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Original Research Article

Reliability Evaluation of Reverse Osmosis System in Water Treatment Using Modified Weibull Distribution

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Abstract

Recently, reliability has become a critical criterion for product quality and decision-making that covers a wide range of subjects, including failure analysis systems. Performing a reliability analysis is essential for the study of operating safety in industrial systems. In this study, we list evaluation methods and perform real-time reliability analyses. The real-time reliability modeling of a Reverse Osmosis system is addressed in this paper. The model will help create effective maintenance while extending the subsystems' lifespan. To achieve our goal, we suggested the 2-parameter modified Weibull distribution. The simulation was performed using Maple software. The evaluation for each subsystem was displayed in the result and analyses section. The conclusion, however, draws a broad conclusion about the study.

Keyword: Reliability; System; Dependability; Weibull; Reverse Osmosis; parameters.

1. List of Abbreviations and Nomenclature

NMW	New modified Weibull
MTTF	Mean time to failure
MTBF	Mean time between failures
MTTR	Mean time to repairs
MDT	Mean downtime
$\boldsymbol{F(t)}$	Cumulative Distribution Function
$\boldsymbol{\beta}$	Shape parameter
$\boldsymbol{\alpha}$	Scale parameter
$\boldsymbol{R(t)}$	Reliability as a function of time
f(t): PDF	Probability density function
CDF	Cumulative probability function
H.P	High pressure
$\boldsymbol{h(t)}$	Failure rate
$\boldsymbol{K_0}$	Number of days in a year
$\boldsymbol{K_s}$	Units survive the test
$\boldsymbol{K_f}$	Units fail the test

2. Introduction

Failure analysis is crucial if the system's dependability is evaluated using a reliability test. If we want to increase dependability, we must understand the precise reasons why units fail because only then can we Figure out what needs to change. Suppose the material cracked, sheared, or experienced other types of material failure. In that case, you can choose a different material or alter the design of the damaged component to make it more resistant. A component should typically have a strength two to four times greater than the stress or strain it will experience during normal operation. A shape parameter determines the shape of the Weibull distribution family of particular frequency distributions. The scale and location parameters impact the precise Weibull function, producing numbers that conform to the Weibull distribution. The Weibull function has other uses in probability computation besides determining the kind of sample design that should be used.

There are numerous disciplines and uses for the Weibull distribution W. A. Weibull [1]. The Weibull distribution has only three possible hazard functions: growing, declining, or constant. As a result, it cannot be used to model lifetime data with a bathtub-shaped hazard

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function, such as human mortality and machine life cycles. Researchers have been developing numerous Weibull distribution extensions and modifications for a very long time, with parameters ranging from 2 to 5. The hazard function of Marshall [2]. The adaptable Weibull extension for two parameters can be growing, descending, or bathtub-shaped. The hazard function of the truncated Weibull distribution, which resembles a bathtub, was examined by Zhang and Xie ([3] to determine its properties and applications. Maihulla and Yusuf [4] study the reverse osmosis machine filtration system using the Copula approach. Maihulla and Yusuf [5] presented the reliability modeling and performance analysis of a reverse osmosis machine in water purification using Gumbel–Hougaard family copula. Maihulla et al. [6]. Reliability, Availability, Maintainability, and Dependability Analysis were used to evaluate a complex reverse osmosis machine system for purifying water reliability.

However, poor operation or upkeep during the utilizing period may reduce reliability. Therefore, all authorities must consider maintaining the system's reliability from the start to the end of its lifespan, Billinton et al. [7]. For improving the effectiveness of industrial processes, Gupta and Tiwari's writings include some case studies based on steady-state analysis of Markov models, Gupta and Tewari [8]. A transient analysis of manufacturing systems' performance was conducted by Narahari and Viswanadham [9]. Butter oil processing and the production of plastic pipes, Gupta et al. [10] under the premise that failure and repair rates are constant, a numerical analysis of the system behavior in transient states using the Runge Kutta fourth-order method was conducted. A numerical approach was put forth by Kaur et al. [11] to research transient analysis for a system with a fixed failure rate and a variable repair rate. The effects of stochastic processes on system performance were found to be based on the exponential distribution, and reliability analysis and steady-state availability analysis were used to conduct further behavior analysis of the process industry. We chose reverse osmosis as the test system for our methodology because we wanted to assess how well our modified Weibull distribution guaranteed reliability and other metrics for optimal maintenance and fault tolerance, as well as to suggest ways to ensure maximum dependability possibly. The water purification industry is currently dealing with a serious problem with purification filter failure, which is the cause of this. As a result, progress has been made in water filtration technology and understanding of its significance in people's lives worldwide. The RO industries are working very hard to keep up with the expanding complexity of the systems. Reverse Osmosis membranes are typically used as cross-flow filters, where the wastewater flowing along the filter at high speed maintains a turbulent flow that helps control the thickness of the solids on the filter and lowers filter plugging.

