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Review Article

On Possibility of Extending the Optimal Replacement time of Series and Parallel Systems

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Abstract

Among all systems, the series system has the lowest optimal replacement time, while the parallel system has the highest optimal replacement time. This paper is comparing the standard age replacement strategy (SARS) with some proposed replacement strategies (strategy A and strategy B) for two multi-unit systems. Two numerical examples are provided for a simple illustration of the proposed replacement cost models under SARS, strategies A and B. The results obtained showed that strategy B can extend the optimal replacement time of a series system.

Keywords: Failure, Optimal, Strategy, System, Unit.

1. Introduction

The failure of an operating unit or system might sometimes be costly, dangerous, negatively affect revenue, the production of defective items, or causes a delay in customer services. It is an important problem to determine when best to preventively replace or maintain an operating unit or system before failure. Under the age replacement policy, the series system has the lowest optimum replacement time, while the parallel system has the highest optimum replacement time. As the series system is the having the lowest optimum replacement time, this may lead to the early replacement of a series system.

There is extensive literature on age replacement models with minimal repair. Cha and Finkelstein [1] introduced a new type of minimal repair to be called conditional statistical minimal repair, and their approach goes further and deals with the corresponding minimal repair processes for systems operating in a random environment. Chang [2] considered a system that suffers one of two types of failures based on a specific random mechanism. Chang and Chen [3] discussed that effective replacement policies should be collaborative once gathering data from the time of operations, mission durations, minimal repairs, and maintenance triggering approaches. Coria et al. [4] proposed an analytical optimization method for preventive maintenance replacement cost rate. Fallahnezhad and Najafian [5] studied the best time for performing preventive maintenance operations for many systems. Gheisary and Goli [6] investigated an efficient method to compute the exact reliability for a multi-state system is a system consisting of n components by using the distribution of bivariate order statistics. Based on the continuous-time Markov theory, Huang and Wang [7] construct a time-replacement policy for multistate systems with aging multistate components so as determine the optimal time to replace the entire system. Jain and Gupta [8] studied optimal replacement policy for a repairable system with multiple vacations and imperfect coverage. Enogwe et al. [9] applied the knowledge of probability distribution of failure times and proposed a replacement model for items that fail suddenly. Lim et al. [10] presented some age replacement policies in which a system is replaced by a new one at the planned age and when a failure occurs before the planned replacement age, it can be either perfectly repaired with random probability p or minimally repaired with random probability $1-p$. Liu et al. [11] established uncertain reliability mathematical models of simple repairable series systems, simple repairable parallel systems, simple repairable series-parallel systems, and simple repairable parallel-series systems, respectively. Malki et al. [12] presented some age replacement policies for a parallel system with stochastic dependence. Mirjalili and Kazempoor [13] investigated three replacement policies including cold standby and minimal repair policies for a system consisting of independent components with increasing failure rate functions. Murthy and Hwang [14] discussed that, in a probabilistic sense, failures can be reduced through effective maintenance actions, and such maintenance actions can occur either at discrete time instants or continuously over time. Nakagawa [15] presented a modified standard age replacement (SAR) model to a discrete-time age replacement model. Nakagawa et al. [16] presented the advantages of some proposed replacement policies. In an approach for analyzing the behavior of an industrial system under the cost-free warranty policy, Niwas and Garg [17] developed a mathematical model of a system based on the Markov process, they also derived various parameters such as reliability, mean time to system failure, availability and expected profit for the system. Rebaiaia and Ait-kadi [18] presented the problem of selecting the best among three maintenance strategies for conducting maintenance planning that is the most economical. Safaei et al. [19] investigated the optimal period for preventive maintenance and the best decision for repair or replacement in terms of some measures. Safaei et al. [20] used the copula framework and present two optimal age replacement policies based on the minimum expected cost function and maximum availability function for series or parallel systems with dependent components. Sanoubar et al. [21] considered an age-replacement policy (without minimal repair) under which the system is replaced at failure or at a prescribed replacement time, whichever occurs first, where it is assumed that replacement costs are non-decreasing in system age. Sheu et al. [22] presented preventive replacement models for a system subjected to shocks that arrive according to a non-homogeneous Poisson process, such that when a shock takes place, the system is either replaced by a new one (type 2 failure) or minimally repaired (type 1 failure). Sudheesh et al. [23] considered the discrete age-replacement model, and then studied the properties of mean time to failure of a system. Tsoukalas and Agrafiotis [24] introduced a new replacement policy for a system with correlated failure and usage time. Waziri et al. [25] explored some characteristics of an age replacement model with minimal repair for a series-parallel system with six units, such that the six units are having non-uniform failure rates. Waziri [26] presented a discrete scheduled replacement model with the discounting rate for a unit that is subjected to three categories of failures. Furthermore, Waziri and Yusuf [27] presented a discrete scheduled replacement cost model for a multi-component system that is subjected to two categories of failures. Wu et al. [28] proposed a new replacement policy and established corresponding replacement models for a deteriorating repairable system with multiple vacations of one repairman. Xie et al. [29] analyzed the impacts of cascading failures on the reliability of series-parallel systems, where they studied the effects of safety barriers on preventing occurring failures. Yaun and Xu [30] studies a cold standby repairable system with two different components and one repairman taking multiple vacations. Yusuf and Ali [31] constructed an age replacement cost model for a parallel system with units under some assumptions. Zhao et al. [32]investigated the problem of which replacement is better between continuous and discrete scheduled replacement times. Zhao et al. [33] developed some analytic replacement cost rates under two proposed policies considering random mission durations time, to avoid preventive replacement during the mission period.

