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Original Research Article

Optimal Sensors Positioning by Using Value of Information Method

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Abstract
This study first discusses the importance of data collection and sensor placement in engineering. The Value of Information (VoI) method is introduced as a new approach for optimizing sensor placement. The decision-making theories, the VoI method, and its foundations are then explained. The application of this method for optimizing sensor placement is also described. Two case examples in the field of sensor placement in engineering are presented and analyzed. The first case involves determining the load-bearing status of land, the associated risks and costs, and the need to install piles. The second case involves monitoring the creep phenomenon in high-pressure vessels and pipes, where sensor placement is determined using the VoI method based on relevant risks. The results are compared with the UNI 11096 standard for pressure and high-temperature vessels.

Keywords: Value of information method, Optimal placement of sensors, Reliability, Bayesian theory, Decision making, Optimization

List of Nomenclature and abbreviations

E_i	i'th event
n_{UNI}	number of sensors as for UNI 11096 sensor positioning
r_{UNI}	VoI ratio for UNI 11096 sensor positioning
$\mathbb{E}L_i(Y)$	posterior cost function
CVoI	Conditional Value of Information
E[C]	expected cost
E[CVoI]	expected value of CVoI
E[U]	expected utility
HAZ	Heat Affected Zone
Pr	probability
SSCs	Systems, Structures, and Components
UNI	Italian National Unification (Ente Nazionale Italiano di Unificazione)
VoI	Value of Information
$\mathrm{VoI}_{\mathrm{UNI}}$	VoI obtained by UNI 11096 sensor positioning
X_i	coordinate
β	reliability index
$C(f.a)$	coast function
$\mathbb{E}L(\emptyset)$	initial expected cost

1. Introduction

There are various problems in various fields of science in which, according to the results of experimental tests and measurements carried out in certain parts of that field, opinions, and judgments are made regarding the distribution of that phenomenon in the field. Usually, as shown in Fig. 1, sets of sensors are used to monitor complex spatio-temporal phenomena, such as temperature and light in a building, pollution in a lake, precipitation over a wide geographical area, traffic conditions in a road network or water quality in the urban water distribution network. The way sensors are placed in the desired field can significantly impact the results, as it relies on a general understanding of the field and the problem at hand. In medical and therapeutic matters, there is always the challenge that the doctor decides based on which test (or tests) and with the least cost to treat the patient. Data collection and observations are done by placing sensors or detector stations in all these issues. Condition monitoring and intelligent analysis of data collected by sensors may help predict degradation escalation and anticipate the risk of failure.[1]

In practice, doing data mining has a cost. For example, the cost of procuring sensors and their deployment, energy consumption, time and effort required to conduct the test, patience and attention of the user or the cost of conducting medical tests, etc., are things that limit data collection.

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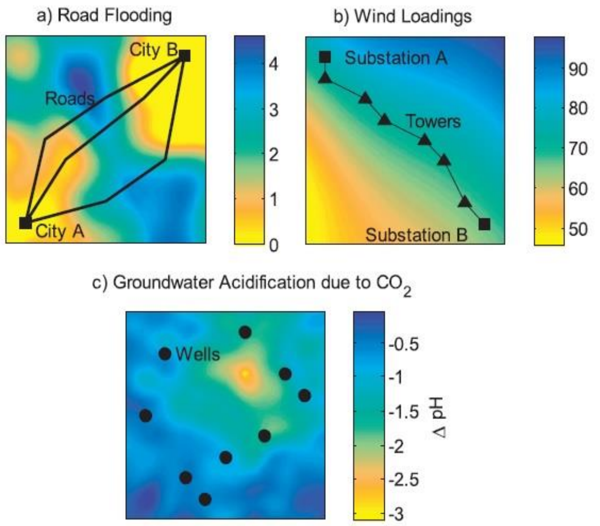

Figure 1. Placement of sensors to observe **a)** roads that pass through the floodplain with different depths, **b)** wind force on the power transmission line, **c)** a group of wells exposed to acidification of underground water due to leakage that is placed from a carbon deposition tank [2].

Therefore, the basic question is how to collect data to obtain the most useful information that is as cost-effective as possible.

There are two approaches to data mining: exploratory approaches, which do not strive for optimality but quickly find solutions with often reasonable performance. These approaches include myopic algorithms such as greedy heuristics or continuous relaxation approaches [3].

Non-heuristic approaches try to find the optimal solution, but usually, it is very difficult to find the optimization criteria and form the objective function for larger problems. These approaches include probabilistic planning using Markov decision processes, the value of information method, etc. [4]–[8].

In many practical applications, choosing between useful but costly observations is important. By exploiting the structure of the problem and paying attention to it, many data acquisition problems can be solved efficiently and almost optimally.

This article investigates data collection and sensor placement optimization using the Value of Information (VoI), a non-exploratory method. VoI is a decision analytic method based on Bayesian theory [9] for quantifying the benefit of acquiring additional information to support such analyses that can be used to help in a wide range of decisions. VoI methods are used to determine where the weakest evidence in a decision model is and what data should be collected to improve it [10].

