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Original Research Article

# Optimal Preventive Maintenance Policy for Non-Identical Components: Traditional Renewal Theory vs Modern Reinforcement Learning

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## Abstract

This paper compares the traditional approach against reinforcement learning algorithms to find the optimal preventive maintenance policy for equipment composed of multi-non-identical components with different time-to-failure distributions. As an application, we used the data from military trucks, which consisted of multiple components with very different failure behavior, such as tires, transmissions, wheel rims, couplings, motors, brakes, steering wheels, and shifting gears. The literature proposes Four different strategies for preventive maintenance of these components. To find the optimal preventive manganocene policy, we used the traditional approach (renewal theory-based) and the conventional reinforcement learning algorithms and compared their performance. The main advantages of the latter approach are that, unlike the traditional approach, they are not required to estimate the model parameters (e.g., transition probabilities). Without any explicit mathematical formula, they converge to the optimal solution. Our results showed that the traditional approach works best when the component time-to-failure distributions are available. However, the reinforcement learning approach outperforms where no such information is available or the distributions are misspecified.

**Keyword:** Opportunistic maintenance; Preventive maintenance; Markov decision process; Monte Carlo; Q-learning; Reinforcement learning

## 1. Introduction

In the recently released European Standards regarding maintenance, maintenance is defined as the combination of all technical, administrative, and managerial actions during the life cycle of an item intended to retain it in, or restore it to, a state in which it can perform the required function; see Marquez and Gupta [1]. Maintenance problems can be solved using traditional approaches and machine learning methods. In recent years, reinforcement learning (RL) algorithms have become very popular and widely used. RL is one of the newer machine learning approaches that has gained prominence in various fields of human life today. In general, RL is a technique that allows a decision-making (agent) to maximize his total reward by interacting with the environment. Ravichandiran [2] introduced the steps of a typical RL algorithm as follows:

1. First, the agent interacts with the environment by performing an action
2. The agent acts and moves from one state to another
3. And then, the agent will receive a reward based on the action it performed
4. Based on the reward, the agent will understand whether the action was good or bad
5. If the action was good, if the agent received a positive reward, then the agent will prefer performing that action again. Otherwise, the agent will try performing another action, resulting in a positive reward. So it is a trial-and-error learning process.

As mentioned earlier, RL algorithms are used a lot in most fields. Wang et al. [3] applied multi-agent RL to solve the maintenance problem for a flow line system consisting of two series machines with an intermediate finite buffer in between. Liang et al. [4]modeled the energy management problem by a Markov decision process and solved it using an Approximate Dynamic Programming (ADP)-based approach to match electricity supply and demand. Yousefi et al. [5] used an RL approach to develop a new dynamic maintenance policy

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for multi-component systems with individually repairable components, where each component is at risk of two competing failure processes of degradation and random shocks. Adsule et al. [6] modeled the Condition-based maintenance (CBM) decision-making as a continuous semi-Markov decision process. They applied an RL algorithm to learn the optimal maintenance decisions and inspection schedules based on the current health state of the component.

In this paper, we consider military trucks composed of multi-non-identical components. Trucks are systems that are used continuously, so the possibility of them breaking down on the road is very high, resulting in financial and life-threatening costs. Additionally, trucks are used in the military, so minimizing the downtime of any truck is essential. It is important to obtain the optimal replacement times for each system component efficiently. Haleem and Yacout [7] and Barde et al. [8] tackled this problem before. They used the truck's eight more important components in their analysis: tires, transmissions, wheel rims, couplings, motors, brakes, steering wheels, and shifting gears. We will use the same set of components here as well. Abdel Haleem and Yacout [7] used renewal theory to estimate the components' replacement times. Barde et al. [8] estimated replacement times by using Monte Carlo reinforcement learning (MCRL). We aimed to obtain the optimal maintenance policy by using the two existing approaches in the literature and employing a time difference (TD) learning approach, a more efficient RL algorithm than MCRL. We will evaluate the performance of all these approaches under two scenarios: when the true failure time distributions are available versus misspecified.

In the next section, we will present the problem assumptions and existing maintenance strategies; In the Method section, we will present different algorithms. Finally, in the Results section, we will report the numerical results comparing the TD-based RL algorithm's performance against the two other methods proposed in the literature.

## 2. Motivation

We consider equipment that contains multiple non-identical components. Our purpose in this article is to find a policy that minimizes the total downtime of the equipment. The downtime is defined as the non-productive time when the system is not operational due to a failure or a preventive action. Each equipment component has a different time-to-failure distribution modeled by a Weibull distribution with its shape and scale parameters. The strategies are based on the following assumptions:

1. If we replace a component due to failure, it takes more time than if we replace a component preventively.
2. If we replace a group of components or a whole system, it takes less time than if we replace each component separately.
3. There are replacement opportunities at regular intervals.

The assumptions mentioned above can be found in many military applications, where the equipment's reliability is essential, downtime must be minimized, and cost considerations are less important; see Haleem and Yacout [7].

Following Haleem and Yacout [7], the following four replacement strategies will be used and compared against one another:

- **Strategy I**: Every component is replaced upon failure. It is corrective maintenance (baseline).
- **Strategy II**: every component is replaced upon failure and at an individual fixed interval, $T_i$, for component i. It is based on preventive maintenance. Haleem and Yacout [7] estimated $T_i$ by minimizing the downtime per unit time, $D_i$, for component i. $D_i$ is calculated from the following expression:

$$argmin_{T_i} D_i = \frac{tp_i R(T_i) + tf_i[1 - R(T_i)]}{(T_i + tp_i)R(T_i) + [tf_i + E(t|t \leq T_i)][1 - R(T_i)]}, \forall i \qquad (1)$$

Where $tp_i$ is time to replace component i preventively, $tf_i$ is time to replace component i upon failure, $R(T_i)$ is the reliability of component i at the time $T_i$ and $E[t|t \leq T_i]$ is the expected time to failure, given that it occurs before $T_i$.

- **Strategy III**: It is based on Strategy II, to which it is added a scheduled overhaul. In other words, as in Strategy II, every component is replaced at failure and replacement intervals $T_i$ for component I, the whole system is replaced at a known fixed time.
- **Strategy IV**: It is a group-based maintenance strategy. Any component i that fails or reaches its replacement interval $T_i$, the components of its group are also replaced with it.

## 3. Markov Decision Process

A Markov decision process (MDP) framework has the following key components:

1. S: Set of states (s ∈ S)
2. A: Set of actions (a ∈ A)
3. $P(s_{t+1}|s_t, a_t)$: Transition probabilities
4. $R(s, a)$: Reward function of doing action in the states.

We use model-free RL for two reasons: the curse of dimensionality and the curse of modeling. The curse of dimensionality arises from the much longer computational time and much larger memory space needed as the state space of a problem becomes larger. The curse of modeling arises from the need to estimate the transition probabilities, which is often difficult, especially when the state space is large; see Powell [9]. We present MDP formulation (state, action, and reward function) for each preventive strategy (i.e., II, III, and IV).

**MDP formulation of strategy II**: Let $G_i$ be the age of component i, $f_i = 1$ denotes that component i is failed and $f_i = 0$ denotes its normal status, so the state of the system at time t is the vector defined as follows:

$$s_t = G_1, \ldots, G_8, f_1, \ldots, f_8.$$

Let $a_i$ = 1 means PM action and $a_i$ = 0 means "do nothing" action, then the action of the system at time t is:

$$a_t = (a_1, \dots, a_8). \tag{2}$$

Based on Barde et al. [8] work, the reward function can be defined as follows:

$$R(s_t, a_t) = \begin{cases} -\alpha_i . tp_i \,, & if\ a_i = 1 \\ -\alpha_i . \Delta . \left\lceil \frac{tf_i}{\Delta} \right\rceil, & a_i = 0\ and\ f_i = 1 \\ \Delta \,, & otherwise \end{cases} \tag{2}$$

Where $\alpha_i = \frac{\Delta}{tp_i}$ is a scale factor as they assumed that $tp_i$ is scaled such that it has at least the same period than Δ (see Barde et al. [8] for more information); Δ is the time interval between two epochs and $\lceil . \rceil$ is the ceiling function.

**MDP formulation of strategy III**: Let $G_i$ be the age of component i, $f_i = 1$ denotes that component i is failed whereas $f_i = 0$ denotes normal status and $O = 1$ means to replace the whole system, whereas $O = 0$ denotes that don't replace the whole system; then, the state of the system at time t is the vector defined as follows:

$$s_t = (G_1, \dots, G_8, O, f_1, \dots, f_8). \tag{3}$$

The action on the system is defined as:

$$a_t = (a_1, \dots, a_8). \tag{4}$$

The reward function is:

$$R(s_t, a_t) = \begin{cases} -\alpha_i . tp_i, & if\ a_i = 1\,, O = 0 \\ -\alpha_i . \Delta . \left\lceil \frac{tf_i}{\Delta} \right\rceil, & if\ a_i = 0, O = 0, f_i = 1 \\ -\alpha_i . \Delta . \left\lceil \frac{\beta . \sum_{i=1}^{8} tp_i}{\Delta} \right\rceil, & if\ O = 1 \\ \Delta, & otherwise \end{cases} \tag{5}$$

Where $\beta \in (0, 1)$ comes from the assumption that the time to replace the whole system is less than the sum of times to replace each component separately.

**MDP formulation of Strategy IV**: states and actions in Strategy IV are the same as those in Strategy II, but the reward function is different due to group structure. The components are grouped as follows:

$$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) = (\{1,3\}, \{3,8\}, \{3,5\}, \{7,6\}, \{4,2\}) \tag{6}$$

The groups are formed based on technical reasons, such as the difficulty or ease of reaching and changing a component when a neighboring component has failed. Then, the reward function is:

$$R(s_t, a_t) = \begin{cases} -\alpha_i . \beta . \sum_{l \in \phi_j} tp_l \,, & if\ a_k = 1\ and\ k \in \phi_j \\ -\alpha_i . \Delta . \left\lceil \frac{\beta . \sum_{l \in \phi_j} tf_l}{\Delta} \right\rceil, & if\ f_k = 1,\ \alpha_k = 0, k \in \phi_j \\ \Delta, & otherwise \end{cases} \tag{7}$$

Where $\alpha_i = \frac{\Delta}{\beta . \Delta . \sum_{l \in \phi_j} tp_l}$ and $\beta \in (0,1)$ is defined similarly as in Strategy III.

# 4. Reinforcement Learning

Barde et al. [8] used the on-policy first-visit MCRL algorithm to find the optimal replacement time $T_i$ for each preventive strategy separately, whereas we will use a TD learning approach that is more efficient than MCRL.

TD learning is a model-free approach that combines sampling and bootstrapping simultaneously. One of the advantages of TD over MCRL is that MCRL can only be used for episodic problems. In other words, MCRL learns from complete episodes only. Unlike MCRL, TD learning employs single steps to learn (be updated after every step) and does not need to wait until the end of an episode. Therefore, TD learning can be applied to both continuing and episodic problems.