The Weibull distribution (Weibull, 1951), It's safe to say that the lifetime distribution, known by the name of the Swedish professor Waloddi Weibull, is the most frequently used for lifetime data analysis. This is because the distribution is more flexible in analyzing a wide range of phenomena than other distributions and is simpler and clearer than other distributions [1].

The Weibull distribution is an extension of the exponential distribution. This distribution is appropriate for complex multi-part systems or components.

A new Weibull extension distribution is then presented in this study. This model is considered an expansion of the Weibull distribution with a bathtub-shaped failure rate function. A breakdown of the model's characteristics is also provided. Jun et al. [12] presented a Parameter Evaluation of 3-parameter Weibull Distribution based on Adaptive Genetic Algorithm. Basheer et al. [13] analyzed the Reliability Estimation of Three Parameters of Weibull Distribution based on Particle Swarm Optimization.

It is clearly stated by Quek and Ang (1086) [14] if the lifetimes follow the Weibull distribution, the p.d.f.

$$f(t) = \beta\alpha^{-\beta}t^{\beta-1}e^{-(\frac{t}{\alpha})^{\beta}} \quad (1)$$

Where α and β are the scale and shape parameters of the Weibull distribution, respectively.

The size of the units where the random variable, t, is measured is reflected by the scale parameter, α. The distribution's form changes depending on the shape parameter, β. We can create a diverse set of curves that reflect real lifetime failure distributions by modifying the value of β.

From (1), the Cumulative Distribution Function is given by:

$$F(t) = 1 - e^{-(\frac{t}{\alpha})^{\beta}} \quad (2)$$

From the relation,

$$R(t) = 1 - F(t) \quad (3)$$

We can substitute (2) into (3), and we have

$$Rt=e-(t\alpha)\beta \quad (4)$$

Where $R(t)$ is the reliability or survival function.

The failure rate function or the hazard function can therefore be derived from the following relation:

$$h(t) = \frac{f(t)}{R(t)} \quad (5)$$

Substituting (1) and (4) into (5), we have:

$$h(t) = \frac{\beta\alpha^{-\beta}t^{\beta-1}e^{-(\frac{t}{\alpha})^{\beta}}}{e^{-(\frac{t}{\alpha})^{\beta}}} \quad (6)$$

$$h(t) = \beta\alpha^{-\beta}t^{\beta-1} \quad (7)$$

If we consider K_0 to be the number of hours in a year (the size of the population). Out of which K_s units (the number of hours that the system is upstate) survive the test. While K_f fail, then reliability function $R(t)$ is given by:

$$R(t) = \frac{K_s}{K_0} = \frac{K_0 - K_f}{K_0} \quad (8)$$

Differentiating both sides of (8) and taking K_0 fixed, the following equation will result

$$\frac{dR(t)}{dt} = \frac{1}{K_0}\frac{dK_f}{dt} \quad (9)$$

$$\frac{dK_f}{dt} = -K_0 \frac{dR(t)}{dt} \tag{10}$$

Dividing both sides of the above equation by K_s, we obtain the instantaneous probability $g(t)$ of failure, this is:

$$g(t) = \frac{1}{K_S}\frac{dK_f}{dt} = -\frac{K_0}{K_S}\frac{dR(t)}{dt} \tag{11}$$

Using equation (8) into (11), we get:

$$g(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{12}$$

Integrating both sides of the equation (12), we have:

$$\int g(t)dt = -\log R(t) \tag{13}$$

From the above equation, the $R(t)$ will be:

$$R(t) = e^{-\int_0^x g(t)dt} \tag{14}$$

Where x is variable.