A Bayesian framework for optimal sensor placement in structures health monitoring applications was proposed in [11], where the optimal sensor placement method optimized a metric related to the probability of damage detection of all regions of the structure. Straub [6] presents the modeling and computation of VoI based on structural reliability methods. Seyed Mojtaba Hoseyni et al. [7] used the VoI-based sensors positioning framework to compare the outcomes with standards/ recommendations/ guidelines for monitoring energy Safety-critical Systems, Structures, and Components (SSCs) issued by regulatory bodies, to confirm their validity or suggest improvements.

This article introduces the Value of Information (VoI) method. The decision-making theories, the VoI method, and its foundations are then explained. The application of this method for optimizing sensor placement is also described. Two case examples in the field of sensor placement in engineering are presented and analyzed.

The first case involves determining the load-bearing status of land, the associated risks and costs, and the need to install piles [12]. In this study, the calculations were repeated according to the defined problem, and the same results were obtained. The second case involves monitoring the creep phenomenon in high-pressure vessels and pipes, where sensor placement is determined using the VoI method based on relevant risks. The results are compared with the UNI 11096 pressure and high-temperature vessel standards [7]. Also, in this problem, the calculations were repeated according to the defined problem, and the same results were obtained.

2. Value of Information method

Value of Information (VoI) analysis is a means of valuing the expected gain from reducing the uncertainty of data collection. This method estimates the expected gain from reducing uncertainty in the input parameters of a decision analytic model.

2.1 The framework of decision-making theory

In Figure 2, a decision tree schematic and a diagram of the effect of a decision in uncertainty are presented. An optimal decision under uncertainty is a decision that maximizes expected utility (E[U]) and minimizes expected cost (E[C]). In addition, it optimizes maintenance, repair, and action costs and the risk associated with failure (for the entire system) [6].

Since the monitoring results improve action decisions, the necessary action (corrective action) should be optimized according to the system state's available (previous) data. This process is called prior decision optimization.

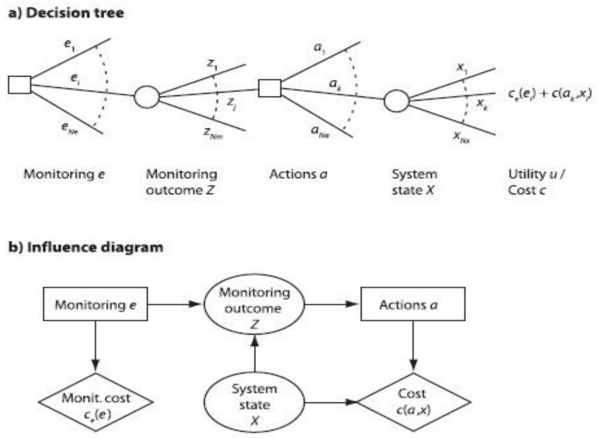

Figure 2. Decision tree and influence diagram [6]

2.2 Perfect information

Perfect information is a hypothetical situation where there is no uncertainty about X. In this case, the decision maker can always choose the best necessary action.

2.3 Imperfect information

In practice, however, monitoring and measurement systems are subject to random errors or uncertainties. For example, when complete knowledge of material parameters is available, the failure event cannot be predicted with certainty if future loads on the item remain uncertain. In addition, most measurements are indirect. Therefore, we are always faced with data that is not certain.

2.4 Bayesian updating

Bayesian probability is defined under uncertainty. Imperfect information can be used to learn about X and thus about the events E_1..., E_m. Bayesian update is a mathematical framework for knowing the probability distribution of X and the probabilities of events E_1..., E_m with new uncertain information. In this method, the probability of event E_i under the condition of observing event Z is expressed as follows:

$$\Pr(E_i|Z) = \frac{\Pr(E_i \cap Z)}{\Pr(Z)} \quad (1)$$

To study in more detail about Bayesian probabilistic logic, refer to [13]

2.5 Conditional Value of Information (CVoI) and VoI

Once Z is observed, the probabilities $\Pr(E_i | Z)$ can be calculated. Optimizing the decision is done based on comparing the results of the previous optimal decision. The difference between the cost of the previous optimal action and the cost of the new (posterior) optimal action is defined as the conditional value of information (CVoI). Note that CVoI is zero if the posterior optimal decision is the same as the prior optimal decision and has a positive value otherwise. These values are comparative and do not have computational value alone [6].

The Monte Carlo method is usually used to generate conditional information numerically (simulation of observing Z) according to sensor placement at different points [12], [14]. In this way, to simulate the measurement results of a point, many random values are created, and the measured value will be the average of the random values. According to Bayesian methods, the matrix of average value and covariance is updated for each of the simulated values. Then, the expected cost function, the average expected cost at each point, is calculated.

As stated, the difference between the cost of the previous optimal action and the cost of the new (posterior) optimal action is defined as the CVoI. VoI is the expected value of CVoI given all possible measurement outcomes.

$$\mathrm{VoI} = \mathrm{E[CVoI]} \quad (2)$$

3. Several applications of sensor placement optimization

In this section, two case examples in the field of sensor placement in engineering are presented and analyzed. The first case involves determining the load-bearing status of land, the associated risks and costs, and the need to install piles. The second case involves monitoring the phenomenon of creep in high-pressure vessels and pipes, where sensor placement is determined using the VoI method based on relevant risks. The results are compared with the UNI 11096 standard for pressure and high-temperature vessels.