In this paper, for the military trucks problem, we will use a Q-learning (QL) algorithm, which is an off-policy TD control algorithm. QL is one of the most popular and efficient algorithms in RL. It is an off-policy RL algorithm because the QL function learns from actions not necessarily taken under the current agent policy. Like other RL algorithms, QL seeks to learn a policy that maximizes a pre-defined total reward in every state. The objective of the QL algorithm is to learn and estimate the optimal action-value function that defined as

$$Q^*(s_t, a_t) = \max_{\pi} Q_\pi(s_t, a_t), \tag{8}$$

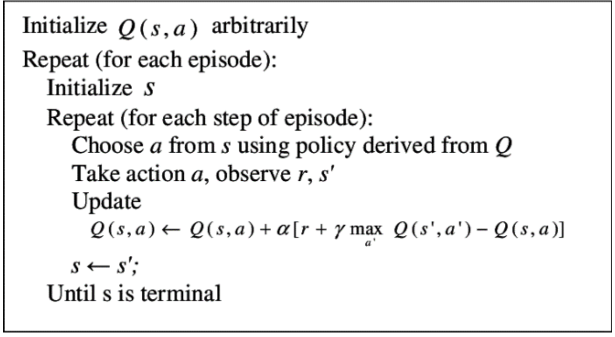
where

$$Q_\pi(s_t, a_t) = \mathbb{E}_\pi[\textstyle\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} |\, s_t, a_t] \tag{9}$$

QL directly approximates the optimal action-value function by taking the best action when bootstrapping:

$$Q(s\_t, a\_t\ ) \leftarrow Q(s\_t, a\_t\ ) + \ \alpha[R\_(t+1) + \gamma \ \ (max)\top a \ \ 〚Q(s\_(t+1), a) - Q(s\_t, a\_t)], 〛 \tag{10}$$

Where $\alpha \in (0,1)$ is the learning rate that controls the importance of the old against learned value, $\gamma \in (0,1)$ is the discount factor determines how much importance we give to future rewards compared to the immediate reward $R_{t+1}$; see Sutton and Barto [10]. The algorithm's steps are shown in Figure 1.



```
Initialize $Q(s, a)$ arbitrarily
Repeat (for each episode):
    Initialize $s$
    Repeat (for each step of episode):
        Choose $a$ from $s$ using policy derived from $Q$
        Take action $a$, observe $r$, $s'$
        Update
            $Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$
        $s \leftarrow s'$;
    Until s is terminal
```

**Figure 1**. Q-learning (off-policy TD control)

One crucial RL element is the trade-off between exploitation and exploration. Exploration consists of the agent trying all the possible actions at least once to make better action selections in the future. In contrast, exploitation consists of the agent using its current knowledge to obtain the highest reward. To achieve this balance, we use an $\varepsilon$-greedy policy to take optimal actions. The $\varepsilon$-greedy approach selects the action with the highest estimated reward most of the time ($(1-\varepsilon)\times 100$ % of the time).

We chose $\varepsilon$ so that in the initial episodes, the agent starts exploring and gathering information. As time passes and the m$^{th}$ agent collects more information about the environment, $\varepsilon$ vanishes. Finally, when the agent acquired "enough knowledge", it will solely take actions to maximize its reward (no exploration). Also, to ensure convergence to the optimal value, we chose $\varepsilon$ as $\varepsilon_t = \frac{1}{t}$ for the $t^{th}$ episode where both assumptions of $\sum_{t=0}^{\infty}\varepsilon_t^2 < \infty$ and $\sum_{t=0}^{\infty}\varepsilon_t = \infty$ are hold; see Tsitsiklis [11].

If the learning rate ($\alpha$) is set to zero, the action-value function is not updated, and therefore, there will be no learning for the agent. If one chooses the learning rate to be near to one, the learning process will be very quick; Therefore, we update the learning rate after each episode as follows:

$$\alpha(t) = \max(0.1,\ \min(1, 1-\log(\tfrac{t+1}{\gamma}))) \qquad (11)$$

Where $\gamma$ is a problem-specific decay parameter that must be chosen by trial and error.

# 5. Results

## Scenario 1: True failure time distributions are available

It is assumed that each component's failure probability is independent from others, the algorithm searches for the optimal action-value function for each component, and $T_i$ that corresponds to the age where the value of the action 'replace preventively' is higher than that of the action 'do nothing.'

Let $P(t,\lambda_i,k_i)$ be the probability density function of Weibull, $\lambda_i$ the scale parameter, and $k_i$ the shape parameter of the distribution for the $i^{th}$ component. Table 1 reports the component-specific Weibull distribution parameters as well as $tp_i$ and $tf_i$.

The interval between every two decision epochs is assumed to be 5 hours. This value is chosen because the probability that two components will fail during this interval is approximately zero. We used a similar simulation setting as the one proposed by Barde et al. [8] to estimate each approach downtime for each strategy. A comparison of the performance of each strategy is performed between the traditional method, the MRCL, and the QL approaches.

In strategy II, the agent learns the optimal maintenance policy to interact with the environment in ~400 episode by applying MCRL. In contrast, the agent learns the optimal policy in interacting with the environment in ~200 episodes by applying the QL algorithm. Table 2 demonstrates the optimal replacement time for strategy II (in weeks) using the three mentioned approaches. As can be seen, there is a slight difference between the optimal replacement times in all three approaches; however, the optimal replacement times of the MC algorithm seem to be slightly higher than those of the other two approaches.

**Table 1**. Components failure time distribution

| | Tire | Transmission | Wheel | Coupling | Motor | brake | Steering | Gears |
|---|---|---|---|---|---|---|---|---|
| **mean** | 14.06 | 5.903 | 4.218 | 8.332 | 2.039 | 23.32 | 4.868 | 12.13 |
| $\boldsymbol{\lambda_i}$ (scale) | 14.076 | 5.934 | 4.248 | 8.373 | 2.046 | 23.41 | 4.93 | 12.148 |
| $\boldsymbol{k_i}$ (shape) | 378.17 | 108.917 | 79.65 | 115.829 | 170.756 | 143.747 | 43.953 | 278.507 |
| $\boldsymbol{tp_i}$ | 0.0024 | 0.032 | 0.0037 | 0.0051 | 0.0074 | 0.0042 | 0.0026 | 0.0052 |
| $\boldsymbol{tf_i}$ | 0.012 | 0.039 | 0.015 | 0.036 | 0.03 | 0.021 | 0.018 | 0.021 |

**Table 2**. Optimal replacement times (in weeks) for Strategy II

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 13.780 | 13.988 |
| **Transmission** | 5.770 | 5.804 | 8.860 |
| **Wheel** | 3.964 | 3.928 | 4.137 |
| **Coupling** | 7.917 | 7.827 | 8.125 |
| **Motor** | 1.970 | 2.024 | 2.024 |
| **Brake** | 22.381 | 22.292 | 22.798 |
| **Steering** | 4.339 | 4.226 | 4.643 |
| **Gears** | 11.875 | 11.875 | 11.905 |

Table 3 and Figure 2 illustrate a performance comparison among the three approaches in Strategy II. It can be seen that the traditional method has a total downtime of 7.454 weeks with 16 failed components and 867 preventive replaced components due to preventive actions. Those numbers are 7.574 weeks, 25 failed components, 860 preventive replaced components for the QL algorithm and 8.129 weeks, 68 failed components, and 806 preventive replaced components for the MC algorithm. The QL approach outperformed the MC approach; its performance seems similar to the traditional approach, with the traditional approach having a slightly lower system downtime and the number of failed components.

**Table 3**. System downtime (in weeks), number of failed and replaced components of each approach for Strategy II

| | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **System downtime** | 7.454 | 7.574 | 8.129 |
| **number of the failed component** | 16 | 25 | 68 |
| **Number of prevention action** | 867 | 860 | 806 |

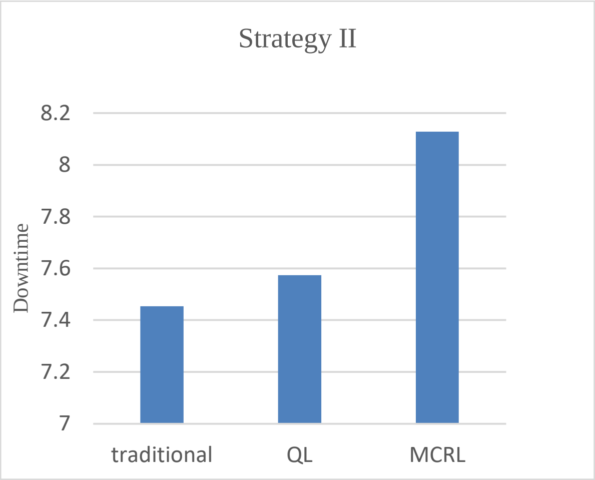


**Figure 2**. System downtime (in weeks) of different approaches for Strategy II

In Strategy III, the agent finds the optimal policy in interacting with the environment in ~400 and ~250 episodes with MC and QL algorithms, respectively. Table 4 reports the optimal replacement times of each component by using different approaches in Strategy III.

**Table 4**. Optimal replacement times (in weeks) for Strategy III

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 13.810 | 13.839 |
| **Transmission** | 5.770 | 5.804 | 5.923 |
| **Wheel** | 3.964 | 3.988 | 4.137 |
| **Coupling** | 7.917 | 7.738 | 8.185 |
| **Motor** | 1.970 | 2.024 | 2.024 |
| **Brake** | 22.381 | 22.530 | 23.036 |
| **Steering** | 4.339 | 4.137 | 4.643 |
| **Gears** | 11.875 | 11.964 | 12.054 |

Table 5 shows the total system downtime of the three approaches in Strategy III. The traditional approach achieves the lowest downtime of 7.657 weeks at a scheduled overhaul of 21 weeks, with 9 failed components and 956 preventive replaced components due to preventive action; The QL algorithm achieves the lowest downtime of 7.763 weeks at a scheduled overhaul of 21 weeks, with 12 and 985 failed and preventive replaced components; Finally, the MC algorithm achieves the lowest downtime of 8.985 weeks at a scheduled overhaul of 15 weeks, with 120 and 912 failed and preventive replaced components. In Strategy III, the traditional method has the lowest overall downtime compared to the other two approaches. The difference between the average downtime of QL and that of the traditional approach was less than one day (~18 hours). However, such difference between the MC and the traditional approaches was as high as one week.

**Table 5**. System downtime (in weeks) obtained by different approaches for Strategy III with different overhaul times

| Schedule overhaul | System downtime | | |
|---|---|---|---|
| | Traditional | Q-learning | MCRL |
| **3** | 10.987 | 11.054 | 11.121 |
| **6** | 10.159 | 10.334 | 11.223 |
| **9** | 8.515 | 8.898 | 9.821 |
| **12** | 8.673 | 8.635 | 9.638 |
| **15** | 7.934 | 8.040 | 8.985 |
| **18** | 8.273 | 8.457 | 9.731 |
| **21** | 7.657 | 7.763 | 9.339 |
| **24** | 8.230 | 8.160 | 9.561 |
| **27** | 7.731 | 7.909 | 9.553 |
| **30** | 7.903 | 7.981 | 9.505 |

Table 6, Table 7, and Figure 3 illustrates similar comparison results among the three approaches for Strategy IV. The agent finds the optimal policy in interacting with the environment in ~1000 and ~200 episodes in MC and QL algorithms. Table 6 reports the optimal replacement times for different components using Strategy IV's three approaches. The replacement times estimated using the RL algorithms are much lower than those obtained through the traditional approach. The reduction in replacement times is due to the group structure in this strategy. Components with a lower mean failure time dominate the overall replacement time of their fellow components. It can be seen in Table 7 that the traditional approach has a total downtime of 8.375 weeks with 27 failed components and 1237 preventive replaced components due to preventive action. The overall system downtime was 9.117 weeks (39 failed and 1348 preventive replaced components) and 10.815 weeks (84 failed and 1618 preventive replaced components) for the QL and MC algorithms. In Strategy IV, the traditional approach has the lowest downtime and number of failed components. It has fewer preventive replacements than the other two approaches (which will lead to lower maintenance costs).

