

Determination of Optimum Sample Size for Lot Acceptance Attribute Sampling under Life Tests Based On Rayleigh Distribution Using Graphical Evaluation Review Technique (GERT)

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Abstract

This paper presents the graphical evaluation and review technique (GERT) exploration of performance measures for lot acceptance sampling procedures having attribute characteristics following life tests based on percentiles of Rayleigh Distribution and henceforth determining optimum sampling size. The advantageous implications of GERT analysis in this framework is primarily to visualize the dynamics of the sampling inspection system and secondly, critical analysis of sampling procedure characteristics. The formula of operating characteristics (OC) function and average sample number (ASN) function is derived and illustrated numerically. Lastly, tables have been provided to determine the optimum sample size assuring certain mean life or quality of the product.

Keywords: Reliability life test sampling plan, Graphical Evaluation Review Technique (GERT), Rayleigh Distribution.

Introduction

Acceptance model schemes are commonly used to determine product acceptance. Lifetime is an important quality attribute of an object. The prototypes used to determine the acceptability of a product for its lifetime are called reliability or life test prototype. When the life test shows that the mean (average) or percentage life of the product is above the desired quality, the submitted lot is accepted; otherwise it is rejected lot.

Reliability sampling is a process that establishes the minimum sample size to be used for testing. This is especially valuable if the quality of an object is defined in its lifetime. A specific reliability model project, in which case, sample observation is subject to the lifetime testing of the products, is intended to demonstrate that the actual population average exceeds the required minimum. Population mean refers to the average lifetime of a product, say θ . If θ_0 is a certain minimum value, one wants to check $\theta \geq \theta_0$; Lots rejected or life test model plan.

The decision-making criterion is naturally based on the number of failures observed in the sample of n

products in a given time T form, which is obtained at the lowest average lifetime unknown. If the number of failures found is large, greater than one number c , the lower limit obtained is smaller than θ_0 , and the hypothesis $\theta \geq \theta_0$ is not verified. So, a lot is unacceptable. Such a model plan is called a reliability model plan, an important feature of the reliability accepting model scheme is that it involves a randomness. The lifetime distribution can be adequately described by the consecutive type distributions such as Normal, Exponential, Weibull, Lognormal and Gamma. Many works have been done in previous years on the reliability model project using this distribution. In recent years, there have been a few other types of literature available, such as Logistics, Log-Logistics, Rayleigh, inverse Rayleigh, Generalized Exponential, Pareto, Marshall-Olk in Extended Lomax, Exponentiated Rayleigh, and, Exponentiated Exponential Distribution.

Reliability Functions

Basic to the definition of reliability functions and other related functions is the length of the variable. The length of life (lifetime) of a component/system is the length of the time interval T , from the initial activation of the unit

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until failure. This variable T is considered a random variable, since the length of life cannot be exactly predicted.

The cumulative (life) distribution function (CDF) of T , denoted by $F(t)$ is the probability that the lifetime does not exceed t .

i.e.,

$$F(t) = \Pr\{T \leq t\}, \quad 0 < t < \infty \tag{1}$$

The lifetime random variable T is called continuous if its CDF is a continuous function of t . The probability density function (PDF) corresponding to $F(t)$ is its derivative (if it exists). We denote the PDF by $f(t)$. This is a non-negative valued function such that

$$F(t) = \int_0^t f(x)dx, \quad 0 < t < \infty \tag{2}$$

The reliability function $R(t)$ of a component/system having a life distribution $F(t)$ is

$$R(t) = 1 - F(t) = \Pr\{T > t\} \tag{3}$$

This is the probability that the lifetime of the component/system will exceed t . another important function related to the life distribution is the failure rate or hazard function $h(t)$. this is the instantaneous failure rate of an element which has survived t units of time.

i.e.,

$$h(t) = \lim_{\Delta \rightarrow 0} \frac{F(t + \Delta) - F(t)}{\Delta \Pr\{T > t\}} = \frac{f(t)}{R(t)} \tag{4}$$

Notice that $h(t)\Delta t$ is approximately, for small Δt , the probability that a unit still functioning at age t will fail during the interval $(t, t + \Delta t)$.

$$h(t) = -\frac{d}{dt} \ln R(t), \tag{5}$$

and

$$R(t) = \exp \left\{ - \int_0^t h(x)dx \right\}. \tag{6}$$

Mean Time to Failure (MTTF)

The average length of time until failure (the expected value of T). The general definition of the expected value of a lifetime random variable T is

$$E\{T\} = \int_0^\infty t f(t)dt, \tag{7}$$

Provided this integral is finite. It can be shown that

$$E\{T\} = \int_0^\infty R(t)dt. \tag{8}$$

The mean time to failure is denoted by MTTF and also it will simply called as μ .