3.1 Investigating the state of bearing capacity of the soil for construction based on the VoI

A series of structures are supposed to be built on a square plot of 250 meters by 250 meters. The locations within this plot are specified by a coordinate system (X_1, X_2), where $250 \geq X_1 \geq 0$ and $250 \geq X_2 \geq 0$. A lognormal random field describes the soil bearing capacity f (X1, X2) at any location in this domain. The lognormal random field describes phenomena such as biological species distribution, minerals distribution in the earth's crust, etc. [15]. Since the bearing capacity of the soil is a function of its constituent elements, the log-normal distribution is chosen to express the random dispersion of this quantity.

The land is divided into 25 squares of 50 x 50 meters. In each square, the bearing capacity of the soil must be more than 80 kPa to allow safe construction (the bearing capacity of the square is assumed to be approximately the same as the bearing capacity at the center of the square).

Two construction options are possible in each square: soil can be used as is to support footing and foundation (corresponding to an action choice a=0 for the square), or it can be piled into the ground (corresponding to a choice of action a=1 for the square), which guarantees sufficient bearing capacity. The use of piles requires an additional construction cost of 5 M€, but if piles are not used and the bearing capacity of the soil is less than 80 kPa, structural failure will occur, and a cost of 200 M€ will be created for the square section. The damage function for each square is expressed as follows.

$$C(f.a) = \begin{cases} 0 & if\ f \geq 80\ [kPa]\ and\ a = 0 \\ 200\ M€ & if\ f < 80\ [kPa]\ and\ a = 0 \\ 5\ M€ & if\ \ a = 1 \end{cases} \tag{3}$$

Using the considered distribution (lognormal), the mean value matrix, standard deviation, and covariance are calculated, and the reliability matrix (β) is calculated from them. According to the explanations provided in [6]–[8], [12], [14], in this simulation, each of the 25 points is considered as a measurement point. According to the coordinates of each measurement point, the correlation of that point with other points is determined. Based on the obtained correlation, new average values and standard deviation are calculated based on Bayesian probabilities. Using the updated mean and standard deviation values and a random error probability of 1%, the reliability index (β) value is calculated 1000 times for each point. Based on the reliability index's value, the probability of failure under the condition of measurement at the assumed point and the expected damage under the condition of measurement at the assumed point is obtained. Finally, the value of the expected posterior damage function will be averaged among all the considered possibilities under the measurement condition at the assumed point (y) as follows.

$$\mathbb{E}L_i(Y) \cong \frac{1}{n_{simulations}} \sum_{j=1}^{n_{simulations}} \mathbb{E}L_i(y_i) \tag{4}$$

Finally, the measurement VoI value relative to the assumed point y is calculated as the difference between the initial expected cost and the posterior cost function from the following relationship.

$$\text{VoI} = \mathbb{E}L(\emptyset) - \sum_{i=1}^{25} \mathbb{E}L_i(Y) \tag{5}$$

By repeating the calculations for all problem points, VoI values are obtained, as shown in Figure 3. It can be seen that the measurement at the points (25m, 125m) and (25m, 75m) has the highest value. Therefore, it is better to measure at these points.

x_2 = 225m	6.8 M€	4.4 M€	0.7 M€	0.0 M€	0.0 M€
x_2 = 175m	9.5 M€	6.2 M€	0.9 M€	0.0 M€	0.0 M€
x_2 = 125m	10.2 M€	6.6 M€	1.0 M€	0.0 M€	0.0 M€
x_2 = 75m	10.1 M€	6.2 M€	0.8 M€	0.0 M€	0.0 M€
x_2 = 25m	7.6 M€	4.6 M€	0.7 M€	0.0 M€	0.0 M€
	x_1 = 25m	x_1 = 75m	x_1 = 125m	x_1 = 175m	x_1 = 225m

Figure 3. VoI values in different parts of the land [12]

Optimum positioning of the sensor in the pressurized equipment

One of the newest methods of optimizing sensor placement is the VoI method. Seyed Mojtabi Hoseyni and his colleagues from Milan Polytechnic [7] have shown in research that using this method can reduce the number of sensors to control the thickness of a high pressure and high-temperature pipe compared to the relevant standard. Long-term exposure to materials under stress and at high temperatures will cause the creep phenomenon to occur and intensify. Figure 4 shows the schematic of the superheated steam content manifold.

Table 1 presents the specifications of the materials and working conditions of the superheated steam manifold. Figure 5 shows how to place the sensors according to the UNI 11096 standard. According to the mentioned standard, 32 sensors should be placed for such a problem.

Researchers have investigated four different geometries for the tube. The first geometry is connecting two pipes in the middle. The second geometry is to roll the sheet and weld its seam to make a tube. The third geometry is the combination of the above two geometries. In other words, it is rolling and welding two tubes in the middle of each other, and the fourth geometry is a seamless tube. Figure 6 presents these geometries.