**Table 6**. Optimal replacement times (in weeks) for Strategy IV

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 4.167 | 4.226 |
| **Transmission** | 5.770 | 5.804 | 5.893 |
| **Wheel** | 3.964 | 2.024 | 1.994 |
| **Coupling** | 7.917 | 5.832 | 5.893 |
| **Motor** | 1.970 | 2.024 | 1.994 |
| **Brake** | 22.381 | 4.375 | 4.643 |
| **Steering** | 4.339 | 4.375 | 4.643 |
| **Gears** | 11.875 | 4.167 | 4.167 |

**Table 7**. System downtime (in weeks), number of failed and replaced components of each approach for Strategy IV

| | **Traditional** | **Q-learning** | **MCRL** |
|---|---|---|---|
| **System downtime** | 8.375 | 9.117 | 10.815 |
| **number of failed components** | 27 | 39 | 84 |
| **Number of prevention action** | 1237 | 1348 | 1618 |

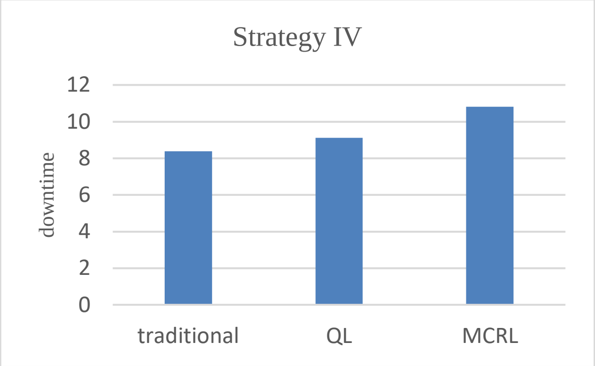


**Figure 3**. System downtime (in weeks) for Strategy IV

According to Table *8*, Table *9*, and Table *10*, Strategy I is the worst strategy as it has the longest downtime. This shows the clear advantage of preventive strategies over corrective maintenance strategies. Overall, the most efficient strategy among these proposed strategies is Strategy II.

Table 11 reports the average execution time for each approach and strategy to obtain the optimal policy. As seen, the traditional approach was the most time-efficient (less than a second). After the traditional approach, the QL algorithm was about twice faster than the on-policy first visit MC algorithm. As expected, the more complex a strategy is, the more time requires to obtain its optimal policy.

**Table 8**. Evaluation of traditional approach for different strategies

| | **Strategy I** | **Strategy II** | **Strategy III** | **Strategy IV** |
|---|---|---|---|---|
| **System downtime** | 21.366 | 7.454 | 7.657 | 8.375 |
| **number of failed components** | 842 | 16 | 9 | 27 |
| **Number of prevention action** | 0 | 867 | 956 | 1237 |

**Table 9**. Evaluation of QL algorithm for different strategies

| | **Strategy II** | **Strategy III** | **Strategy IV** |
|---|---|---|---|
| **System downtime** | 7.574 | 7.762 | 9.117 |
| **number of failed components** | 25 | 12 | 39 |
| **Number of prevention action** | 860 | 985 | 1348 |

**Table 10**. Evaluation of MC algorithm for different strategies

| | **Strategy II** | **Strategy III** | **Strategy IV** |
|---|---|---|---|
| **System downtime** | 8.129 | 8.985 | 10.815 |
| **number of failed components** | 68 | 120 | 84 |
| **Number of prevention action** | 806 | 912 | 1618 |

**Table 11**.Execution time to converge to the optimal policy by different approaches in different strategies

| | **Strategy II** | **Strategy III** | **Strategy IV** |
|---|---|---|---|
| **Traditional** | < 1 sec | < 1 sec | < 1 sec |
| **QL** | 2 min | 2 min | 22 min |
| **MC** | 4 min | 3 min | 53 min |

## Scenario 2: Misspecified failure time distributions are available

The results reported in Scenario 1 are under the assumption of knowing the environment and, therefore, the true failure time distribution of different components was available. However, under a more realistic scenario, the true failure time distribution of the components may not be available.

In this section, we evaluate the performance of the three approaches under the components' misspecified failure time distribution. More specifically, we assumed the components' true distribution of failure time remains Weibull with the same parameters reported in Table 1. However, a misspecified Weibull distribution (either its shape or scale parameter is overestimated by different degrees) is assumed for each component while finding the optimal policy by each approach.

The RL free-model algorithms require no environmental assumptions, and the agent interacts with the environment directly (data-driven). That is, the misspecified failure time distribution will not impact them.

Table 12 reports the optimal replacement times obtained through "Eq. (1)" by assuming an overestimated Weibull shape parameter. As shown in Figure 4, Figure 5, and Figure 6, the misspecification of the Weibull shape parameter does not seem to have a large impact on the estimated optimal replacement time of the components by the traditional approach. When using the optimal replacement time of the traditional approach, the estimated system downtime seemed to be impacted differently in different strategies. Specifically, the traditional system downtime in Strategy II and III was slightly impacted (increased by 1.5-3 days) only when the shape parameter was overestimated by at least 20%.

**Table 12**.Optimal replacement times with minor changes in the Weibull shape parameters.

| Component Name | Shape (real) | Shape +10% | Shape +20% | Shape +30% | Shape +40% | Shape +50% |
|---|---|---|---|---|---|---|
| **Tire** | 13.81 | 13.85 | 13.86 | 13.88 | 13.89 | 13.90 |
| **Transmission** | 5.77 | 8.78 | 5.79 | 5.80 | 5.80 | 5.81 |
| **Wheel** | 3.96 | 3.99 | 4.01 | 4.02 | 4.03 | 4.04 |
| **Coupling** | 7.92 | 7.95 | 7.98 | 8 | 8.02 | 8.04 |
| **Motor** | 1.97 | 1.98 | 1.99 | 1.99 | 1.99 | 1.99 |
| **Brake** | 22.38 | 22.47 | 22.54 | 22.60 | 22.67 | 22.68 |
| **Steering** | 4.34 | 4.39 | 4.42 | 4.45 | 4.48 | 4.50 |
| **Gears** | 11.88 | 11.89 | 11.90 | 11.92 | 11.94 | 11.95 |

Figure 7, Figure 8 and Figure 9 illustrate system downtime obtained by the traditional approach compared to RL algorithms. As the figures show, the Weibull scale parameter overestimation from 2% to 5% increased the system downtime by 1 to 8 weeks in different strategies.

Moreover, Table 13 shows how minor changes in the Weibull scale parameter affect the optimal replacement times obtained through "Eq. (1)".

**Table 13**.Optimal replacement times with minor changed in the Weibull scale parameter.

| Component Name | Scale (real) | Scale +1% | Scale +2% | Scale +3% | Scale +4% |
|---|---|---|---|---|---|
| **Tire** | 13.81 | 13.96 | 14.11 | 14.23 | 14.38 |
| **Transmission** | 5.77 | 5.83 | 5.89 | 5.92 | 6 |
| **Wheel** | 3.96 | 3.99 | 4.05 | 4.08 | 4.12 |
| **Coupling** | 7.92 | 7.98 | 8.07 | 8.16 | 8.21 |
| **Motor** | 1.97 | 1.99 | 2.01 | 2.03 | 2.05 |
| **Brake** | 22.38 | 22.62 | 22.86 | 23.07 | 23.27 |
| **Steering** | 4.34 | 4.39 | 4..43 | 4.49 | 4.51 |
| **Gears** | 11.88 | 11.99 | 12.11 | 12.20 | 12.34 |

