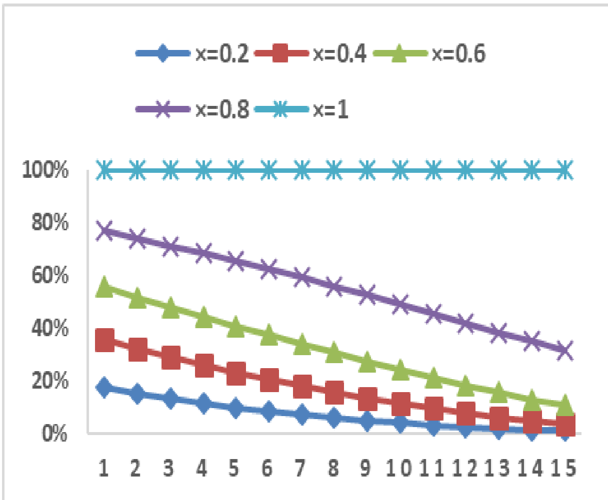

Figure 6. The Plot of Cost Rate $C(T, x)$ Versus the Scheduled Replacement Time T for Different Values of x

Table 3. The Cost Rate $C(T,x)$ of the System as the Cost of Unscheduled Replacement Increases with $x = 0.2$

T	$C_F = 25$	$C_F = 30$	$C_F = 35$	$C_F = 40$
0.1	3687.05	4973.33	5086.66	5199.99
0.2	1989.47	3538.23	3675.78	3813.34
0.3	1467.44	3381.05	3551.39	3721.73
0.4	1248.60	3681.36	3897.240	4113.11
0.5	1161.66	4352.36	4633.31	4914.26
0.6	1153.52	5482.52	5859.38	6236.247
0.7	1206.70	7298.98	7821.978	8344.97
0.8	1319.68	10236.73	10990.52	11744.31
0.9	1501.90	15111.54	16244.06	17376.57
1	1774.20	23488.85	25268.76	27048.67
1.1	2172.95	38480.22	41415.76	44351.30
1.2	2758.96	66522.78	71617.48	76712.18
1.3	3634.03	121517.16	130843.93	140170.69
1.4	4971.97	234870.17	252917.87	270965.58
1.5	7077.59	480985.07	517965.98	554946.88

Table 4. The Cost Rate $C(T,x)$ of the System as the Cost of Scheduled Replacement (C_P) Decreases with $x = 0.2$

T	$C_P = 15$	$C_P = 12$	$C_P = 9$	$C_P = 5$
0.1	3687.05	4491.99	4123.99	3755.99
0.2	1989.47	3168.14	2935.60	2703.07
0.3	1467.44	3008.51	2806.31	2604.11
0.4	1248.60	3260.96	3056.44	2851.91
0.5	1161.66	3842.83	3614.26	3385.69
0.6	1153.52	4829.54	4553.43	4277.31
0.7	1206.70	6419.32	6062.66	5706.01
0.8	1319.68	8993.17	8503.40	8013.63
0.9	1501.90	13266.19	12553.35	11840.51
1	1774.20	20610.99	19513.04	18415.10
1.1	2172.95	33756.08	31967.48	30178.88
1.2	2758.96	58346.25	55264.43	52182.61
1.3	3634.03	106571.26	100952.13	95332.99
1.4	4971.97	205972.41	195122.36	184272.31
1.5	7077.59	421795.63	399587.09	377378.54

Table 5. The Optimal Replacement Time of the System as the Cost of Unscheduled Replacement (C_F) Increases (Extracted from Table 3)

C_F	$C_F = 25$	$C_F = 30$	$C_F = 35$	$C_F = 40$
T^*	$T^* = 0.6$	$T^* = 0.3$	$T^* = 0.3$	$T^* = 0.3$

Table 6. The Optimal Replacement Time of the System as the Cost of Scheduled Replacement (C_P) Increases (Extracted from Table 4)