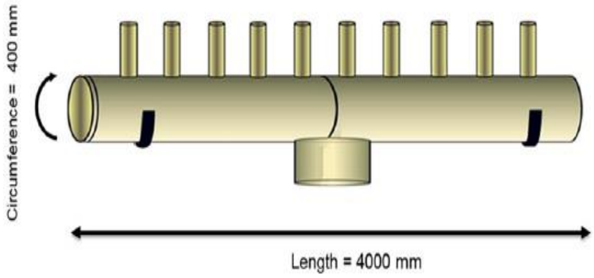

Figure 4. manifold containing superheated steam [7]

Table 1. Operating conditions of the pipe [7]

Design pressure	189 [barg]
Design temperature	Inlet: 778 [K] Outlet: 723 [K]
Material	9Cr-IMo-V-Nb (Plate) ASME SA-387/SA-387M Grade 91
Percentage of life spent	35%
Operating hours	100,000 [hours]
Tensile strength	475 [MPa]
Thickness	20 [mm]

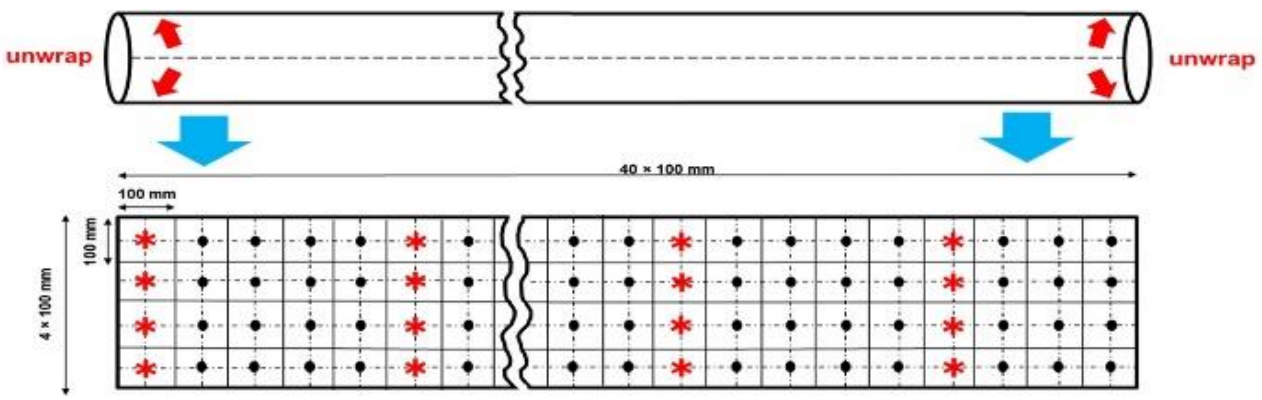

Figure 5. Schematic view of the unwrapped manifold with sensor locations in line with UNI 11096 [7]

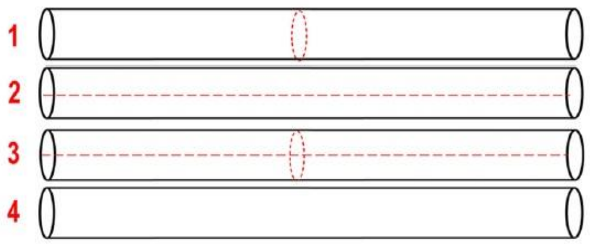

Figure 6. Schematic view of the four case studies [7]

Further, by conducting finite element analysis, they have obtained the deviation range from the permissible strength. In this investigation, the initial thickness of the pipes is 20 mm, and the minimum allowed thickness (final limit) is 16.9 mm. The pipe's thickness is considered a normal distribution with a mean value of 20 mm and a standard deviation of 1 mm. In the heat-affected zone (HAZ) by welding, a standard deviation of 2 mm is considered.

In the first step, the researchers have preliminarily analyzed the probability of failure. This analysis is estimated according to the average value, standard deviation, and normal distribution relations. The failure criterion is when the pipe thickness reaches 16.9 mm due to creep. The results of this analysis are given in Figure 7 In the first geometry, the maximum probability of failure is 6.06%, and the minimum is 0.1%.