The literature review presented in this paper did not capture a method or strategy for extending the optimal replacement time of a multi-component system. This paper will come up with some replacement cost models under some proposed strategies, to see the possibility of extending the optimum replacement time of series and parallel systems, and this will be achieved through the following objectives:

1. By constructing an age replacement cost model for series and parallel systems under the standard age replacement strategy (SARS).
2. By constructing age replacement cost models for series and parallel systems under two proposed strategies.
3. By providing some two numerical examples for a simple illustration of the constructed replacement cost models.

2. Notations and systems description

2.1 Notations

$r_i(t)$	Type I failure rate of unit A_i ,for $i = 1, 2, 3, 4, 5, 6$
$r_i^*(t)$	Type II failure rate of unit A_i, for $i = 1, 2, 3, 4, 5, 6$.
$R_i^*(t)$	Reliability function of Type II failure of unit A_i, for $i = 1, 2, 3, 4, 5, 6$.
$SARS$	Standard age replacement strategy
$CS_i(T)$	Cost rate of system $S_i$under SARS, for $i = 1, 2$.
$CYS_i(T)$	Cost rate of system $S_i$under strategy A, for $i = 1, 2$
$CZS_i(T)$	Cost rate of system $S_i$under strategy B, for $i = 1, 2$.
X_{Si}^*	Optimal replacement time of system S_i under SARS, for $i = 1, 2$.
Y_{Si}^*	Optimal replacement time of system $S_i$under strategy A, for $i = 1, 2$.
Z_{Si}^*	Optimal replacement time of system $S_i$under strategy B, for $i = 1, 2$.
C_{ir}	Cost of unplanned replacement of failed $A_i$due to Type II failure, for $i = 1, 2, 3, 4, 5, 6$.
C_{im}	Cost of minimal repair of failed unit $A_i$due to Type II failure, for $i = 1, 2, 3, 4, 5, 6$.

C_{sp}	Cost of planned replacement of system S_i at planned replacement time T, for $i = 1, 2$.
C_{sr}	Cost of un-planned replacement of system S_i due to Type II failure, for $i = 1, 2$.

2.2 Systems description

Consider six units A_1, A_2, A_3, A_4, A_5 and A_6, arranged to form two different systems, series system (S_1) and parallel system (S_2). It is assumed that all the six units are subjected to Type I and Type II failures, such that, Type I failure is a repairable one, while Type II failure is a non-repairable failure. Now, since all the six units are subjected to Type I and Type II failures, then we can say that all the two systems are also subjected to Type I and Type II failures. See the Figures 1 and 2 for the diagram of the two systems (S_1 and S_2).