Censoring

Censoring is a major issue, especially in survival analysis. Censoring distinguishes survival analysis from conventional statistical problems. Censoring is done when an observation is incomplete for some random reasons. The reason for censorship usually depends on the occurrence of interest.

Censoring differs from Censoring in that the incompleteness of the observations for reduction occurs due to a systematic selection process inherent in the study design. There are five types of Censoring, based on the directions in which the incompleteness in the observations comes from

- 1) Type I Censoring
- 2) Type II Censoring
- 3) Random Censoring
- 4) Progressively censoring:
- 5) Hybrid censoring

Type I Censoring: Sometimes tests are performed within a certain period of time. Three the exact life span of an object is known only if it is less than some predetermined value. In that case, data are said to be type I censored (from right). More precisely a type I censored sample is one that arises when n items numbered say $1, 2, \dots, n$ are subject to limited periods of observations, and let L_1, \dots, L_n be those periods \exists i th item's lifetime L_i is observable only if $T_i \leq L_i$. L_i : called fixed censoring time for i^{th} item If all L_i are equal, data are said to be single type I censored.

Type II censoring: Suppose n random sample units are set on life-testing experimentation. But due to some reasons the experiment terminates after smallest r readings. Let these be denoted by the order statistics $T_{(1)}, \dots, T_{(r)}$. Here integer r is prefixed i.e. nonrandom. Since the remaining $n - r$ random sample value are at least as high as high as $T_{(r)}$: the sampling scheme is a censored one. Such a censoring is known as Type II censoring. Type II censoring are frequently used in life- testing experiments. Here say total of n items are placed on test.

Right censoring: The general form of censoring here is the lifetime of an object until the event (i.e. failure or death) has not yet occurred, but after that time this event will not participate in the further study.

Left censoring: This occurs when the event of interest has already occurred at the time observed, but the exact time at which the event occurred is unknown.

Progressively censoring: A sample of randomly selected n units is placed in a life test. In the event of a failure, r_1 the units are approximately removed from the remaining n_1 units. At the time of the second failure r_2 units are approximately removed from the remaining $n - 2 - r_1$ units during the second failure. At any time the test continues until m^{th} fails, all remaining $r_m = n - m - r_1 - r_2 - \dots - r_{m-1}$ units are removed.

Hybrid censoring: Combination of Type I and Type II censoring schemes. The sample life of approximately selected n units is subjected to testing. If a fixed number r of n items fails or the pre-determined time reaches t during the test, the test will be stopped.

Life Distribution models and their characteristics

Types of Failure Observations

A typical test of equipment life testing involves installing a sample of n identical units on the appropriate equipment and subjecting the units to operating under specific conditions until the failure of equipment is detected. In this case, we have accurate information about the lifespan or failure time T of that unit. The observed random variable, T is a continuous variable, i.e. it can take any value at a given time interval. The second type of data arises when units are observed only at separate time points t_1, t_2, \dots, N . The number of failures in the tested units is recorded for each of inter-inspection time interval. Let N_1, N_2, \dots , Indicate the number of failed units at time intervals $[0, t_1), [t_1, t_2) \dots$. these are unique random variables for the number of failures.

Proper analysis of the data depends on the observations available. Tests must often be stopped before all units of the test have failed. In such cases, we only have complete information about the time until failure (if monitoring is continuous) in a part of the model. We have only partial information on all failed units. Such data is called time censoring. If all the units start operating at the same time, we say that the censoring is single. Also known as one-time censoring type-I censoring. Some tests end in the event of r -th failure, where r is smaller than the predetermined integer n . In these cases the data is failed- censoring. The single failure censoring is called Type-II censoring. If different units start operating at different time points at intervals of $[0, t^*]$, and the test is stopped at t^* , we have multiple data censoring. We are different from censoring on the left and censoring on the right. If some units start operating before the official time, we have censoring. The other type of censoring information that the unit is still in operation at the end of the monitoring is called proper censoring.