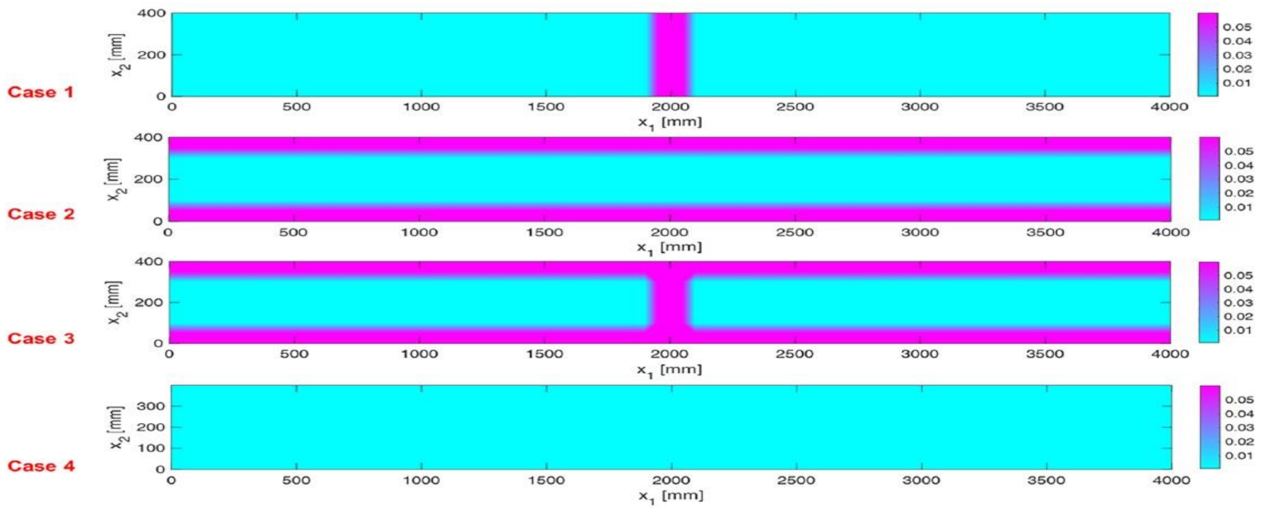

Figure 7. Prior probability of failure for the four case studies [7]

In Figure 8, the optimization algorithm of sensor deployment points is presented. According to the explanation presented in the research of Seyed Mojtabi Hosseini et al. [6], each of the 160 points is considered a measurement point in this simulation. According to the coordinates of each measurement point, the correlation of that point with other points is determined. The mean values and the new standard deviation are calculated based on the obtained correlation. Using the updated mean and standard deviation values and a random error probability of 1%, the reliability index (β) value is calculated for each point 10,000 times. Based on the reliability index's value, the probability of failure under the condition of measurement at the assumed point and the expected damage under the condition of measurement at the assumed point is obtained. Ten thousand calculations will be done for each point, and the average will be taken. Now, the VoI value obtained by assuming measurement at a known point is compared with the VoI value obtained by assuming measurement at points according to the instructions of the UNI 11096 standard. If the VoI value is higher, this point is selected as the optimal point for placing the measurement sensor. The calculation process is described in the flowchart below.

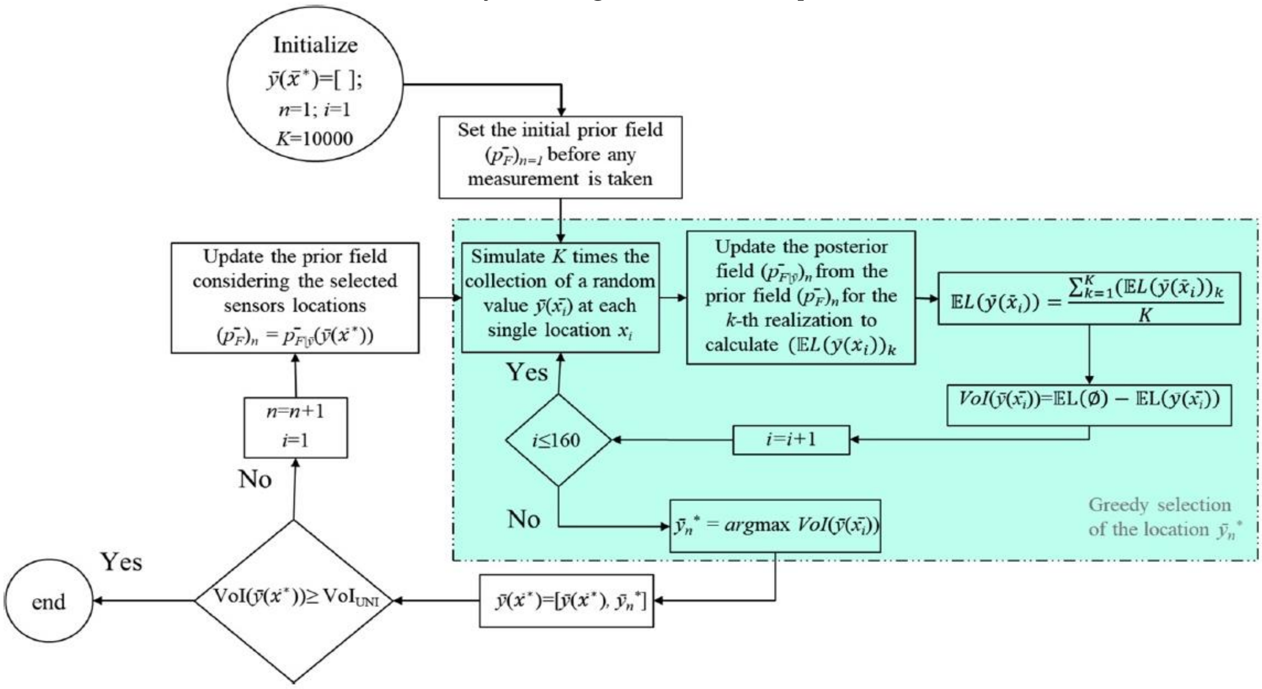

Figure 8. Algorithm for optimization of sensor placement points [7]