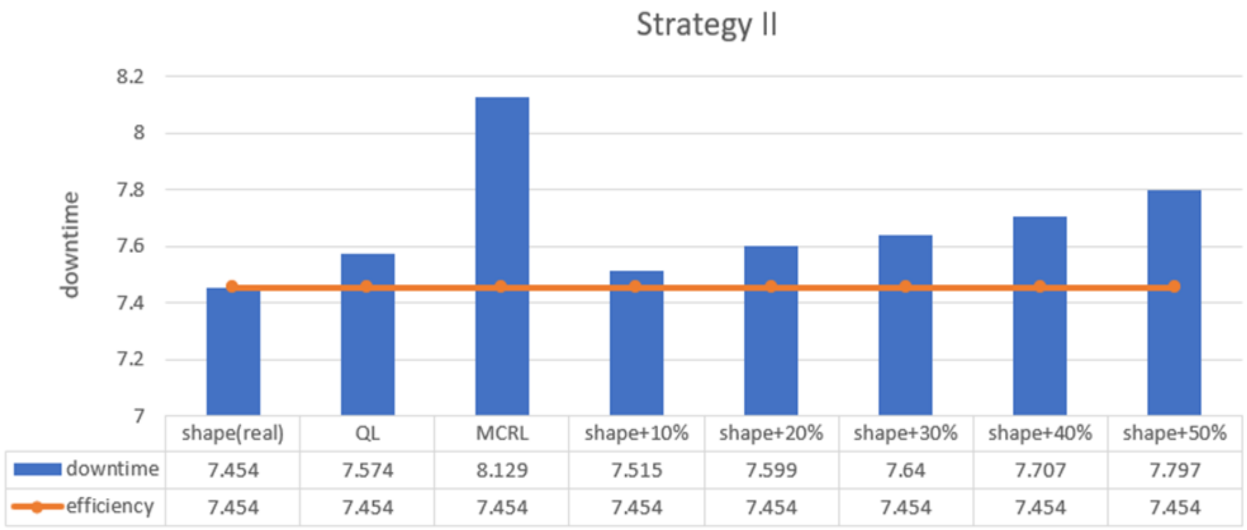


**Figure 4**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy II

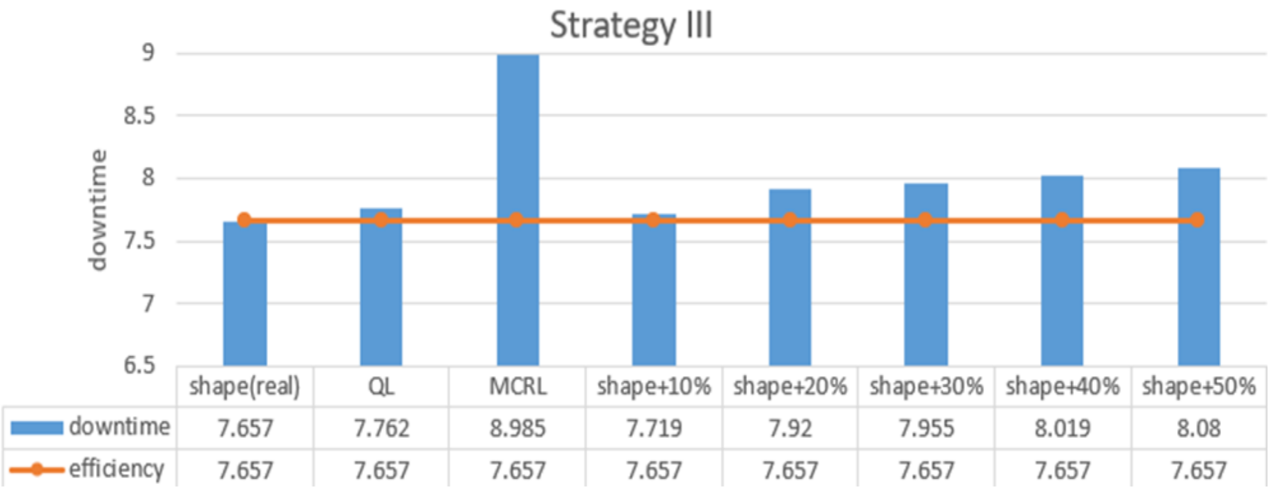
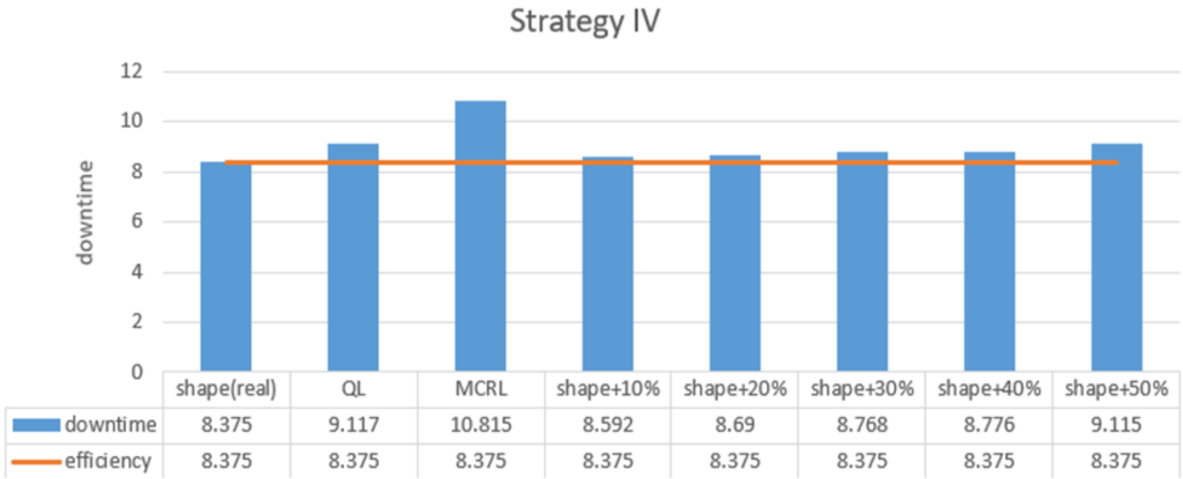
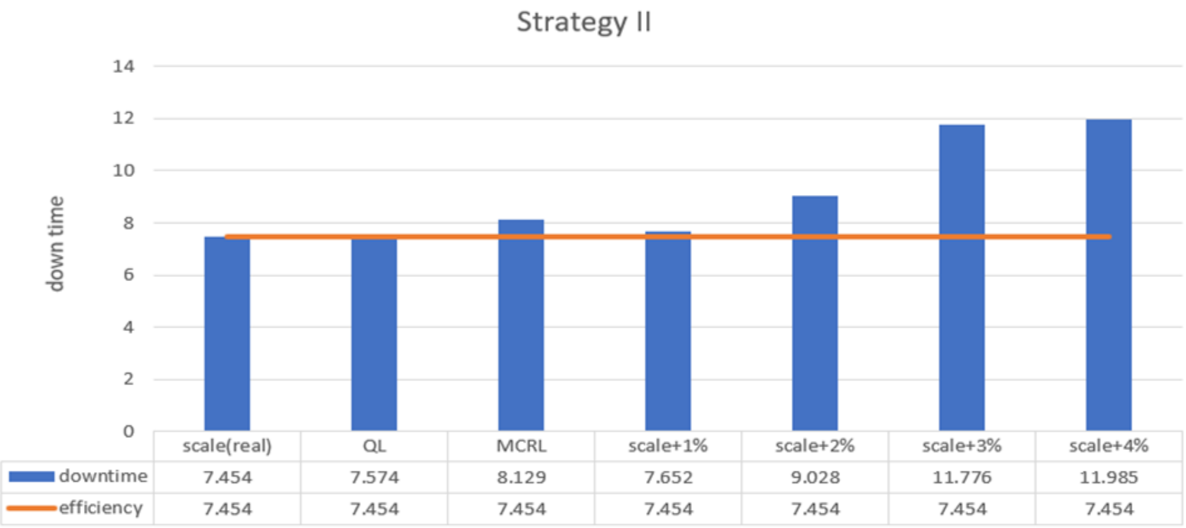


**Figure 5**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy III



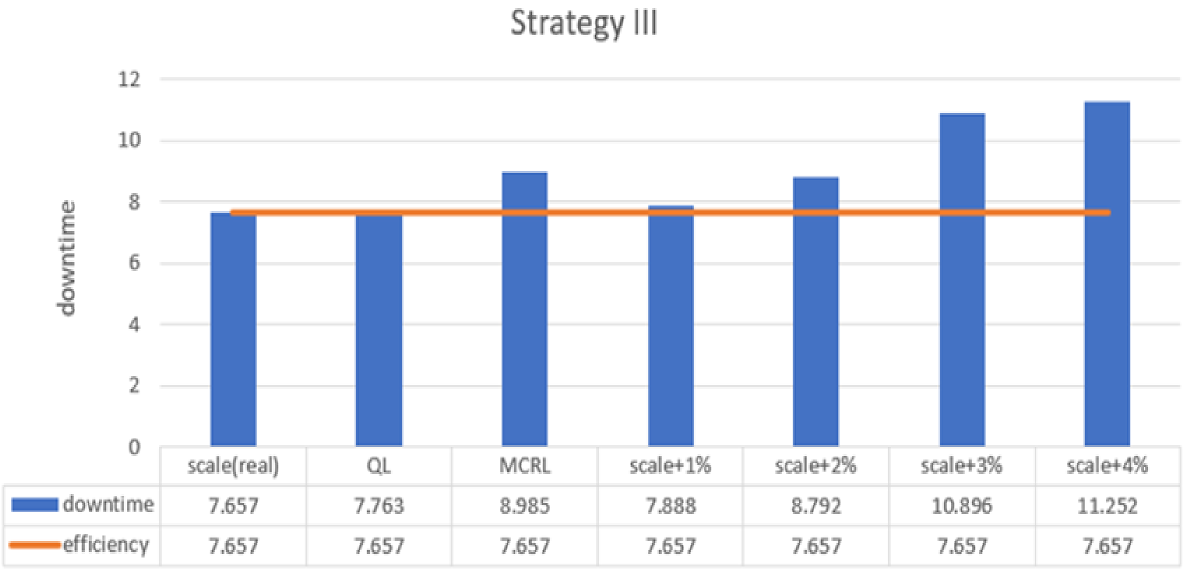
| | shape(real) | QL | MCRL | shape+10% | shape+20% | shape+30% | shape+40% | shape+50% |
|---|---|---|---|---|---|---|---|---|
| downtime | 8.375 | 9.117 | 10.815 | 8.592 | 8.69 | 8.768 | 8.776 | 9.115 |
| efficiency | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 |

**Figure 6**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy IV



| | scale(real) | QL | MCRL | scale+1% | scale+2% | scale+3% | scale+4% |
|---|---|---|---|---|---|---|---|
| downtime | 7.454 | 7.574 | 8.129 | 7.652 | 9.028 | 11.776 | 11.985 |
| efficiency | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 |

**Figure 7**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy II



| | scale(real) | QL | MCRL | scale+1% | scale+2% | scale+3% | scale+4% |
|---|---|---|---|---|---|---|---|
| downtime | 7.657 | 7.763 | 8.985 | 7.888 | 8.792 | 10.896 | 11.252 |
| efficiency | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 |

**Figure 8**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy III

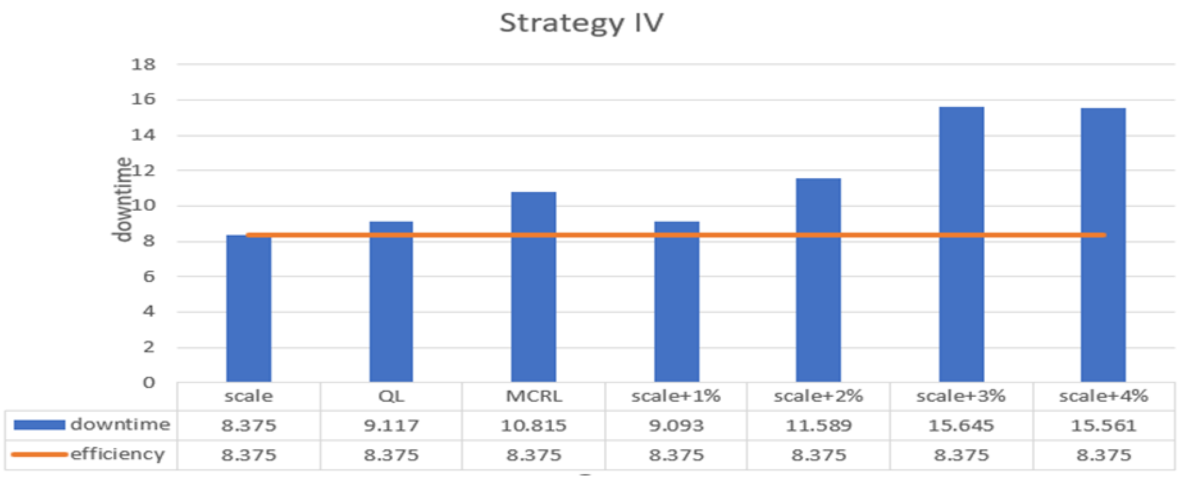


**Figure 9**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy IV

## 6. Discussion

In this work, we employed three traditional (renewal theory) approaches, MCRL and TDRL, to find the optimal preventive maintenance policy for equipment composed of multi-non-identical components. Three preventive maintenance strategies, along with a corrective maintenance strategy (as baseline), were studied. Our results confirmed that preventive maintenance strategies perform better than the corrective maintenance policy, as expected, for our system. More importantly, our results showed that the traditional approach (renewal theory) is sensitive to the misspecification of the components' failure time distribution. More specifically, under the assumption of the components Weibull distributed failure times, the optimal policy and, consequently, the performance of the traditional approach seem to be impacted only slightly by misspecifying the shape parameters up to 50% (downtime increased by < 3 days). However, even minor misspecification in the scale parameter (up to 5%) can lead to a huge increase in the system downtime following the traditional approach optimal policy by up to 8 weeks. On the other hand, since the model-free RL algorithms are data-driven with no requirements of prior assumption on the environment distribution (e.g., failure time distributions), they can be minimally impacted by such misspecifications.

Different RL algorithms, however, can potentially perform very differently. Under the assumptions of our study, the QL algorithm outperformed the MC algorithm dramatically. Given the quick progress in developing RL algorithms nowadays, a natural next step to our work might be evaluating different RL algorithms for different systems with different assumptions.