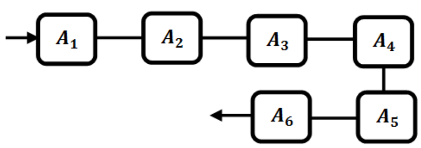

Figure 1. Reliability block diagram of system $\boldsymbol{S_1}$

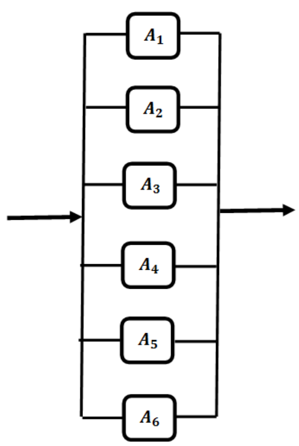

Figure 2. Reliability block diagram of system $\boldsymbol{S_2}$

3. Formulation of cost model under SARS

3.1 Some assumptions under SARS

1. If a system fails due to Type I failure, then the system is minimally repaired.
2. If a system fails due to Type II failure, then the whole system is replaced completely with a new one.
3. Both the two types of failures for the six units arrive according to a non-homogeneous Poisson process.
4. The rate of Type II failure follows the order: $r_1^*(t) \geq r_3^*(t) \geq r_5^*(t) \geq r_2^*(t) \geq r_4^*(t) \geq r_6^*(t)$.
5. The rate of Type I failure follows the order: $r_1(t) \geq r_3(t) \geq r_5(t) \geq r_2(t) \geq r_4(t) \geq r_6(t)$.
6. A system is replaced at a planned time $T(T > 0)$ after its installation or at Type II failure, whichever arrives first.
7. The cost of the planned replacement of a system is less than the cost of un-planned replacement.
8. The cost of repair of a failed unit is less than the cost of replacement of a unit.
9. All costs are positive numbers.

The probability that system S_1 will be replaced at planned replacement time T, before Type II failure occurs, is

$$R_{S1}^*(T) = R_1^*(T)R_2^*(T)R_3^*(T)R_4^*(T)R_5^*(T)R_6^*(T) \qquad (1)$$

The probability that system S_2 will be replaced at planned replacement time T, before Type II failure occurs, is

$$R_{S2}^*(T) = 1 - \prod_{i=1}^{6}(1 - R_i^*(T)) \qquad (2)$$

In the meantime systems of S_1 and S_2 under SARS, is

$$Mean\ time = \int_0^T R_{Si}^*(t)dt \text{ , for } i = 1, 2 . \qquad (3)$$

The cost of un-planned replacement (failure due to Type II failure) of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ unplanned\ replacement = C_{sr}\left(1 - R_{Si}^*(T)\right) \text{ , for } \ i = 1, 2 . \qquad (4)$$

The cost of planned replacement at time T of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ planned\ replacement = C_{sp}R_{Si}^*(T), \text{ for } i = 1, 2 . \qquad (5)$$

The cost of minimal repair of units A_1, A_2, A_3, A_4, A_5 and A_6 due to Type I failure in one replacement cycle, is

$$Cost\ of\ repair = \int_0^T C_{1m}r_1(t)R_{Si}^*(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^*(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^*(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^*(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^*(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^*(t)dt. \qquad (6)$$

The replacement cost rate of S_1 and S_2 under SARS, is

$$CS_i(T) = \frac{C_{sr}\left(1-R_{Si}^*(T)\right)+C_{sp}R_{Si}^*(T)+\int_0^T J(t)R_{Si}^*(t)dt}{\int_0^T R_{Si}^*(t)dt}, \text{ for } i = 1, 2, \qquad (7)$$

where

$$J(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t). \qquad (8)$$

4. Formulation of cost model under strategy A

Strategy A is a preventive maintenance strategy, in which the un-planned replacement of a whole system depends on the failure of units A_1, A_3 and A_5 due to Type II. Noting that, the reliability function of a system due to strategy A, depends on the location of units

A_1, A_3 and A_5 in a system. But when any of the units A_2, A_4 or A_6 fails due to Type II failure, the failed unit is replaced completely with a new one and allows the system to continue operating from where it stopped.