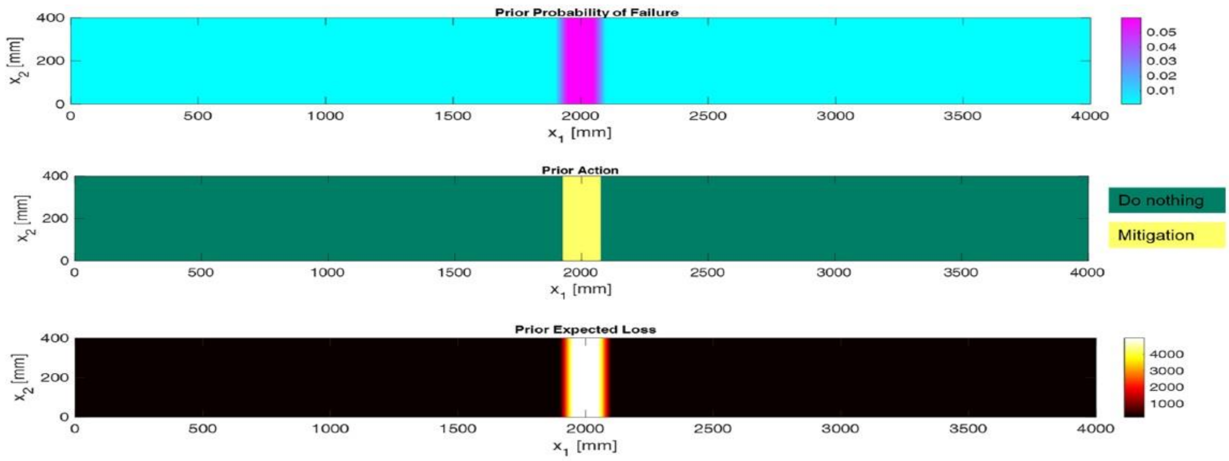

Figure 9. Prior probability, prior actions, and the prior expected loss for the case study 1 [7]

In Figure 9, values of initial failure probability, points requiring initial action (requiring sensor installation), and initial expected damage are reported. In the calculations to estimate the initial expected damage, the maximum amount of damage is 5000, and the minimum is 194 €. Figure 10 shows the expected damage assuming measurement at the point (150, 1950). The performed calculations show that the expected damage amount has been reduced to 351 €. Because this point is one of the critical points and by installing the sensor on it, the probability of its failure before corrective actions will be significantly reduced.

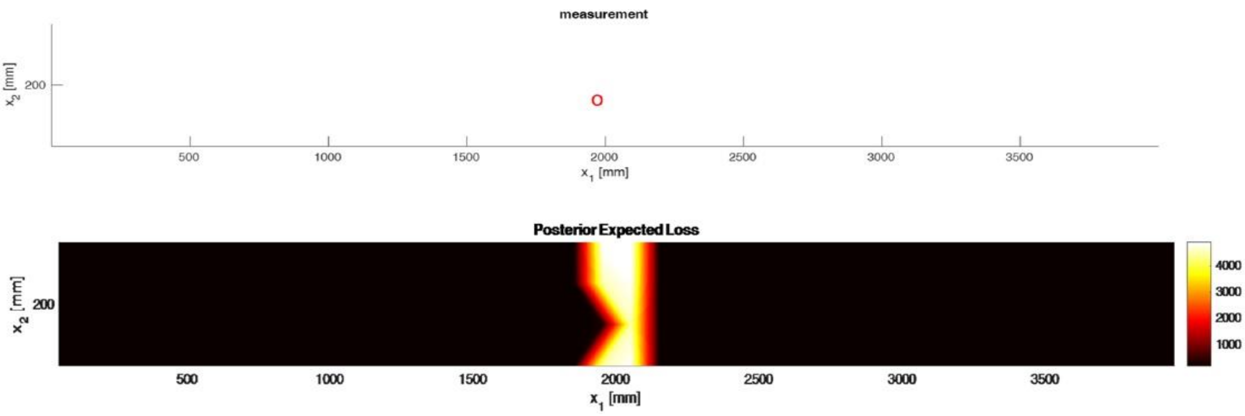

Figure 10. The plot of the posterior expected loss field, conditioned on a specific observation at location xo [7]