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Original Research Article

# Optimal Preventive Maintenance Policy for Non-Identical Components: Traditional Renewal Theory vs Modern Reinforcement Learning

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## Abstract

This paper compares the traditional approach against reinforcement learning algorithms to find the optimal preventive maintenance policy for equipment composed of multi-non-identical components with different time-to-failure distributions. As an application, we used the data from military trucks, which consisted of multiple components with very different failure behavior, such as tires, transmissions, wheel rims, couplings, motors, brakes, steering wheels, and shifting gears. The literature proposes Four different strategies for preventive maintenance of these components. To find the optimal preventive manganocene policy, we used the traditional approach (renewal theory-based) and the conventional reinforcement learning algorithms and compared their performance. The main advantages of the latter approach are that, unlike the traditional approach, they are not required to estimate the model parameters (e.g., transition probabilities). Without any explicit mathematical formula, they converge to the optimal solution. Our results showed that the traditional approach works best when the component time-to-failure distributions are available. However, the reinforcement learning approach outperforms where no such information is available or the distributions are misspecified.

**Keyword:** Opportunistic maintenance; Preventive maintenance; Markov decision process; Monte Carlo; Q-learning; Reinforcement learning

## 1. Introduction

In the recently released European Standards regarding maintenance, maintenance is defined as the combination of all technical, administrative, and managerial actions during the life cycle of an item intended to retain it in, or restore it to, a state in which it can perform the required function; see Marquez and Gupta [1]. Maintenance problems can be solved using traditional approaches and machine learning methods. In recent years, reinforcement learning (RL) algorithms have become very popular and widely used. RL is one of the newer machine learning approaches that has gained prominence in various fields of human life today. In general, RL is a technique that allows a decision-making (agent) to maximize his total reward by interacting with the environment. Ravichandiran [2] introduced the steps of a typical RL algorithm as follows:

1. First, the agent interacts with the environment by performing an action
2. The agent acts and moves from one state to another
3. And then, the agent will receive a reward based on the action it performed
4. Based on the reward, the agent will understand whether the action was good or bad
5. If the action was good, if the agent received a positive reward, then the agent will prefer performing that action again. Otherwise, the agent will try performing another action, resulting in a positive reward. So it is a trial-and-error learning process.

As mentioned earlier, RL algorithms are used a lot in most fields. Wang et al. [3] applied multi-agent RL to solve the maintenance problem for a flow line system consisting of two series machines with an intermediate finite buffer in between. Liang et al. [4]modeled the energy management problem by a Markov decision process and solved it using an Approximate Dynamic Programming (ADP)-based approach to match electricity supply and demand. Yousefi et al. [5] used an RL approach to develop a new dynamic maintenance policy

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for multi-component systems with individually repairable components, where each component is at risk of two competing failure processes of degradation and random shocks. Adsule et al. [6] modeled the Condition-based maintenance (CBM) decision-making as a continuous semi-Markov decision process. They applied an RL algorithm to learn the optimal maintenance decisions and inspection schedules based on the current health state of the component.

In this paper, we consider military trucks composed of multi-non-identical components. Trucks are systems that are used continuously, so the possibility of them breaking down on the road is very high, resulting in financial and life-threatening costs. Additionally, trucks are used in the military, so minimizing the downtime of any truck is essential. It is important to obtain the optimal replacement times for each system component efficiently. Haleem and Yacout [7] and Barde et al. [8] tackled this problem before. They used the truck's eight more important components in their analysis: tires, transmissions, wheel rims, couplings, motors, brakes, steering wheels, and shifting gears. We will use the same set of components here as well. Abdel Haleem and Yacout [7] used renewal theory to estimate the components' replacement times. Barde et al. [8] estimated replacement times by using Monte Carlo reinforcement learning (MCRL). We aimed to obtain the optimal maintenance policy by using the two existing approaches in the literature and employing a time difference (TD) learning approach, a more efficient RL algorithm than MCRL. We will evaluate the performance of all these approaches under two scenarios: when the true failure time distributions are available versus misspecified.

In the next section, we will present the problem assumptions and existing maintenance strategies; In the Method section, we will present different algorithms. Finally, in the Results section, we will report the numerical results comparing the TD-based RL algorithm's performance against the two other methods proposed in the literature.

## 2. Motivation

We consider equipment that contains multiple non-identical components. Our purpose in this article is to find a policy that minimizes the total downtime of the equipment. The downtime is defined as the non-productive time when the system is not operational due to a failure or a preventive action. Each equipment component has a different time-to-failure distribution modeled by a Weibull distribution with its shape and scale parameters. The strategies are based on the following assumptions:

1. If we replace a component due to failure, it takes more time than if we replace a component preventively.
2. If we replace a group of components or a whole system, it takes less time than if we replace each component separately.
3. There are replacement opportunities at regular intervals.

The assumptions mentioned above can be found in many military applications, where the equipment's reliability is essential, downtime must be minimized, and cost considerations are less important; see Haleem and Yacout [7].

Following Haleem and Yacout [7], the following four replacement strategies will be used and compared against one another:

- **Strategy I**: Every component is replaced upon failure. It is corrective maintenance (baseline).
- **Strategy II**: every component is replaced upon failure and at an individual fixed interval, $T_i$, for component i. It is based on preventive maintenance. Haleem and Yacout [7] estimated $T_i$ by minimizing the downtime per unit time, $D_i$, for component i. $D_i$ is calculated from the following expression:

$$argmin_{T_i} D_i = \frac{tp_i R(T_i)+tf_i[1-R(T_i)]}{(T_i+tp_i)R(T_i)+[tf_i+E(t|t\leq T_i)][1-R(T_i)]}, \forall i \qquad (1)$$

Where $tp_i$ is time to replace component i preventively, $tf_i$ is time to replace component i upon failure, $R(T_i)$ is the reliability of component i at the time $T_i$ and $E[t|t \leq T_i]$ is the expected time to failure, given that it occurs before $T_i$.

- **Strategy III**: It is based on Strategy II, to which it is added a scheduled overhaul. In other words, as in Strategy II, every component is replaced at failure and replacement intervals $T_i$ for component I, the whole system is replaced at a known fixed time.
- **Strategy IV**: It is a group-based maintenance strategy. Any component i that fails or reaches its replacement interval $T_i$, the components of its group are also replaced with it.

## 3. Markov Decision Process

A Markov decision process (MDP) framework has the following key components:

1. S: Set of states ($s \in S$)
2. A: Set of actions ($a \in A$)
3. $P(s_{t+1}|s_t, a_t)$: Transition probabilities
4. $R(s, a)$: Reward function of doing action in the states.

We use model-free RL for two reasons: the curse of dimensionality and the curse of modeling. The curse of dimensionality arises from the much longer computational time and much larger memory space needed as the state space of a problem becomes larger. The curse of modeling arises from the need to estimate the transition probabilities, which is often difficult, especially when the state space is large; see Powell [9]. We present MDP formulation (state, action, and reward function) for each preventive strategy (i.e., II, III, and IV).

**MDP formulation of strategy II**: Let $G_i$ be the age of component i, $f_i = 1$ denotes that component i is failed and $f_i = 0$ denotes its normal status, so the state of the system at time t is the vector defined as follows:

$$s_t = G_1, \ldots, G_8, f_1, \ldots, f_8.$$

Let $a_i$ = 1 means PM action and $a_i$ = 0 means "do nothing" action, then the action of the system at time t is:

$$a_t = (a_1, \dots, a_8). \tag{2}$$

Based on Barde et al. [8] work, the reward function can be defined as follows:

$$R(s_t, a_t) = \begin{cases} -\alpha_i.\, tp_i\,, & if\ a_i = 1 \\ -\alpha_i.\Delta.\left\lceil \frac{tf_i}{\Delta} \right\rceil, & a_i = 0\ and\ f_i = 1 \\ \Delta\,, & otherwise \end{cases} \tag{2}$$

Where $\alpha_i = \frac{\Delta}{tp_i}$ is a scale factor as they assumed that $tp_i$ is scaled such that it has at least the same period than $\Delta$ (see Barde et al. [8] for more information); $\Delta$ is the time interval between two epochs and $\lceil . \rceil$ is the ceiling function.

**MDP formulation of strategy III**: Let $G_i$ be the age of component i, $f_i = 1$ denotes that component i is failed whereas $f_i = 0$ denotes normal status and $O = 1$ means to replace the whole system, whereas $O = 0$ denotes that don't replace the whole system; then, the state of the system at time t is the vector defined as follows:

$$s_t = (G_1, \dots, G_8, O, f_1, \dots, f_8). \tag{3}$$

The action on the system is defined as:

$$a_t = (a_1, \dots, a_8). \tag{4}$$

The reward function is:

$$R(s_t, a_t) = \begin{cases} -\alpha_i.\, tp_i, & if\ a_i = 1\,, O = 0 \\ -\alpha_i.\Delta.\left\lceil \frac{tf_i}{\Delta} \right\rceil, & if\ a_i = 0, O = 0, f_i = 1 \\ -\alpha_i.\Delta.\left\lceil \frac{\beta.\sum_{i=1}^{8} tp_i}{\Delta} \right\rceil, & if\ O = 1 \\ \Delta, & otherwise \end{cases} \tag{5}$$

Where $\beta \in (0, 1)$ comes from the assumption that the time to replace the whole system is less than the sum of times to replace each component separately.

**MDP formulation of Strategy IV**: states and actions in Strategy IV are the same as those in Strategy II, but the reward function is different due to group structure. The components are grouped as follows:

$$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) = (\{1{,}3\}, \{3{,}8\}, \{3{,}5\}, \{7{,}6\}, \{4{,}2\}) \tag{6}$$

The groups are formed based on technical reasons, such as the difficulty or ease of reaching and changing a component when a neighboring component has failed. Then, the reward function is:

$$R(s_t, a_t) = \begin{cases} -\alpha_i.\beta.\sum_{l\in\phi_j} tp_l\,, & if\ a_k = 1\ and\ k \in \phi_j \\ -\alpha_i.\Delta.\left\lceil \frac{\beta.\sum_{l\in\phi_j} tf_l}{\Delta} \right\rceil, & if\ f_k = 1,\ \alpha_k = 0, k \in \phi_j \\ \Delta, & otherwise \end{cases} \tag{7}$$

Where $\alpha_i = \frac{\Delta}{\beta.\Delta.\sum_{l\in\phi_j} tp_l}$ and $\beta \in (0{,}1)$ is defined similarly as in Strategy III.

# 4. Reinforcement Learning

Barde et al. [8] used the on-policy first-visit MCRL algorithm to find the optimal replacement time $T_i$ for each preventive strategy separately, whereas we will use a TD learning approach that is more efficient than MCRL.

TD learning is a model-free approach that combines sampling and bootstrapping simultaneously. One of the advantages of TD over MCRL is that MCRL can only be used for episodic problems. In other words, MCRL learns from complete episodes only. Unlike MCRL, TD learning employs single steps to learn (be updated after every step) and does not need to wait until the end of an episode. Therefore, TD learning can be applied to both continuing and episodic problems.