General Characteristics of Life Distributions

We consider here the continuous random variable, T , which denotes the length of lie, or the length of time failure, in a continuous operation of the equipment. We denote by $F(t)$ the cumulative distribution function (CDF) of T , i.e.,

$$F(t) = \Pr\{T \leq t\}. \tag{9}$$

Obviously, $F(t) = 0$ for all $t \leq 0$. We assume here that initially the equipment is in proper operating condition. Thus, we eliminate from consideration here defective or inoperative units. The CDF $F(t)$ is assumed to be continuous, satisfying the conditions.

- 1) $F(0) = 0$;
- 2) $\lim_{t \rightarrow \infty} F(t) = 1$;
- 3) If $t_1 < t_2$ then $F(t_2) \leq F(t_1)$

The reliability at time t is the probability that the life length of the equipment exceeds t [time units]. The survival function is the same as the reliability function.

The probability density function (PDF) of a random variable, T , having a CDF $F(t)$, is a non-negative function, $f(t)$, such that

$$F(t) = \int_0^t f(x)dx, \quad 0 \leq t < \infty \tag{10}$$

According to this definition, $f(t)$ can be determined, at almost all points of t , as the derivative of $F(t)$.

The p^{th} percentile point of a life distribution $F(t)$, for a value of p in $(0,1)$, is the value of t , denoted by t_p , for which $F(t) = p$; i.e.,

$$F(t_p) = p. \tag{11}$$

If there is more than one value of t satisfying the above equation, we define t_p to be the smallest one. The median, $t_{.50}$, and the lower and upper quartiles, $t_{.25}$ and $t_{.75}$, respectively, are important characteristics of a life distribution.

Moments of order r of the life distribution are defined as

$$\mu_r = \int_0^\infty t^r f(t)dt, \quad r = 1, 2, \dots \tag{12}$$

Moments μ_r may not be finite.

If the PDF, $f(t)$, is symmetric around a point \bar{t} , then $\mu = \bar{t}$ (provided μ is finite). Moreover, if $f(t)$ is symmetric then the median is equal to the MTTF.

Another important relationship is that

$$\mu = \int_0^\infty R(t)dt \tag{13}$$

Where $R(t)$ is the reliability function.

The failure rate function, associated with a life distribution $F(t)$, is

$$h(t) = \frac{f(t)}{R(t)}, \quad 0 \leq t < \infty. \tag{14}$$

The function $H(t) = \int_0^t h(x)dx$ is called the cumulative hazard function.

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Appendix

$$M = \begin{pmatrix} -(2\beta_0 + 2\beta_1) & 2\beta_0 & 2\beta_1 & 0 & 0 & 0 \\ \alpha_0 & -(\alpha_0 + \beta_0 + 2\beta_1) & 0 & 2\beta_1 & 0 & 0 \\ \alpha_1 & 0 & -(\alpha_1 + 2\beta_0 + 2\beta_1) & 2\beta_0 & 2\beta_1 & 0 \\ 0 & \alpha_1 & \alpha_0 & -(\beta_0 + 2\beta_1 + \alpha_0 + \alpha_1) & 0 & 2\beta_1 \\ 0 & 0 & \alpha_1 & 0 & -(\alpha_1 + 2\beta_0 + \beta_1) & 2\beta_0 \\ 0 & 0 & 0 & \alpha_1 & \alpha_0 & -(\beta_0 + \beta_1 + \alpha_0 + \alpha_1) \end{pmatrix}$$

Table 1. States of the system

| State | Client 1 | Client 2 | Client 3 | Server 1 | Server 2 | System's Status |
|-----------------|------------|------------|------------|------------|-------------|-----------------|
| S ₀ | Functional | Functional | Functional | Functional | Replication | Operative |
| S ₁ | Functional | Functional | Functional | Failed | Functional | Operative |
| S ₂ | Failed | Functional | Functional | Functional | Replication | Operative |
| S ₃ | Failed | Functional | Functional | Failed | Functional | Operative |
| S ₄ | Failed | Failed | Functional | Functional | Replication | Operative |
| S ₅ | Failed | Failed | Functional | Failed | Functional | Operative |
| S ₆ | Failed | Failed | Failed | Idle | Idle | Down |
| S ₇ | Failed | Failed | Failed | Failed | Idle | Down |
| S ₈ | Idle | Idle | Idle | Failed | Failed | Down |
| S ₉ | Failed | Idle | Idle | Failed | Failed | Down |
| S ₁₀ | Failed | Failed | Idle | Failed | Failed | Down |