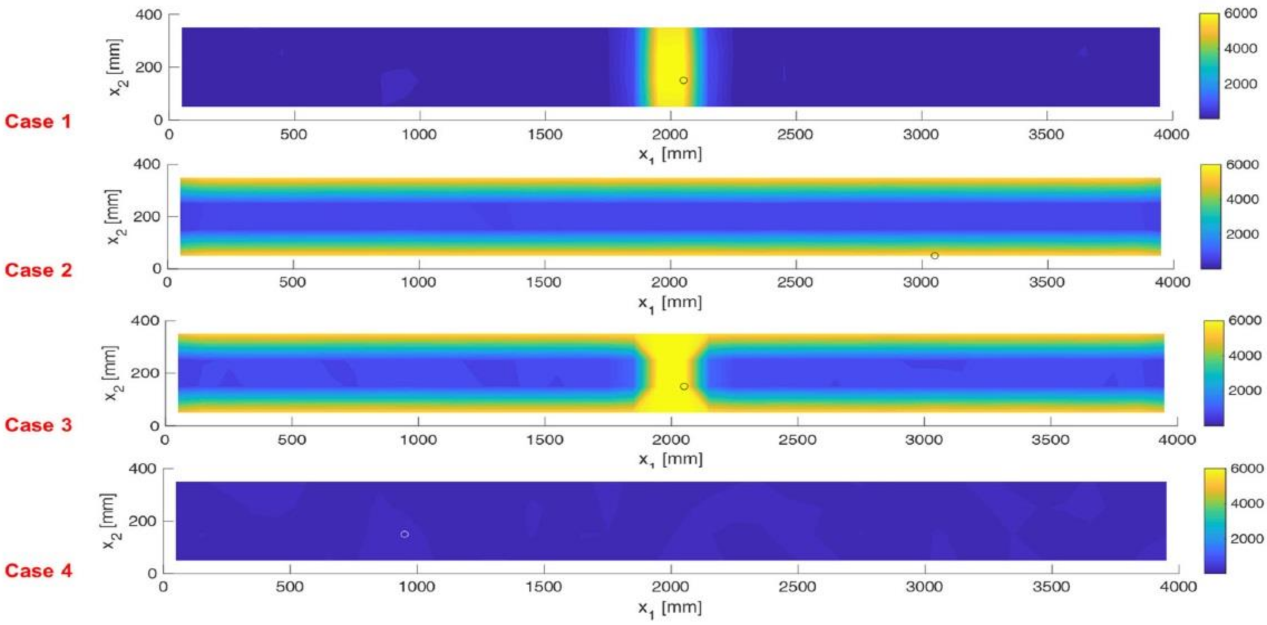

Figure 11. VoI contours for the four case studies at the first iteration n=1 [7]

By running the described algorithm, the value of the VoI function has been calculated at different points of each geometry. The results are presented in Figure 11 For the first geometry, the VoI value based on the measurement at the point (2050,150) was obtained with 6354 €, and for other points affected by welding, between 5446 and 6354 €.

As the solution continues, the optimal points according to the greedy optimization algorithm are reported as follows. According to Figure 12, the first geometry has 5 points in which the VoI value is higher than the VoI value based on the standard target points.

Comparing the results with the UNI 11096 standard shows that more complete information (or at least information of the same level of importance) can be extracted from the pipe condition with fewer sensors. By defining the ratio of the value of information to the number of sensors (n) as follows, a scale for the value of the resulting information can be defined.

$$r = \frac{\mathrm{VoI}}{n} \tag{6}$$

Similarly, the same scaling factor is defined as follows for the placement of sensors according to the UNI 11096 standard, with the difference that the number of sensors is fixed at 32. This comparison is presented in Table 2.

$$r_{UNI} = \frac{\mathrm{VoI}_{UNI}}{n_{UNI}} \tag{7}$$

Figure 13 shows the position of 5 points whose VoI is higher than $\mathrm{VoI}_{\mathrm{UNI}}$ for the first geometry. Now, the question may be raised that the points identified as points with high VoI are all geometrically symmetrical, and the information of one point will logically determine the conditions of other points. The investigation carried out in the research of Seyed Mojtabi Hoseyni et al. [7] shows that if the calculations are repeated 1000 times, the resulting VoIs will be 62± 5%. This means that the VoI of these points have almost the same value, and they are one of the optimal sensor selection points due to the geometric symmetry. The result is shown in Figure 14.

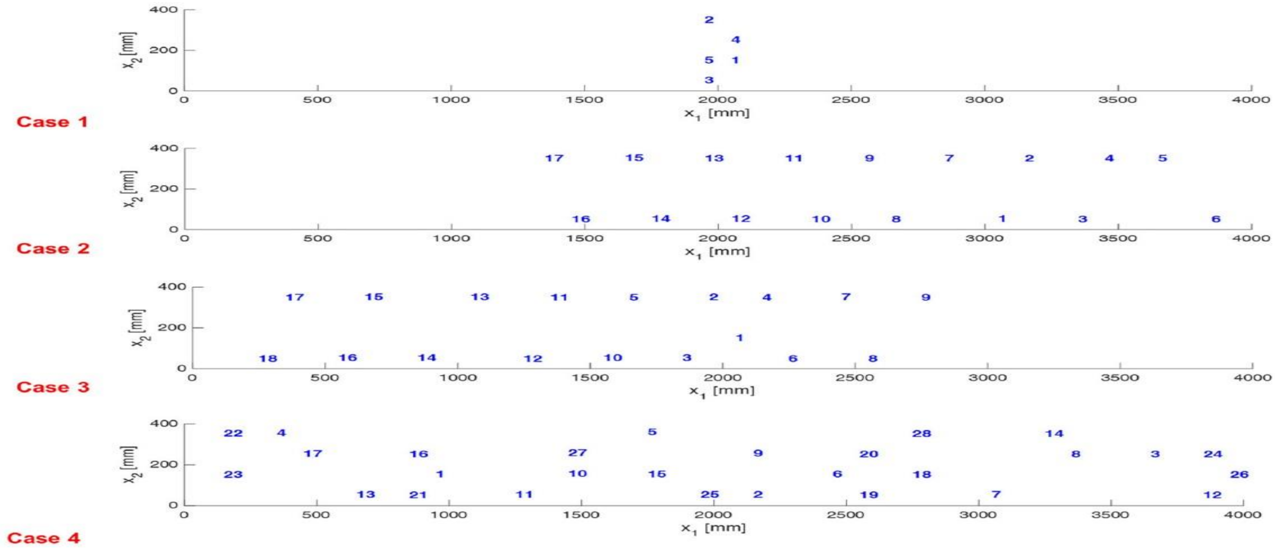

Figure 12. Sensor positioning using the greedy optimization algorithm for the four case studies [7]