Under strategy A, we have the following reliability functions:

1. System S_1: the system is replaced completely with a new one when at least one of the units A_1, A_3 or A_5 fails due to Type II failure. Now, the probability that system S_1 will be replaced at planned replacement time T, before Type II failure occurs under strategy A, is

$$R_{S1}^{a*}(T) = R_1^*(T)R_3^*(T)R_5^*(T) \quad (9)$$

2. System S_2 : the system is replaced completely with a new one when all the three units A_1, A_3 and A_5 fails due to Type II failure. Now, the probability that system S_2 will be replaced at planned replacement time T, before Type II failure occurs under strategy A, is

$$R_{S2}^{a*}(T) = 1 - (1 - R_1^*(T))(1 - R_3^*(T))(1 - R_5^*(T)) \quad (10)$$

In the meantime systems of S_1 and S_2 in one replacement cycle under strategy A, is

$$Mean\ time = \int_0^T R_{Si}^{a*}(t)dt, \text{ for } i = 1, 2. \quad (11)$$

The cost of un-planned replacement (failure due to Type II failure) of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ unplanned\ replacement = C_{sr}\left(1 - R_{Si}^{a*}(T)\right), \text{ for } i = 1, 2. \quad (12)$$

The cost of planned replacement at time T of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ planned\ replacement = C_{sp}R_{Si}^{a*}(T), \text{ for } i = 1, 2. \quad (13)$$

The cost of minimal repair of units A_1, A_2, A_3, A_4, A_5 and A_6 due to Type I failure in one replacement cycle, is

$Cost\ of\ repair$

= ∫ _0^T▒ 〖C_1m r_1 (t) R_Si^(a *) (t)dt〗

$$+ \int_0^T C_{2m}r_2(t)R_{Si}^{a*}(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^{a*}(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^{a*}(t)dt \quad (14)$$

The cost of replacement of units A_2, A_4 and A_6 due to Type II failure in one replacement cycle, is

$$Cost\ of\ replacement = \int_0^T C_{2r}r_2^*(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4r}r_4^*(t)R_{Si}^{a*}(t)dt + \int_0^T C_{6r}r_6^*(t)R_{Si}^{a*}(t)dt. \quad (15)$$

The replacement cost rate of S_1 and S_2 under strategy A, is

$$CYS_i(T) \frac{C_{sr}\left(1 - R_{Si}^{a*}(T)\right) + C_{sp}R_{Si}^{a*}(T) + \int_0^T K(t)R_{Si}^{a*}(t)dt + \int_0^T L(t)R_{Si}^{a*}(t)dt}{\int_0^T R_{Si}^{a*}(t)dt}, \quad (16)$$

for $i = 1, 2$,

where

$$K(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t), \quad (17)$$

and

$$L(t) = C_{2r}r_2^*(t) + C_{4r}r_4^*(t) + C_{6r}r_6^*(t) \quad (18)$$

5. Formulation of cost model under strategy B

Strategy B is a preventive maintenance strategy, in which the un-planned replacement of a whole system depends on the failure of units A_2, A_4 and A_6 due to Type II. Noting that, the reliability function of a system due to strategy B, depends on the location of units A_2, A_4 and A_6 in a system. But when any of the units A_1, A_3 or A_5 fails due to Type II failure, the failed units are replaced completely with new ones and allow the system to continue operating from where it stopped. Under strategy B, we have the following reliability functions:

1. System S_1: the system is replaced completely with a new one when at least one of the units A_2, A_4 or A_6 fails due to Type II failure. Now, the probability that system S_1 will be replaced at planned replacement time T, before Type II failure occurs under strategy B, is

$$R_{S1}^{b*}(T) = R_2^*(T)R_4^*(T)R_6^*(T) \quad (19)$$

2. System S_2 : the system is replaced completely with a new one when all the three units A_2, A_4 or A_6 fails due to Type II failure. Now, the probability that system S_2 will be replaced at planned replacement time T, before Type II failure occurs under strategy B, is

$$R_{S2}^{b*}(T) = 1 - \left(1 - R_2^*(T)\right)\left(1 - R_4^*(T)\right)\left(1 - R_6^*(T)\right). \quad (20)$$

The mean time of systems of S_1 and S_2 in one replacement cycle under strategy B, is

$$Mean\ time = \int_0^T R_{Si}^{b*}(t)dt, \text{ for } i = 1, 2. \quad (21)$$

The cost of un-planned replacement (failure due to Type II failure) of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ unplanned\ replacement = C_{sr}\left(1 - R_{Si}^{b*}(T)\right), \text{ for } i = 1, 2. \quad (22)$$

The cost of planned replacement at time T of S_1 and S_2 in one replacement cycle, is

$$Cost\ of\ planned\ replacement = C_{sp}R_{Si}^{b*}(T), \text{ for } i = 1, 2. \quad (23)$$