Table 2. Transition Table

| | S ₀ | S ₁ | S ₂ | S ₃ | S ₄ | S ₅ | S ₆ | S ₇ | S ₈ | S ₉ | S ₁₀ |
|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|-----------------|
| S ₀ | - | 3β ₀ | 2β ₁ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| S ₁ | α ₀ | - | 0 | 2β ₁ | 0 | 0 | 0 | 0 | β ₀ | 0 | 0 |
| S ₂ | α ₁ | 0 | - | 2β ₀ | 2β ₁ | 0 | 0 | 0 | 0 | 0 | 0 |
| S ₃ | 0 | α ₁ | α ₀ | - | 0 | 2β ₁ | 0 | 0 | 0 | β ₀ | 0 |
| S ₄ | 0 | 0 | α ₁ | 0 | - | 2β ₀ | β ₁ | 0 | 0 | 0 | 0 |
| S ₅ | 0 | 0 | 0 | α ₁ | α ₀ | - | 0 | β ₁ | 0 | 0 | β ₀ |
| S ₆ | 0 | 0 | 0 | 0 | α ₁ | 0 | - | 0 | 0 | 0 | 0 |
| S ₇ | 0 | 0 | 0 | 0 | 0 | α ₁ | 0 | - | 0 | 0 | 0 |
| S ₈ | 0 | α ₀ | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 |
| S ₉ | 0 | 0 | 0 | α ₀ | 0 | 0 | 0 | 0 | 0 | - | 0 |
| S ₁₀ | 0 | 0 | 0 | 0 | 0 | α ₀ | 0 | 0 | 0 | 0 | - |

Table 3. Variation of availability, profit and MTTF with α_1 and β_1

| β_1 | | | | α_1 | | | |
|-----------|--------------|---------------------|---------|------------|--------------|---------------------|---------|
| | Availability | Profit *1.0e+004 | MTTF | | Availability | Profit *1.0e+004 | MTTF |
| 0 | 0.7241 | 7.1379 | 11.2500 | 0 | 0 | -0.2000 | 8.7111 |
| 0.0714 | 0.7056 | 6.9196 | 11.0154 | 0.0714 | 0.2854 | 2.6327 | 9.1805 |
| 0.1429 | 0.6608 | 6.4435 | 10.0376 | 0.1429 | 0.4572 | 4.3597 | 9.5689 |
| 0.2143 | 0.6052 | 5.8679 | 8.6729 | 0.2143 | 0.5536 | 5.3387 | 9.8780 |
| 0.2857 | 0.5492 | 5.2943 | 7.3537 | 0.2857 | 0.6086 | 5.9030 | 10.1203 |
| 0.3571 | 0.4976 | 4.7695 | 6.2445 | 0.3571 | 0.6418 | 6.2456 | 10.3101 |
| 0.4286 | 0.4520 | 4.3076 | 5.3559 | 0.4286 | 0.6628 | 6.4654 | 10.4595 |
| 0.5000 | 0.4124 | 3.9077 | 4.6512 | 0.5000 | 0.6769 | 6.6134 | 10.5781 |
| 0.5714 | 0.3782 | 3.5632 | 4.0890 | 0.5714 | 0.6868 | 6.7173 | 10.6733 |
| 0.6429 | 0.3486 | 3.2659 | 3.6354 | 0.6429 | 0.6939 | 6.7929 | 10.7505 |
| 0.7143 | 0.3229 | 3.0085 | 3.2644 | 0.7143 | 0.6992 | 6.8495 | 10.8138 |
| 0.7857 | 0.3006 | 2.7842 | 2.9570 | 0.7857 | 0.7032 | 6.8930 | 10.8662 |
| 0.8571 | 0.2809 | 2.5878 | 2.6991 | 0.8571 | 0.7063 | 6.9270 | 10.9099 |
| 0.9286 | 0.2636 | 2.4146 | 2.4802 | 0.9286 | 0.7088 | 6.9543 | 10.9468 |
| 1.0000 | 0.2482 | 2.2609 | 2.2924 | 1.0000 | 0.7108 | 6.9763 | 10.9781 |