In this paper, for the military trucks problem, we will use a Q-learning (QL) algorithm, which is an off-policy TD control algorithm. QL is one of the most popular and efficient algorithms in RL. It is an off-policy RL algorithm because the QL function learns from actions not necessarily taken under the current agent policy. Like other RL algorithms, QL seeks to learn a policy that maximizes a pre-defined total reward in every state. The objective of the QL algorithm is to learn and estimate the optimal action-value function that defined as

$$Q^*(s_t, a_t) = \max_{\pi} Q_\pi(s_t, a_t), \tag{8}$$

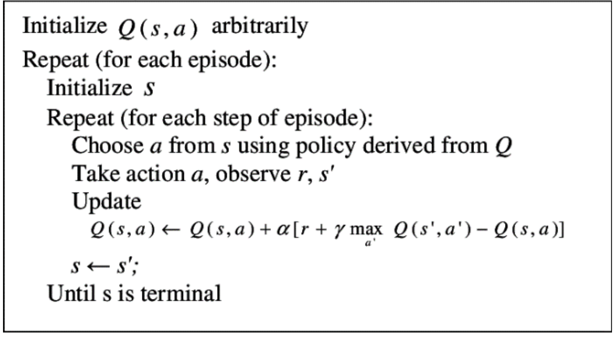
where

$$Q_\pi(s_t, a_t) = \mathbb{E}_\pi[\textstyle\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} |\ s_t, a_t] \tag{9}$$

QL directly approximates the optimal action-value function by taking the best action when bootstrapping:

$$Q(s\_t, a\_t\ ) \leftarrow Q(s\_t, a\_t\ ) +\ \alpha[R\_(t+1) + \gamma\ \ (max)\top a\ \ 〖Q(s\_(t+1), a) - Q(s\_t, a\_t)],〗 \tag{10}$$

Where $\alpha \in (0{,}1)$ is the learning rate that controls the importance of the old against learned value, $\gamma \in (0{,}1)$ is the discount factor determines how much importance we give to future rewards compared to the immediate reward $R_{t+1}$; see Sutton and Barto [10]. The algorithm's steps are shown in Figure 1.



```
Initialize $Q(s,a)$ arbitrarily
Repeat (for each episode):
  Initialize $s$
  Repeat (for each step of episode):
    Choose $a$ from $s$ using policy derived from $Q$
    Take action $a$, observe $r$, $s'$
    Update
      $Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$
    $s \leftarrow s'$;
  Until s is terminal
```

**Figure 1**. Q-learning (off-policy TD control)

One crucial RL element is the trade-off between exploitation and exploration. Exploration consists of the agent trying all the possible actions at least once to make better action selections in the future. In contrast, exploitation consists of the agent using its current knowledge to obtain the highest reward. To achieve this balance, we use an $\varepsilon$-greedy policy to take optimal actions. The $\varepsilon$-greedy approach selects the action with the highest estimated reward most of the time ($(1-\varepsilon) \times 100$ % of the time).

We chose $\varepsilon$ so that in the initial episodes, the agent starts exploring and gathering information. As time passes and the m$^{th}$ agent collects more information about the environment, $\varepsilon$ vanishes. Finally, when the agent acquired "enough knowledge", it will solely take actions to maximize its reward (no exploration). Also, to ensure convergence to the optimal value, we chose $\varepsilon$ as $\varepsilon_t = \frac{1}{t}$ for the $t^{th}$ episode where both assumptions of $\sum_{t=0}^{\infty} \varepsilon_t^2 < \infty$ and $\sum_{t=0}^{\infty} \varepsilon_t = \infty$ are hold; see Tsitsiklis [11].

If the learning rate ($\alpha$) is set to zero, the action-value function is not updated, and therefore, there will be no learning for the agent. If one chooses the learning rate to be near to one, the learning process will be very quick; Therefore, we update the learning rate after each episode as follows:

$$\alpha(t) = \max(0.1,\ \min(1, 1 - \log(\tfrac{t+1}{\gamma}))) \qquad (11)$$

Where $\gamma$ is a problem-specific decay parameter that must be chosen by trial and error.

# 5. Results

## Scenario 1: True failure time distributions are available

It is assumed that each component's failure probability is independent from others, the algorithm searches for the optimal action-value function for each component, and $T_i$ that corresponds to the age where the value of the action 'replace preventively' is higher than that of the action 'do nothing.'

Let $P(t, \lambda_i, k_i)$ be the probability density function of Weibull, $\lambda_i$ the scale parameter, and $k_i$ the shape parameter of the distribution for the $i^{th}$ component. Table 1 reports the component-specific Weibull distribution parameters as well as $tp_i$ and $tf_i$.

The interval between every two decision epochs is assumed to be 5 hours. This value is chosen because the probability that two components will fail during this interval is approximately zero. We used a similar simulation setting as the one proposed by Barde et al. [8] to estimate each approach downtime for each strategy. A comparison of the performance of each strategy is performed between the traditional method, the MRCL, and the QL approaches.

In strategy II, the agent learns the optimal maintenance policy to interact with the environment in ~400 episode by applying MCRL. In contrast, the agent learns the optimal policy in interacting with the environment in ~200 episodes by applying the QL algorithm. Table 2 demonstrates the optimal replacement time for strategy II (in weeks) using the three mentioned approaches. As can be seen, there is a slight difference between the optimal replacement times in all three approaches; however, the optimal replacement times of the MC algorithm seem to be slightly higher than those of the other two approaches.

**Table 1**. Components failure time distribution

| | Tire | Transmission | Wheel | Coupling | Motor | brake | Steering | Gears |
|---|---|---|---|---|---|---|---|---|
| **mean** | 14.06 | 5.903 | 4.218 | 8.332 | 2.039 | 23.32 | 4.868 | 12.13 |
| $\lambda_i$ (scale) | 14.076 | 5.934 | 4.248 | 8.373 | 2.046 | 23.41 | 4.93 | 12.148 |
| $k_i$ (shape) | 378.17 | 108.917 | 79.65 | 115.829 | 170.756 | 143.747 | 43.953 | 278.507 |
| $tp_i$ | 0.0024 | 0.032 | 0.0037 | 0.0051 | 0.0074 | 0.0042 | 0.0026 | 0.0052 |
| $tf_i$ | 0.012 | 0.039 | 0.015 | 0.036 | 0.03 | 0.021 | 0.018 | 0.021 |

**Table 2**. Optimal replacement times (in weeks) for Strategy II

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 13.780 | 13.988 |
| **Transmission** | 5.770 | 5.804 | 8.860 |
| **Wheel** | 3.964 | 3.928 | 4.137 |
| **Coupling** | 7.917 | 7.827 | 8.125 |
| **Motor** | 1.970 | 2.024 | 2.024 |
| **Brake** | 22.381 | 22.292 | 22.798 |
| **Steering** | 4.339 | 4.226 | 4.643 |
| **Gears** | 11.875 | 11.875 | 11.905 |

Table 3 and Figure 2 illustrate a performance comparison among the three approaches in Strategy II. It can be seen that the traditional method has a total downtime of 7.454 weeks with 16 failed components and 867 preventive replaced components due to preventive actions. Those numbers are 7.574 weeks, 25 failed components, 860 preventive replaced components for the QL algorithm and 8.129 weeks, 68 failed components, and 806 preventive replaced components for the MC algorithm. The QL approach outperformed the MC approach; its performance seems similar to the traditional approach, with the traditional approach having a slightly lower system downtime and the number of failed components.

**Table 3**. System downtime (in weeks), number of failed and replaced components of each approach for Strategy II

| | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **System downtime** | 7.454 | 7.574 | 8.129 |
| **number of the failed component** | 16 | 25 | 68 |
| **Number of prevention action** | 867 | 860 | 806 |

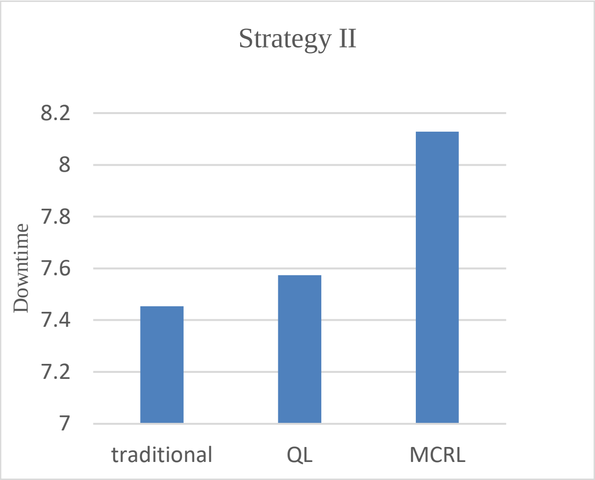


**Figure 2**. System downtime (in weeks) of different approaches for Strategy II

In Strategy III, the agent finds the optimal policy in interacting with the environment in ~400 and ~250 episodes with MC and QL algorithms, respectively. Table 4 reports the optimal replacement times of each component by using different approaches in Strategy III.

**Table 4**. Optimal replacement times (in weeks) for Strategy III

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 13.810 | 13.839 |
| **Transmission** | 5.770 | 5.804 | 5.923 |
| **Wheel** | 3.964 | 3.988 | 4.137 |
| **Coupling** | 7.917 | 7.738 | 8.185 |
| **Motor** | 1.970 | 2.024 | 2.024 |
| **Brake** | 22.381 | 22.530 | 23.036 |
| **Steering** | 4.339 | 4.137 | 4.643 |
| **Gears** | 11.875 | 11.964 | 12.054 |

Table 5 shows the total system downtime of the three approaches in Strategy III. The traditional approach achieves the lowest downtime of 7.657 weeks at a scheduled overhaul of 21 weeks, with 9 failed components and 956 preventive replaced components due to preventive action; The QL algorithm achieves the lowest downtime of 7.763 weeks at a scheduled overhaul of 21 weeks, with 12 and 985 failed and preventive replaced components; Finally, the MC algorithm achieves the lowest downtime of 8.985 weeks at a scheduled overhaul of 15 weeks, with 120 and 912 failed and preventive replaced components. In Strategy III, the traditional method has the lowest overall downtime compared to the other two approaches. The difference between the average downtime of QL and that of the traditional approach was less than one day (~18 hours). However, such difference between the MC and the traditional approaches was as high as one week.

**Table 5**. System downtime (in weeks) obtained by different approaches for Strategy III with different overhaul times

| Schedule overhaul | System downtime | | |
|---|---|---|---|
| | Traditional | Q-learning | MCRL |
| **3** | 10.987 | 11.054 | 11.121 |
| **6** | 10.159 | 10.334 | 11.223 |
| **9** | 8.515 | 8.898 | 9.821 |
| **12** | 8.673 | 8.635 | 9.638 |
| **15** | 7.934 | 8.040 | 8.985 |
| **18** | 8.273 | 8.457 | 9.731 |
| **21** | 7.657 | 7.763 | 9.339 |
| **24** | 8.230 | 8.160 | 9.561 |
| **27** | 7.731 | 7.909 | 9.553 |
| **30** | 7.903 | 7.981 | 9.505 |

Table 6, Table 7, and Figure 3 illustrates similar comparison results among the three approaches for Strategy IV. The agent finds the optimal policy in interacting with the environment in ~1000 and ~200 episodes in MC and QL algorithms. Table 6 reports the optimal replacement times for different components using Strategy IV's three approaches. The replacement times estimated using the RL algorithms are much lower than those obtained through the traditional approach. The reduction in replacement times is due to the group structure in this strategy. Components with a lower mean failure time dominate the overall replacement time of their fellow components. It can be seen in Table 7 that the traditional approach has a total downtime of 8.375 weeks with 27 failed components and 1237 preventive replaced components due to preventive action. The overall system downtime was 9.117 weeks (39 failed and 1348 preventive replaced components) and 10.815 weeks (84 failed and 1618 preventive replaced components) for the QL and MC algorithms. In Strategy IV, the traditional approach has the lowest downtime and number of failed components. It has fewer preventive replacements than the other two approaches (which will lead to lower maintenance costs).