The cost of minimal repair of units A_1, A_2, A_3, A_4, A_5 and A_6 due to Type I failure in one replacement cycle, is

$$Cost\ of\ repair = \int_0^T C_{1m}r_1(t)R_{Si}^{b*}(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^{b*}(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^{b*}(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^{b*}(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^{b*}(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^{b*}(t)dt. \quad (24)$$

The cost of replacement of units A_1, A_3 and A_5 due to Type II failure in one replacement cycle, is

$$Cost\ of\ replacement = \int_0^T C_{1r}r_1^*(t)R_{Si}^{b*}(t)dt + \int_0^T C_{3r}r_3^*(t)R_{Si}^{b*}(t)dt + \int_0^T C_{5r}r_5^*(t)R_{Si}^{b*}(t)dt. \quad (25)$$

The replacement cost rate of systemsS_1 and S_2 under strategy B, is

$$CZS_i(T) = \frac{C_{sr}\left(1 - R_{Si}^{b*}(T)\right) + C_{sp}R_{Si}^{b*}(T) + \int_0^T M(t)R_{Si}^{b*}(t)dt + \int_0^T N(t)R_{Si}^{b*}(t)dt}{\int_0^T R_{Si}^{b*}(t)dt} \quad (26)$$

, for $i = 1, 2$,

where

$$M(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t), \quad (27)$$

and

$$N(t) = C_{1r}r_1^*(t) + C_{3r}r_3^*(t) + C_{5r}r_5^*(t) \quad (28)$$

5.1 Numerical examples

In this section, two numerical examples were provided to illustrate the characteristics of the proposed replacement cost models constructed above.

Example 1

Let the failure time of Type l I failure for the six units follows the Weibull distribution:

$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1}$, for $i = 1, 2, 3, 4, 5, 6,$ (29)

where $\propto_i > 1$ and $t \geq 0$.

Also, let the failure time of Type II failure for the six units follows the Weibull distribution:

$r_i^*(t) = \lambda_i^* \propto_i^* t^{\propto_i^* - 1}$, for $i = 1, 2, 3, 4, 5, 6,$ (30)

where $\propto_i > 1$, and $t \geq 0$.

Let the set of parameters and cost of repair/replacement be used throughout this particular example:

1. $\propto_1 = 4$, $\propto_2 = 3$, $\propto_3 = 3$, $\propto_4 = 3$, $\propto_5 = 4$ and $\propto_6 = 2$.
2. $\lambda_1 = 0.03$, $\lambda_2 = 0.002$, $\lambda_3 = 0.03$, $\lambda_4 = 0.001$, $\lambda_5 = 0.001$ and $\lambda_6 = 0.001$.
3. $\propto_1^* = 4, \propto_2^* = 3.5, \propto_3^* = 4$, $\propto_4^* = 3.5, \propto_5^* = 4$, and $\propto_6^* = 3.5$.
4. $\lambda_1^* = 0.00033$, $\lambda_2^* = 0.00025$, $\lambda_3^* = 0.00030$, $\lambda_4^* = 0.00023$, $\lambda_5^* = 0.00025$ and $\lambda_6^* = 0.0002$.
5. $C_{sr} = 70$, $C_{sp} = 45$ and $C_{im} = 0.4$, for $i = 1, 2, 3, 4, 5, 6$.

By substituting the parameters of equations (29) and (30), the rates of Type I and Type II failures are obtained as follows:

$r_1(t) = 0.12t^3.$ (31)

$r_2(t) = 0.06t.$ (32)

$r_3(t) = 0.09t^2.$ (33)

$r_4(t) = 0.003t^2.$ (34)

$r_5(t) = 0.004t^3.$ (35)

$r_6(t) = 0.002t.$ (36)

$r_1^*(t) = 0.00132t^3.$ (37)

$r_2^*(t) = 0.000875t^{2.5}.$ (38)

$r_3^*(t) = 0.00012t^3.$ (39)

$r_4^*(t) = 0.000805t^{2.5}.$ (40)

$r_5^*(t) = 0.001t^3.$ (41)

$r_6^*(t) = 0.0007t^{2.5}.$ (42)

The tables below in this example are obtained by substituting all the rates of the two failures (Type I and Type II failures), and costs of replacement and repair in equations (7), (16), and (26).