Table 4. Variation of availability, profit and MTTF with α_0 and β_0

| β_0 | | | | α_0 | | | |
|-----------|--------------|---------------------|----------|------------|--------------|---------------------|---------|
| | Availability | Profit *1.0e+004 | MTTF | | Availability | Profit *1.0e+004 | MTTF |
| 0 | 0.9494 | 9.4304 | 155.0000 | 0 | 0 | -0.1500 | 7.3209 |
| 0.0714 | 0.8824 | 8.7107 | 38.8944 | 0.0714 | 0.2838 | 2.6759 | 8.1412 |
| 0.1429 | 0.7723 | 7.5843 | 16,3659 | 0.1429 | 0.4703 | 4.5422 | 8.9525 |
| 0.2143 | 0.6710 | 6.5597 | 9.8243 | 0.2143 | 0.5933 | 5.7772 | 9.7548 |
| 0.2857 | 0.5873 | 5.7162 | 6.9014 | 0.2857 | 0.6766 | 6.6161 | 10.5484 |
| 0.3571 | 0.5194 | 5.0346 | 5.2816 | 0.3571 | 0.7347 | 7.2040 | 11.3334 |
| 0.4286 | 0.4643 | 3.3816 | 4.2629 | 0.4286 | 0.7766 | 7.6286 | 12.1100 |
| 0.5000 | 0.4190 | 4.0279 | 3.5668 | 0.5000 | 0.8076 | 7.9439 | 12.8781 |
| 0.5714 | 0.3813 | 3.6510 | 3.0626 | 0.5714 | 0.8311 | 8.1836 | 13.6381 |
| 0.6429 | 0.3496 | 3.3339 | 2.6814 | 0.6429 | 0.8493 | 8.3898 | 14.3900 |
| 0.7143 | 0.3226 | 3.0639 | 2.3834 | 0.7143 | 0.8637 | 8.5171 | 15.1340 |
| 0.7857 | 0.2994 | 2.8317 | 2.1443 | 0.7857 | 0.8752 | 8.6355 | 15.8701 |
| 0.8571 | 0.2792 | 2.6300 | 1.9483 | 0.8571 | 0.8845 | 8.7322 | 16.5985 |
| 0.9286 | 0.2615 | 2.4533 | 1.7849 | 0.9286 | 0.8922 | 8.8120 | 17.3194 |
| 1.0000 | 0.2458 | 2.2974 | 1.6465 | 1.0000 | 0.8987 | 8.8787 | 18.0327 |

Table 5. Variation of availability, profit and MTTF with respect to α_0 for different values of β_0

| α_0 | $A_T(\infty)$ | | | $P_F(\infty)$ | | | MTTF | | |
|------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|-----------------|-----------------|-----------------|
| | $\beta_0 = 0.1$ | $\beta_0 = 0.5$ | $\beta_0 = 0.9$ | $\beta_0 = 0.1$ | $\beta_0 = 0.5$ | $\beta_0 = 0.9$ | $\beta_0 = 0.1$ | $\beta_0 = 0.5$ | $\beta_0 = 0.9$ |
| | | | | *10 ⁴ | *10 ⁴ | *10 ³ | | | |
| 0 | 0 | 0 | 0 | -0.0150 | -0.0150 | -0.1500 | 14.1457 | 2.9827 | 1.6635 |
| 0.1111 | 0.6042 | 0.1899 | 0.1113 | 0.8928 | 0.2696 | 1.5165 | 18.6661 | 3.1995 | 1.7315 |
| 0.2222 | 0.7843 | 0.3369 | 0.2082 | 1.1654 | 0.4903 | 2.9704 | 22.9054 | 3.4158 | 1.7994 |

| | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|---------|--------|--------|
| 0.3333 | 0.8545 | 0.4498 | 0.2922 | 1.2723 | 0.6600 | 4.2306 | 26.8890 | 3.6314 | 1.8673 |
| 0.4444 | 0.8881 | 0.5367 | 0.3647 | 1.3240 | 0.7909 | 5.3197 | 30.6394 | 3.8465 | 1.9351 |
| 0.5556 | 0.9067 | 0.6042 | 0.4272 | 1.3526 | 0.8928 | 6.2605 | 34.1765 | 4.0610 | 2.0028 |
| 0.6667 | 0.9180 | 0.6573 | 0.4813 | 1.3701 | 0.9730 | 7.0741 | 37.5179 | 4.2749 | 2.0706 |
| 0.7778 | 0.9253 | 0.6996 | 0.5281 | 1.3816 | 1.0370 | 7.7795 | 40.6795 | 4.4882 | 2.1382 |
| 0.8889 | 0.9304 | 0.7337 | 0.5688 | 1.3895 | 1.0886 | 8.3928 | 43.6754 | 4.7009 | 2.2058 |
| 1.0000 | 0.9340 | 0.7614 | 0.6042 | 1.3952 | 1.1307 | 8.9279 | 46.5183 | 4.9131 | 2.2734 |

Table 6. Variation of availability, profit and MTTF with respect to β_0 for different values of α_0

| β_0 | $A_T(\infty)$ | | | $P_F(\infty)$ | | | $MTTF$ | | |
|-----------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | $\alpha_0 = 0.1$ | $\alpha_0 = 0.5$ | $\alpha_0 = 0.9$ | $\alpha_0 = 0.1$ | $\alpha_0 = 0.5$ | $\alpha_0 = 0.9$ | $\alpha_0 = 0.1$ | $\alpha_0 = 0.5$ | $\alpha_0 = 0.9$ |
| | | | | $P_F * 10^4$ | $P_F * 10^4$ | $P_F * 10^4$ | | | |
| 0 | 0.9494 | 0.9494 | 0.9494 | 1.4209 | 1.4209 | 1.4209 | 155.0000 | 155.0000 | 155.0000 |
| 0.1111 | 0.5405 | 0.8893 | 0.9270 | 0.7966 | 1.3258 | 1.3842 | 16.2065 | 28.2793 | 38.4200 |
| 0.2222 | 0.3401 | 0.7869 | 0.8786 | 0.4951 | 1.1693 | 1.3093 | 7.5491 | 11.1872 | 14.6504 |
| 0.3333 | 0.2458 | 0.6079 | 0.8215 | 0.3535 | 1.0223 | 1.2221 | 4.8796 | 6.5778 | 8.2382 |
| 0.4444 | 0.1920 | 0.5405 | 0.7642 | 0.2727 | 0.8984 | 1.1349 | 3.5970 | 4.5727 | 5.5362 |
| 0.5556 | 0.1574 | 0.4850 | 0.7102 | 0.2208 | 0.7966 | 1.0531 | 2.8458 | 3.4774 | 4.1041 |
| 0.6667 | 0.1333 | 0.4391 | 0.6608 | 0.1846 | 0.7131 | 0.9783 | 2.3531 | 2.7948 | 3.2341 |
| 0.7778 | 0.1156 | 0.4006 | 0.6163 | 0.1581 | 0.6439 | 0.9110 | 2.0054 | 2.3313 | 2.6561 |
| 0.8889 | 0.1020 | 0.3680 | 0.5763 | 0.1377 | 0.5860 | 0.8506 | 1.7469 | 1.9973 | 2.2469 |
| 1.0000 | 0.0913 | 0.3680 | 0.5405 | 0.1217 | 0.5370 | 0.7966 | 1.5473 | 1.7456 | 1.9435 |

Table 7. Variation of availability, profit and MTTF with respect to α_1 for different values of β_1