**Table 6**. Optimal replacement times (in weeks) for Strategy IV

| Component Name | Traditional | Q-learning | MCRL |
|---|---|---|---|
| **Tire** | 13.809 | 4.167 | 4.226 |
| **Transmission** | 5.770 | 5.804 | 5.893 |
| **Wheel** | 3.964 | 2.024 | 1.994 |
| **Coupling** | 7.917 | 5.832 | 5.893 |
| **Motor** | 1.970 | 2.024 | 1.994 |
| **Brake** | 22.381 | 4.375 | 4.643 |
| **Steering** | 4.339 | 4.375 | 4.643 |
| **Gears** | 11.875 | 4.167 | 4.167 |

**Table 7**. System downtime (in weeks), number of failed and replaced components of each approach for Strategy IV

| | Traditional | Q-learning | MCRL |
|---|---|---|---|
| System downtime | 8.375 | 9.117 | 10.815 |
| number of failed components | 27 | 39 | 84 |
| Number of prevention action | 1237 | 1348 | 1618 |

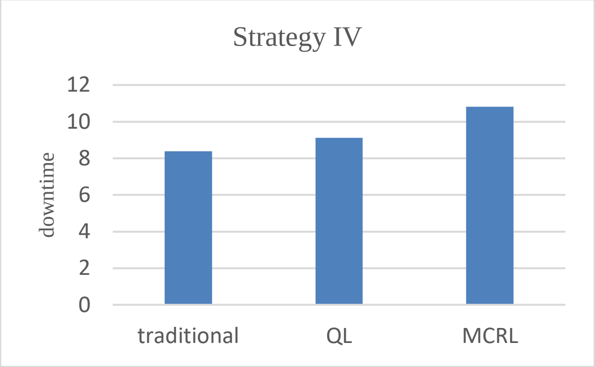


**Figure 3**. System downtime (in weeks) for Strategy IV

According to Table *8*, Table *9*, and Table *10*, Strategy I is the worst strategy as it has the longest downtime. This shows the clear advantage of preventive strategies over corrective maintenance strategies. Overall, the most efficient strategy among these proposed strategies is Strategy II.

Table 11 reports the average execution time for each approach and strategy to obtain the optimal policy. As seen, the traditional approach was the most time-efficient (less than a second). After the traditional approach, the QL algorithm was about twice faster than the on-policy first visit MC algorithm. As expected, the more complex a strategy is, the more time requires to obtain its optimal policy.

**Table 8**. Evaluation of traditional approach for different strategies

| | Strategy I | Strategy II | Strategy III | Strategy IV |
|---|---|---|---|---|
| System downtime | 21.366 | 7.454 | 7.657 | 8.375 |
| number of failed components | 842 | 16 | 9 | 27 |
| Number of prevention action | 0 | 867 | 956 | 1237 |

**Table 9**. Evaluation of QL algorithm for different strategies

| | Strategy II | Strategy III | Strategy IV |
|---|---|---|---|
| System downtime | 7.574 | 7.762 | 9.117 |
| number of failed components | 25 | 12 | 39 |
| Number of prevention action | 860 | 985 | 1348 |

**Table 10**. Evaluation of MC algorithm for different strategies

| | Strategy II | Strategy III | Strategy IV |
|---|---|---|---|
| System downtime | 8.129 | 8.985 | 10.815 |
| number of failed components | 68 | 120 | 84 |
| Number of prevention action | 806 | 912 | 1618 |

**Table 11**.Execution time to converge to the optimal policy by different approaches in different strategies

| | Strategy II | Strategy III | Strategy IV |
|---|---|---|---|
| Traditional | < 1 sec | < 1 sec | < 1 sec |
| QL | 2 min | 2 min | 22 min |
| MC | 4 min | 3 min | 53 min |

## Scenario 2: Misspecified failure time distributions are available

The results reported in Scenario 1 are under the assumption of knowing the environment and, therefore, the true failure time distribution of different components was available. However, under a more realistic scenario, the true failure time distribution of the components may not be available.

In this section, we evaluate the performance of the three approaches under the components' misspecified failure time distribution. More specifically, we assumed the components' true distribution of failure time remains Weibull with the same parameters reported in Table 1. However, a misspecified Weibull distribution (either its shape or scale parameter is overestimated by different degrees) is assumed for each component while finding the optimal policy by each approach.

The RL free-model algorithms require no environmental assumptions, and the agent interacts with the environment directly (data-driven). That is, the misspecified failure time distribution will not impact them.

Table 12 reports the optimal replacement times obtained through "Eq. (1)" by assuming an overestimated Weibull shape parameter. As shown in Figure 4, Figure 5, and Figure 6, the misspecification of the Weibull shape parameter does not seem to have a large impact on the estimated optimal replacement time of the components by the traditional approach. When using the optimal replacement time of the traditional approach, the estimated system downtime seemed to be impacted differently in different strategies. Specifically, the traditional system downtime in Strategy II and III was slightly impacted (increased by 1.5-3 days) only when the shape parameter was overestimated by at least 20%.

**Table 12**.Optimal replacement times with minor changes in the Weibull shape parameters.

| Component Name | Shape (real) | Shape +10% | Shape +20% | Shape +30% | Shape +40% | Shape +50% |
|---|---|---|---|---|---|---|
| **Tire** | 13.81 | 13.85 | 13.86 | 13.88 | 13.89 | 13.90 |
| **Transmission** | 5.77 | 8.78 | 5.79 | 5.80 | 5.80 | 5.81 |
| **Wheel** | 3.96 | 3.99 | 4.01 | 4.02 | 4.03 | 4.04 |
| **Coupling** | 7.92 | 7.95 | 7.98 | 8 | 8.02 | 8.04 |
| **Motor** | 1.97 | 1.98 | 1.99 | 1.99 | 1.99 | 1.99 |
| **Brake** | 22.38 | 22.47 | 22.54 | 22.60 | 22.67 | 22.68 |
| **Steering** | 4.34 | 4.39 | 4.42 | 4.45 | 4.48 | 4.50 |
| **Gears** | 11.88 | 11.89 | 11.90 | 11.92 | 11.94 | 11.95 |

Figure 7, Figure 8 and Figure 9 illustrate system downtime obtained by the traditional approach compared to RL algorithms. As the figures show, the Weibull scale parameter overestimation from 2% to 5% increased the system downtime by 1 to 8 weeks in different strategies.

Moreover, Table 13 shows how minor changes in the Weibull scale parameter affect the optimal replacement times obtained through "Eq. (1)".

**Table 13**.Optimal replacement times with minor changed in the Weibull scale parameter.

| Component Name | Scale (real) | Scale +1% | Scale +2% | Scale +3% | Scale +4% |
|---|---|---|---|---|---|
| **Tire** | 13.81 | 13.96 | 14.11 | 14.23 | 14.38 |
| **Transmission** | 5.77 | 5.83 | 5.89 | 5.92 | 6 |
| **Wheel** | 3.96 | 3.99 | 4.05 | 4.08 | 4.12 |
| **Coupling** | 7.92 | 7.98 | 8.07 | 8.16 | 8.21 |
| **Motor** | 1.97 | 1.99 | 2.01 | 2.03 | 2.05 |
| **Brake** | 22.38 | 22.62 | 22.86 | 23.07 | 23.27 |
| **Steering** | 4.34 | 4.39 | 4..43 | 4.49 | 4.51 |
| **Gears** | 11.88 | 11.99 | 12.11 | 12.20 | 12.34 |