Table 1. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under SARS.

T	$CS_1(T)$	$CS_2(T)$
1	240.42	240.04
2	122.77	120.16
3	88.58	80.36
4	79.17	60.69
5	82.73	49.16
6	91.47	41.80
7	94.78	36.93
8	95.87	33.87
9	97.93	32.50
10	99.00	32.92
11	99.52	34.91
12	100.22	37.87

Table 2. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under strategy A.

T	$CYS_1(T)$	$CYS_2(T)$
1	240.78	240.57
2	122.87	121.18
3	87.58	81.96
4	76.01	62.89
5	76.62	52.13
6	83.71	46.27
7	89.44	44.88
8	90.24	48.34
9	92.73	55.49
10	93.99	60.79
11	95.45	62.77
12	98.00	64.57

Table 3. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under strategy B.

T	$CZS_1(T)$	$CZS_2(T)$
1	240.80	240.64
2	122.54	121.62
3	85.61	83.11
4	70.10	65.11
5	63.97	55.61
6	62.95	50.64
7	64.67	49.55
8	67.21	48.55
9	67.67	50.20
10	68.74	53.05
11	70.46	56.43
12	72.97	59.34

Table 4. The optimal replacement times of systems S_1 and S_2 under SARS, strategies A and B from Tables 1, 2 and 3.

System	SARS	Strategy A	Strategy B
S_1	$X_{S1}^* = 4.00$	$Y_{S1}^* = 4.00$	$Z_{S1}^* = 6.00$
S_2	$X_{S2}^* = 9.00$	$Y_{S2}^* = 7.00$	$Z_{S2}^* = 8.00$

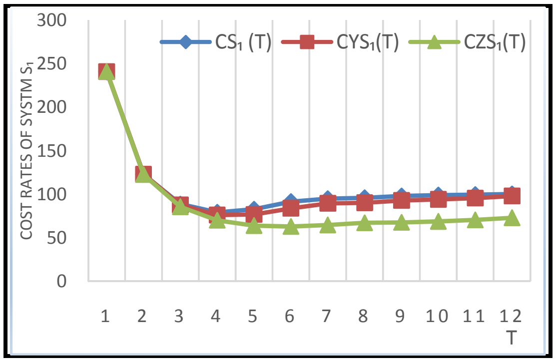

Figure 3. The plot of cost rates of system S_1 under SARS, strategy A and strategy B against planned replacement time T.

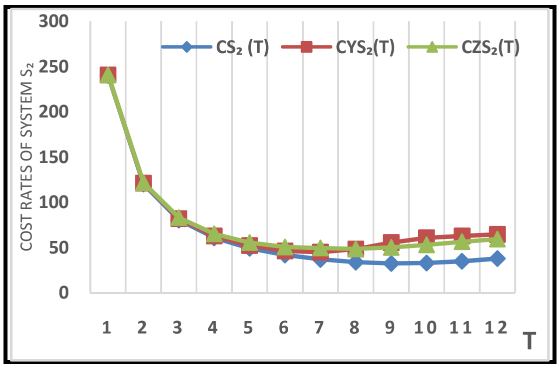

Figure 4. The plot of cost rates of system S_2 under SARS, strategy A and strategy B against planned replacement time T.

Some observations of results obtained from example 1 are as follows

1. From Table 4, observe that, the optimal replacement time of the system S_1 obtained under strategy B is higher than that of SARS and strategy A, while the optimal replacement time of system S_2 obtained under SARS is higher than that of strategies A and B.
2. From Figure 3, observe that, the cost rate of the system S_1 obtained under strategy B is lower than that of SARS and strategy A.
3. From Figure 4, observe that, the cost rate of the system S_2 obtained under SARS is lower than that of strategies A and B.

Example 2

Let the failure time of Type I failure for the six units follows Power law distribution:

$$r_i(t) = \lambda_i \propto_i (\lambda_i t)^{\propto_i - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \quad (43)$$

where $\propto_i > 1$ and $t \geq 0$.

Also, let the failure time of Type II failure for the six units follows Power law distribution:

$$r_i^*(t) = \lambda_i^* \propto_i^* (\lambda_i^* t)^{\propto_i^* - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \quad (44)$$

where $\propto_i > 1$, and $t \geq 0$.