C_P	$C_P = 15$	$C_P = 12$	$C_P = 9$	$C_P = 6$
T^*	$T^* = 0.6$	$T^* = 0.3$	$T^* = 0.3$	$T^* = 0.3$

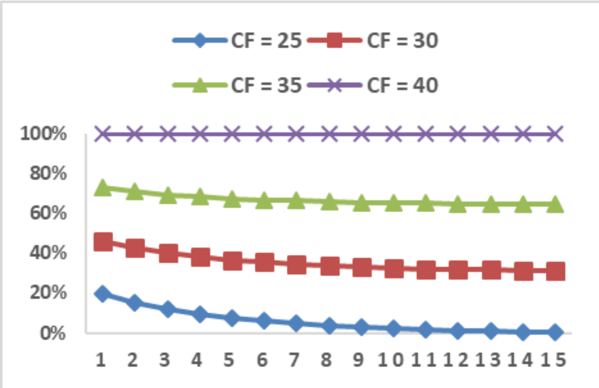

Figure 7. Comparing the Cost Rate $C(T,x)$ of the System as the Cost of Unscheduled Replacement (C_F) Increases.

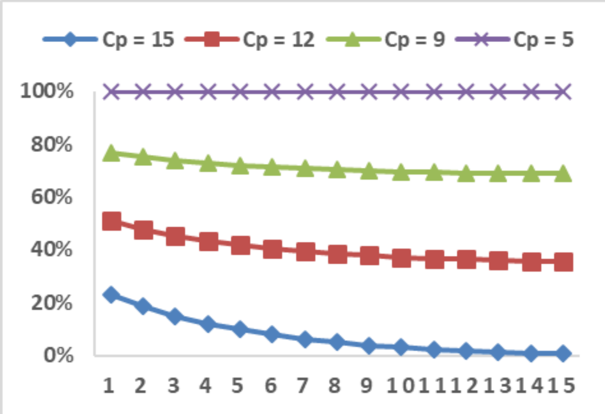

Figure 8. Comparing the Cost Rate $C(T,x)$ of the System as the Cost of Scheduled Replacement (C_P) Decreases.

5.1 Some observations from the results obtained are as follows:

1. Observed from Table 1 and Table 2, the optimal replacement time of the system is 0.6. That is, $C(T^* = 0.6, x = 0.2) = 1153.52$. Figure 1 below is the plot of $C(T, x = 0.2)$ against the scheduled replacement time T.
2. Observed from Table 1 and Table 2, the optimal replacement time of the system is 0.5. That is, $C(T^* = 0.5, x = 0.4) = 1527.53$. Figure 2 below is the plot of $C(T, x = 0.4)$ against the scheduled replacement time T.
3. Observed from Table 1 and Table 2, the optimal replacement time of the system is 0.4. That is, $C(T^* = 0.4, x = 0.6) = 1988.43$. Figure 3 below is the plot of $C(T, x = 0.6)$ against the scheduled replacement time T.
4. Observed from Table 1 and Table 2, the optimal replacement time of the system is 0.3. That is, $C(T^* = 0.3,\ x = 0.8) = 2566.91$. Figure 4

below is the plot of $C(T, x = 0.8)$ against the scheduled replacement time T.

5. Observed from Table 1 and Table 2, the optimal replacement time of the system is 0.3. That is, $C(T^* = 0.3, x = 1) = 3210.71$. Figure 5 below is the plot of $C(T, x = 1)$ against the scheduled replacement time T.
6. As observed from Table 2, as the specified age x increases, the optimal replacement time of the system decreases.
7. As observed from Table 1 and Figure 6, as the specified age increases, the also increases. That is, $C(T, x = 0.2) < C(T, x = 0.4) < C(T, x = 0.6) < C(T, x = 0.8) < C(T, x = 1)$.
8. As observed from Table 3, as the cost of the system's unscheduled replacement increases, the system's optimal replacement time decreases.
9. As observed from Table 3 and Figure 7, as the cost of the unscheduled replacement of the system increases, the cost rate also increases.
10. As observed from Table 4, as the cost of the scheduled system replacement decreases, the optimal replacement time also decreases.
11. As seen from Table 4 and Figure 8, as the cost of the scheduled system replacement decreases, the cost rate also increases.

6. Summary and conclusion

In this paper, an age replacement model for a used series system of age x based on some proposed assumptions was constructed. It is assumed that all the components that made the system are subjected to two types of failures (Type I and Type II failures), such that Type I failure is repairable, while Type II failure is unrepairable. Finally, a numerical example was provided to show the particular properties of the replacement model constructed for the used system. Thus, from the result obtained, one can see the various changes in the optimal replacement times of the system with the changes of x. The model and some research results in this paper are both attractive in reliability theory and reliability engineering.

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