| α_1 | $A_T(\infty)$ | | | $P_F(\infty)$ | | | $MTTF$ | | |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $\beta_1 = 0.1$ | $\beta_1 = 0.5$ | $\beta_1 = 0.9$ | $\beta_1 = 0.1$ | $\beta_1 = 0.5$ | $\beta_1 = 0.9$ | $\beta_1 = 0.1$ | $\beta_1 = 0.5$ | $\beta_1 = 0.9$ |
| | | | | $P_F * 10^4$ | $P_F * 10^4$ | $P_F * 10^3$ | | | |
| 0 | 0 | 0 | 0 | -0.0100 | -0.1000 | -0.1000 | 8.7111 | 3.4003 | 2.0638 |
| 0.1111 | 0.3926 | 0.1030 | 0.0591 | 0.5742 | 1.4261 | 0.7747 | 9.4066 | 3.6112 | 2.1471 |
| 0.2222 | 0.5613 | 0.1926 | 0.1136 | 0.8274 | 2.7571 | 1.5830 | 9.9080 | 3.8350 | 2.2349 |
| 0.3333 | 0.6324 | 0.2702 | 0.1641 | 0.9349 | 3.9136 | 2.3333 | 10.2519 | 4.0686 | 2.3270 |
| 0.4444 | 0.6665 | 0.3366 | 0.2109 | 0.9866 | 4.9044 | 3.0289 | 10.4882 | 4.3088 | 2.4231 |
| 0.5556 | 0.6849 | 0.3926 | 0.2540 | 1.0146 | 5.7424 | 3.6713 | 10.6539 | 4.5529 | 2.528 |
| 0.6667 | 0.6958 | 0.4396 | 0.2936 | 1.0313 | 6.4451 | 4.2618 | 10.7730 | 4.7986 | 2.6258 |
| 0.7778 | 0.7028 | 0.4787 | 0.3298 | 1.0421 | 7.0319 | 4.8021 | 10.8608 | 5.0436 | 2.7318 |
| 0.8889 | 0.7075 | 0.5113 | 0.3627 | 1.0493 | 7.5215 | 5.2947 | 10.9271 | 5.2862 | 2.8405 |
| 1.0000 | 0.7108 | 0.5385 | 0.3926 | 1.0545 | 7.9308 | 5.7424 | 10.9781 | 5.5247 | 2.9517 |

Table 8. Variation of availability, profit and MTTF with respect to β_1 for different values of α_1

| β_1 | $A_T(\infty)$ | | | $P_F(\infty)$ | | | $MTTF$ | | |
|-----------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| | $\alpha_1 = 0.1$ | $\alpha_1 = 0.5$ | $\alpha_1 = 0.9$ | $\alpha_1 = 0.1$ | $\alpha_1 = 0.5$ | $\alpha_1 = 0.9$ | $\alpha_1 = 0.1$ | $\alpha_1 = 0.5$ | $\alpha_1 = 0.9$ |
| | | | | $P_F * 10^4$ | $P_F * 10^3$ | $P_F * 10^3$ | | | |
| 0 | 0.7241 | 0.7241 | 0.7241 | 1.0759 | 1.0759 | 1.0759 | 11.2500 | 11.2500 | 11.2500 |
| 0.1111 | 0.3397 | 0.6677 | 0.7043 | 0.4950 | 0.9884 | 1.0444 | 9.0313 | 10.3983 | 10.8348 |

| | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.2222 | 0.1947 | 0.5638 | 0.6568 | 0.2788 | 0.8313 | 0.9719 | 6.4848 | 8.1911 | 9.2809 |
| 0.3333 | 0.1358 | 0.4695 | 0.5989 | 0.1913 | 0.6895 | 0.8841 | 4.9354 | 6.2731 | 7.4395 |
| 0.4444 | 0.1042 | 0.3958 | 0.5412 | 0.1444 | 0.5790 | 0.7972 | 3.9529 | 4.9334 | 5.9189 |
| 0.5556 | 0.0846 | 0.3397 | 0.4887 | 0.1152 | 0.4950 | 0.7183 | 3.2850 | 4.0104 | 4.7903 |
| 0.6667 | 0.0712 | 0.2964 | 0.4427 | 0.0954 | 0.4304 | 0.6493 | 2.8045 | 3.3548 | 3.9658 |
| 0.7778 | 0.0614 | 0.2625 | 0.4031 | 0.0809 | 0.3798 | 0.5898 | 2.4436 | 2.8719 | 3.3548 |
| 0.8889 | 0.0540 | 0.2353 | 0.3690 | 0.0700 | 0.3393 | 0.5388 | 2.1632 | 2.5044 | 2.8916 |
| 1.0000 | 0.0482 | 0.2131 | 0.3390 | 0.0614 | 0.3062 | 0.4950 | 1.9393 | 2.2167 | 2.5320 |