The cost and the parameters of both Type I and Type II failures in example 1 were adopted. Similarly, the tables below in this example are obtained by substituting all the failure rates of Type I, Type II, and costs of replacement and repair in equations (7), (16), and (26).

Table 5. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under SARS.

T	$CS_1(T)$	$CS_2(T)$
10	224.67	224.29
20	107.02	104.41
30	72.83	64.61
40	63.42	44.94
50	66.98	33.41
60	75.72	26.05
70	79.03	21.18
80	80.12	18.12
90	82.18	16.75
100	83.25	17.17
110	83.77	19.16
120	84.47	22.12

Table 6. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under strategy A.

T	$CYS_1(T)$	$CYS_2(T)$
10	225.03	224.82
20	107.12	105.43
30	71.83	66.21
40	60.26	47.14
50	60.87	36.38
60	67.96	30.52
70	73.69	29.13
80	74.49	32.59
90	76.98	39.74
100	78.24	46.04
110	79.7	47.02
120	82.25	48.82

Table 7. Results obtained from evaluating the replacement cost rates of systems S_1 and S_2 under strategy B.

T	$CZS_1(T)$	$CZS_2(T)$
10	225.05	240.64
20	106.79	121.62
30	69.86	83.11
40	54.35	65.11
50	48.22	55.61
60	47.20	50.64
70	48.92	48.55
80	51.46	48.00
90	51.92	50.20
100	52.99	53.05
110	54.71	56.43
120	57.22	59.34

Table 8. The optimal replacement times of systems S_1 and S_2 under SARS, strategy A and strategy B from tables 5, 6 and 7.

System	SARS	Strategy A	Strategy B
S_1	$X_{S1}^* = 40.00$	$Y_{S1}^* = 40.00$	$Z_{S1}^* = 60.00$
S_2	$X_{S2}^* = 90.00$	$Y_{S2}^* = 70.00$	$Z_{S2}^* = 80.00$

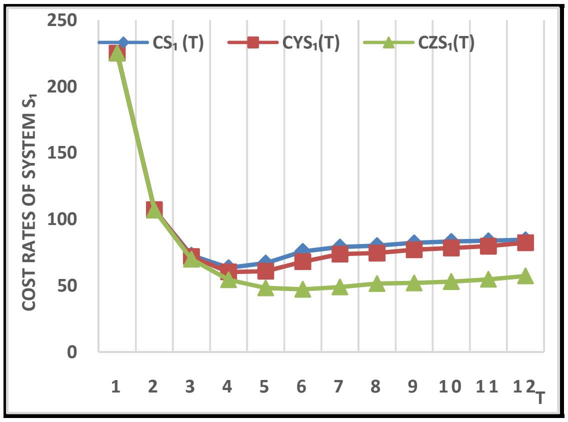

Figure 5. The plot of cost rates of systemS_1 under SARS, strategy A and strategy B against planned replacement time T.

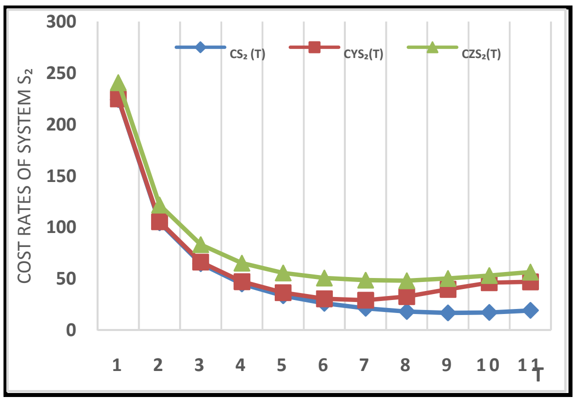

Figure 6. The plot of cost rates of systemS_2 under SARS, strategy A and strategy B against planned replacement time T.

Some observations of results obtained from example 2

1. From Table 8, observe that, the optimal replacement time of the system S_1 under the strategy, B is higher than that of SARS and strategy A, while the optimal replacement time of the system S_2 under SARS is higher than that of strategies A and B.
2. From Figure 5, observe that, the cost rate of the system S_1 under the strategy, B is lower than that of SARS and strategy A.
3. From Figure 6, observe that, the cost rate of the system S_2 under SARS is lower than that of strategies A and B.