Table 2. Comparison between the UNI 11096 and the proposed method for sensor positioning[7]

Case study	VoI	n	r	VoI_{UNI}	n_{UNI}	r_{UNI}
1	26495	5	5299	26386	32	824.56
2	99772	17	5868.94	94014	32	2937.94
3	107270	18	5959.44	104200	32	3256.23
4	6156.1	28	219.86	5984.7	32	187.02

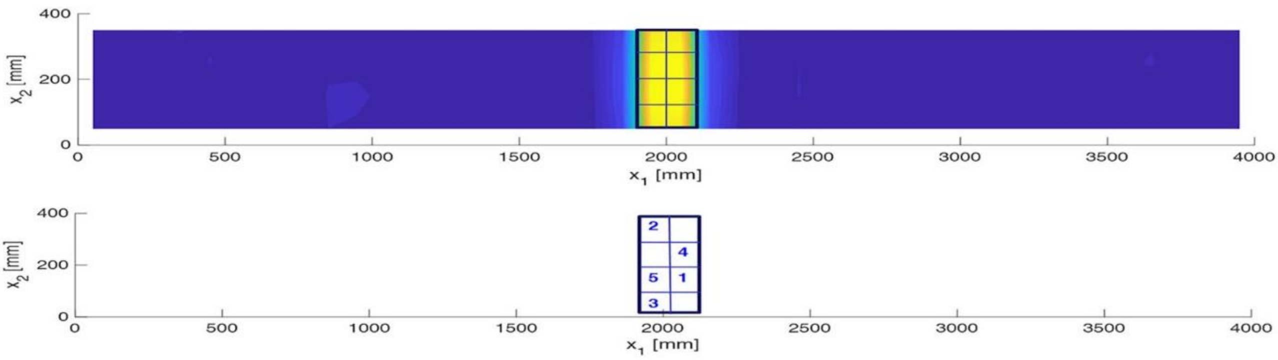

Figure 13. Sensor positioning for case study 1[7]

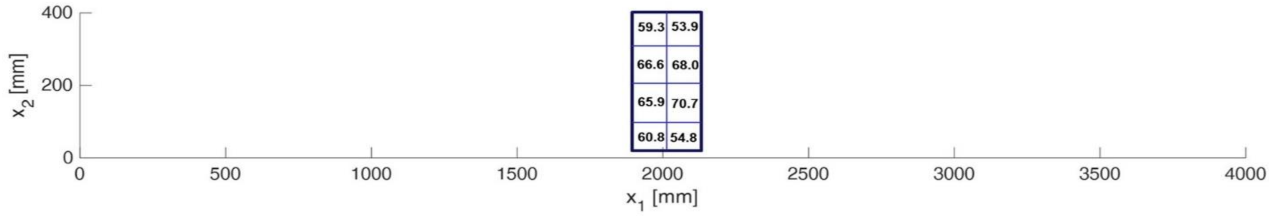

Figure 14. Probability of the locations of the sensors to be selected as one of the optimal positions (%) [7]

4. Conclusion and summary

The problem of optimal sensor placement in all data collection types is important. In aerospace engineering, preliminary analysis of sensor placement before starting experimental tests can lead to more accurate and complete results while reducing testing costs and risks. However, sensor placement is not limited to engineering, as it is relevant in many fields, including urban planning, national and international infrastructure, medicine and therapy, security, and more.

The VoI method is a decision-making method based on Bayesian probabilities, and with any amount of primary information, the decision results can be estimated. Of course, the more complete and accurate the initial information is, the more accurate the results of using this method will be. Using this method, it is possible to establish a balance between the number of tests performed, the number of sensors, the cost of installing and maintaining sensors, the cost of performing the test, and the damage of possible failures and make the optimal decision. This is important for all kinds of research data collection in the field of aerospace engineering, which is very expensive, and the risk of repeating them is very high.

In this review, the importance of the problem of optimal placement of sensors was expressed. In the following, the basics of the VoI method and its basic concepts were described and introduced. Two related types of research were described in the field of sensor placement optimization with the VoI method.

In the first problem, two points were found where the best results can be obtained if soil bearing capacity is measured at those points.

In the second problem, the sensors' positioning obtained using the proposed framework gives results that justify the standard's positioning and require fewer sensors to reach the VoI obtained by duly implementing the current guidelines or standards.

Optimal sensor placement using the proposed framework provides results that can be used to prepare, validate, and develop standards and operational guidelines. It is especially useful in cases where there is limited operational information, or its acquisition is costly.

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