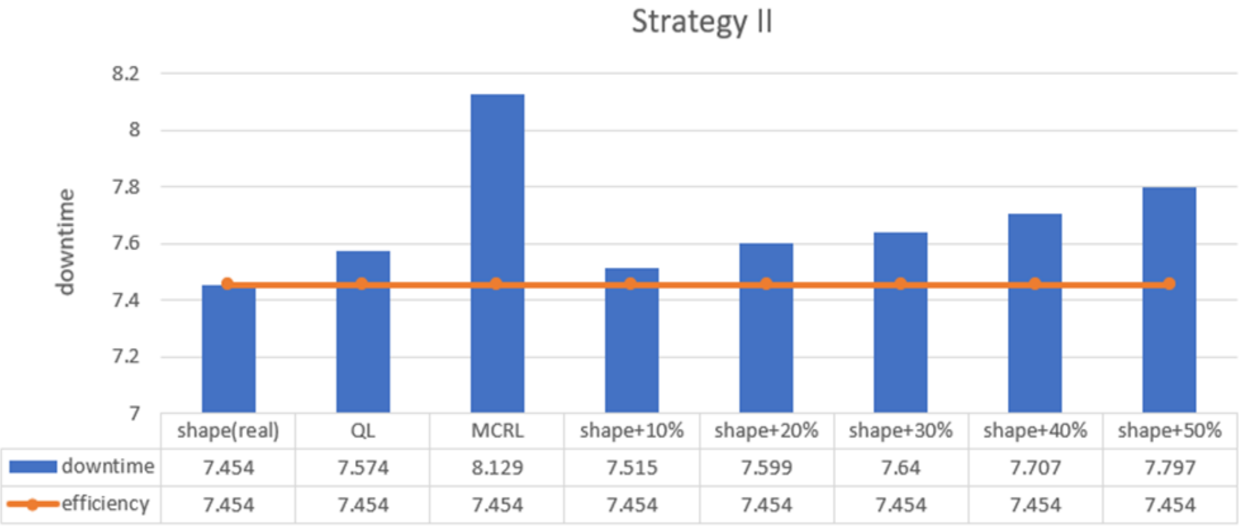


**Figure 4**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy II

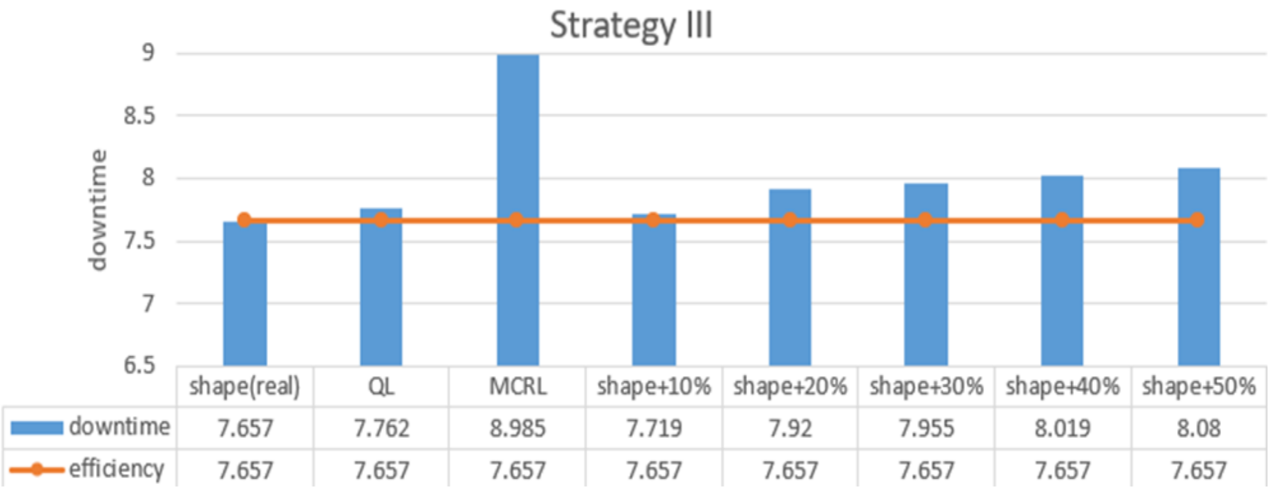
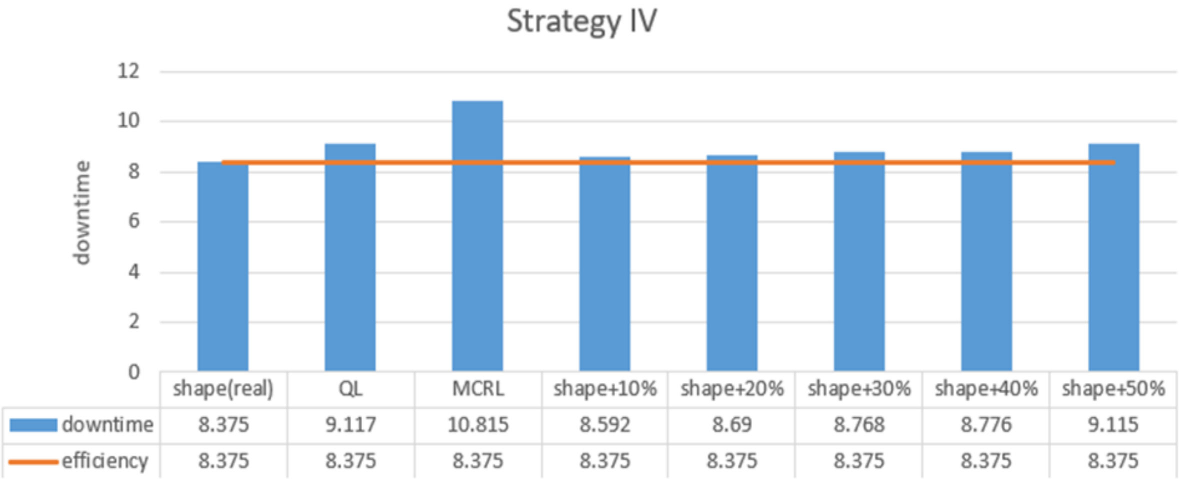
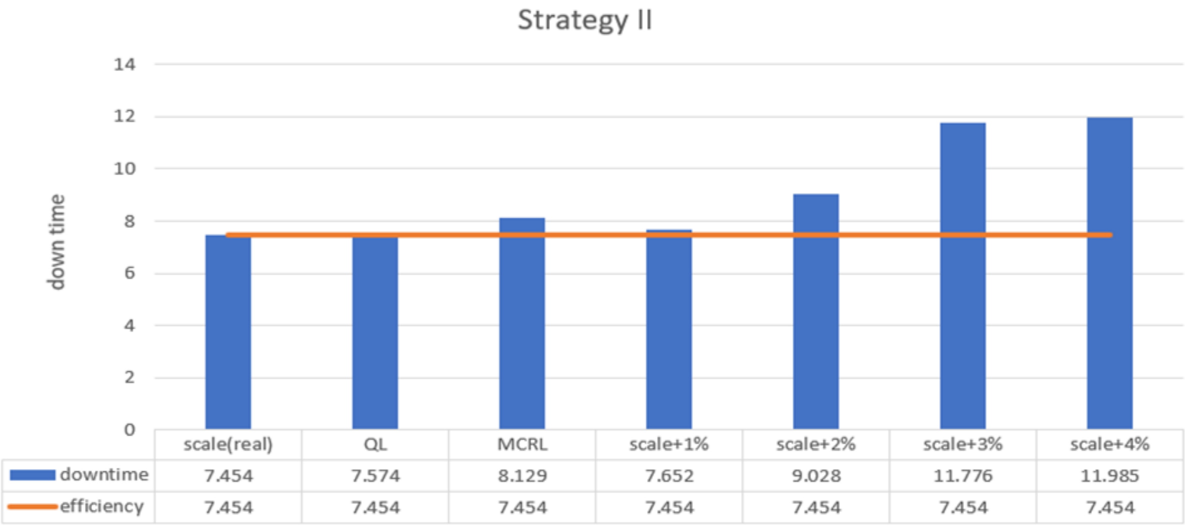


**Figure 5**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy III



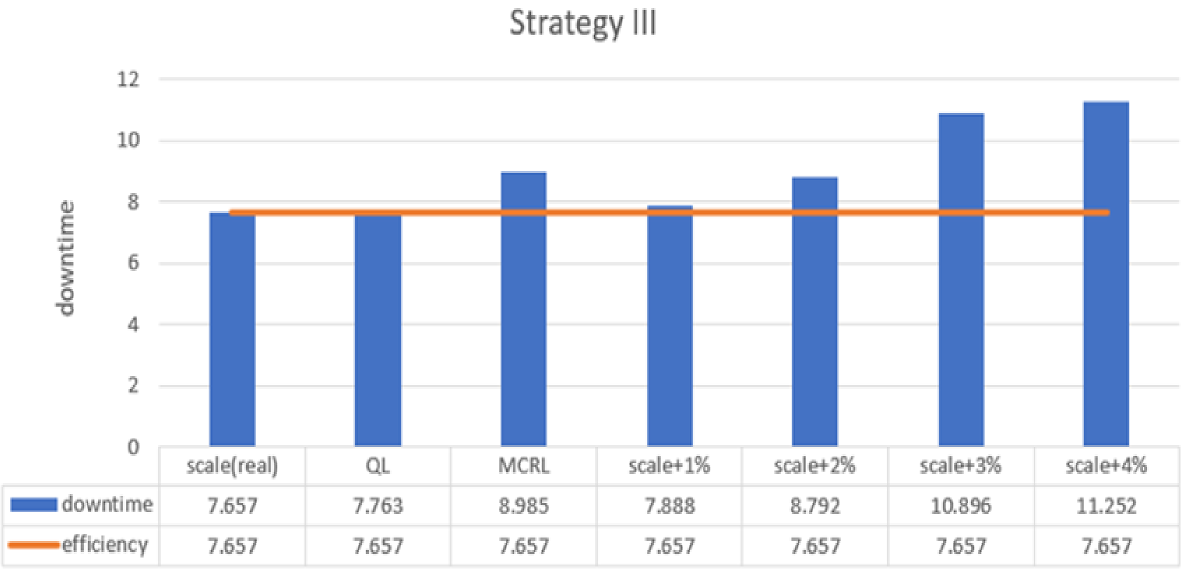
| | shape(real) | QL | MCRL | shape+10% | shape+20% | shape+30% | shape+40% | shape+50% |
|---|---|---|---|---|---|---|---|---|
| downtime | 8.375 | 9.117 | 10.815 | 8.592 | 8.69 | 8.768 | 8.776 | 9.115 |
| efficiency | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 | 8.375 |

**Figure 6**. System downtime of different approaches under minor changes in the Weibull shape parameters for Strategy IV



| | scale(real) | QL | MCRL | scale+1% | scale+2% | scale+3% | scale+4% |
|---|---|---|---|---|---|---|---|
| downtime | 7.454 | 7.574 | 8.129 | 7.652 | 9.028 | 11.776 | 11.985 |
| efficiency | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 | 7.454 |

**Figure 7**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy II



| | scale(real) | QL | MCRL | scale+1% | scale+2% | scale+3% | scale+4% |
|---|---|---|---|---|---|---|---|
| downtime | 7.657 | 7.763 | 8.985 | 7.888 | 8.792 | 10.896 | 11.252 |
| efficiency | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 | 7.657 |

**Figure 8**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy III

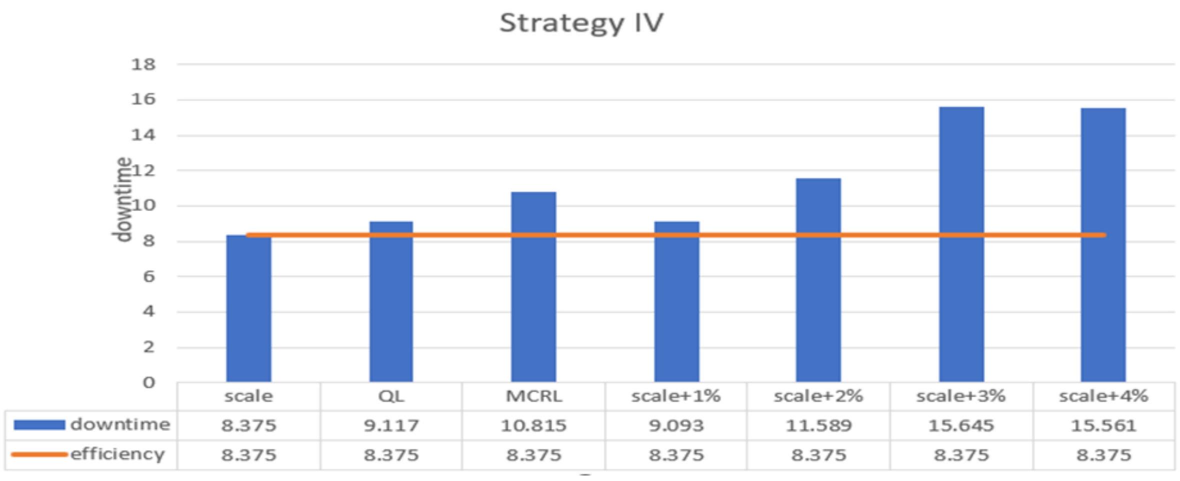


**Figure 9**. System downtime of different approaches under minor changes in the Weibull scale parameters for Strategy IV

## 6. Discussion

In this work, we employed three traditional (renewal theory) approaches, MCRL and TDRL, to find the optimal preventive maintenance policy for equipment composed of multi-non-identical components. Three preventive maintenance strategies, along with a corrective maintenance strategy (as baseline), were studied. Our results confirmed that preventive maintenance strategies perform better than the corrective maintenance policy, as expected, for our system. More importantly, our results showed that the traditional approach (renewal theory) is sensitive to the misspecification of the components' failure time distribution. More specifically, under the assumption of the components Weibull distributed failure times, the optimal policy and, consequently, the performance of the traditional approach seem to be impacted only slightly by misspecifying the shape parameters up to 50% (downtime increased by < 3 days). However, even minor misspecification in the scale parameter (up to 5%) can lead to a huge increase in the system downtime following the traditional approach optimal policy by up to 8 weeks. On the other hand, since the model-free RL algorithms are data-driven with no requirements of prior assumption on the environment distribution (e.g., failure time distributions), they can be minimally impacted by such misspecifications.

Different RL algorithms, however, can potentially perform very differently. Under the assumptions of our study, the QL algorithm outperformed the MC algorithm dramatically. Given the quick progress in developing RL algorithms nowadays, a natural next step to our work might be evaluating different RL algorithms for different systems with different assumptions.

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