6. General observation of results

From the results obtained from examples 1 and 2, one can clearly see that strategy B can extend the optimal replacement time of the series system, while it cannot extend the optimal replacement time of the parallel system. In terms of the cost rate, the results showed that the cost rate of the series system is lower than that of SARM and strategy A, while the cost rate of the parallel system under SARM is lower than that of the strategies A and B.

7. Significance of results

From the results obtained in this research, one can clearly see that the strategy B is a good preventive maintenance plan for maintaining series multi-unit systems because strategy B has the following advantages over SARS and strategy A:

1. The optimal replacement time of the series system obtained under strategy B has a higher optimal replacement time than that of SARS and strategy A. Thus, this will reduce the chances of early replacement of the series systems at an early stage.
2. The cost of maintenance of the series system under strategy B is lower than that of SARS and strategy A.

Also, from the other way round of the findings, maintenance managers and plant management are advised to adopt SARS as a good preventive maintenance strategy for maintaining the parallel multi-unit system, because preventive maintenance under SARS, has the following advantages over strategies A and B:

1. The optimal replacement time of a parallel system obtained under SARS has a higher optimal replacement time than that of strategies A and B.
2. The cost of maintenance of the parallel system under SARS is lower than that of strategies A and B.

One can relate the findings of these results obtained to real life, one can use the results to select the best strategy for maintaining the following:

1. Series and parallel configurations of a combined heat and power (CHP) plant coupled to thermal networks.
2. Subsystems of industrial plants.
3. Subsystems of air crafts

8. Summary and conclusion

This research covered the age replacement policy with the concept of repair at failure. In trying to explore some possible ways of extending the optimal replacement time of some multi-component systems, this paper presented some proposed age replacement cost models under standard age replacement strategy (SARS), strategy A and strategy B for series and parallel systems. It is assumed that the two systems are subjected to Type I and Type II failures. Below are the tables that compare the three proposed strategies.

Table 9. Comparing SARS, Strategy A and Strategy B for system S_1.

SARS	If all the six units fails due Type II failure, then replace the whole system
	If all the six units fails due Type I failure, then repair the failed units, minimally.
Strategy A	If at least one of A_1, A_3 and A_5 fails due Type II failure, then replace the whole system
	If at least one of A_2, A_4 and A_6 fails due Type II failure, then replace the failed unit(s)
	If all the six units fails due Type I failure, then repair the failed unit, minimally.
Strategy B	If at least one of A_2, A_4 and A_6 fails due Type II failure, then replace the whole system
	If at least one of A_1, A_3 and A_5 fails due Type II failure, then replace the failed unit(s)
	If all the six units fails due Type I failure, then repair the failed units, minimally.

Table 10. Comparing SARS, Strategy A and Strategy B for system S_2.

SARS	If all the six units fails due Type II failure, then replace the whole system
	If all the six units fails due Type I failure, then repair the failed units, minimally.
Strategy A	If all the three units A_1, A_3 and A_5 fails due Type II failure, then replace the whole system
	If all the three units A_2, A_4 and A_6 fails due Type II failure, then replace the failed unit
	If all the six units fails due Type I failure, then repair the failed units, minimally.
Strategy B	If all the three units A_2, A_4 and A_6 fails due Type II failure, then replace the whole system
	If all the three units A_1, A_3 and A_5 fails due Type II failure, then replace the failed units
	If all the six units fails due Type I failure, then repair the failed units, minimally.

The results obtained in this research showed that the preventive replacement of the series system under strategy B is optimal over SARS and strategy A. While, whereas the preventive replacement of the parallel system under SARS, is optimal over strategies A and B. Therefore, the main contribution of this research is, that it showed that preventive replacement under strategy B is better than preventive replacement under strategy A and SARS because strategy B extends the optimum replacement time of a series system. For future extension and modification of this research, one can see the following cases:

1. By applying the proposed strategy A and strategy B, to series-parallel and parallel-series systems, to see the possible extension of their optimal replacement time.
2. By considering periodic replacement time NT $(N = 1, 2, 3, \ldots)$ for a fixed T, to see the possible extension of the optimal replacement time of multi-component systems.
3. By in-cooperating a warranty period in the proposed replacement models under strategies and B.

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