The function $g(t)$ is called the hazard function or failure rate. Equation (14) can be considered a generic expression of failure as it applies to both exponential and non-exponential failure distribution.

For our modified Weibull distribution, we compared (14) with (4)

$$(\frac{t}{\alpha})^{\beta} = \int_0^x g(t)dt \tag{15}$$

For,

$$g(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{16}$$

and the below facts:

The derivative of a definite integral of a function is the function itself only when the lower limit of the integral is a constant and the upper limit is a variable concerning which we are differentiating. To summarize Ulrich et al. [15]:

- The derivative of an indefinite integral of a function is the function itself. i.e., $\frac{d}{dx}\int f(x)dx = f(x)$
- The derivative of a definite integral with constant limits is 0. i.e., $\frac{d}{dx}\int_a^b f(x)dx = 0$
- The derivative of a definite integral where the lower limit is a constant, and the upper limit is a variable is a function itself in terms of the given variable (upper bound).

  i.e., $\frac{d}{dx}\int_a^x f(x)dx = f(x)$ where 'a' is a constant and 'x' is a variable.

$$(\frac{t}{\alpha})^{\beta} = -\frac{K_0}{K_S}(R(t)) \tag{17}$$

The NMW proposed for the present research follows;

$$R(t) = -\frac{K_S}{K_0}(\frac{t}{\alpha})^{\beta} \tag{18}$$

3. System Description

3.1 Pre-treatment

Making feed water for the RO compatible with the membrane is the main goal of pretreatment. Pretreatment is necessary to reduce membrane fouling, scaling, and degradation and extend the effectiveness and lifespan of the membrane elements. The term "fouling" describes the entrapment of particles on the surface, or, in the worst case scenario, inside the membrane pores, such as silt, clay, suspended solids, biological slime, algae, silica, iron flocs, and other matter. Fouling typically starts in the lead components of the first stage and progresses through the remaining components. Some metals, such as soluble iron and manganese, oxidize once inside the membrane system and can precipitate anywhere throughout the RO system, depending on the operating conditions and water chemistry. Microbes can also multiply and spread throughout the whole RO system. Microbiological and organic fouling are the most prevalent and hardest to control types of foulants in surface water and wastewater applications. For continuous, consistent, and dependable operation, the proper pretreatment of raw water to make it suitable as feed water to RO must involve a total system approach.

3.2 High-pressure pump

High-pressure pumps also supply water to the spraying equipment, creating working pressure. Positive displacement pumps and hydraulic intensifiers are the two main categories. Positive displacement pumps are frequently used in hydro demolition applications. For instance, positive displacement pumps power about 90% of all on-site machinery in Germany. The main parts of a positive displacement pump are the pressure regulator valve, switch valve, high-pressure plunger conversion set, pump head with low-pressure inlet valves, high-pressure output valves, and safety devices, according to Andreas [16].

3.3 RO Membrane

Reverse osmosis (RO) membranes are crucial in wastewater treatment facilities because they effectively remove salts and other contaminants. RO membranes' performance is affected by various factors, such as feed quality and operational parameters.

The water supply contains total dissolved solids (TDSs), a type of contaminant that RO membranes can typically remove 90%–99% of. The membranes are typically created as a flat sheet of thin composite membranes made of an active polyamide layer with high permeability but impervious to dissolved salts and particulate debris. A porous polysulphone layer supports this layer looped around a central collection tube.

Osmosis takes place when a semipermeable membrane separates two solutions with different concentrations. Hydraulic pressure is delivered via a pump to the concentrated side to overcome the osmotic pressure in RO water purification systems. Water is then extracted from the concentrated solution and gathered beneath the membrane. When sizing RO membranes, [17-18] consider the desired hourly or daily rate of water use.

3.4 Filtration

Water impurities are removed from water using a specialized filtration system during the highly effective water purification process known as reverse osmosis (RO). The RO system's ability to produce high-quality, clean drinking water or treat various types of water for particular purposes makes it a popular choice for residential and industrial applications. Let's examine the main elements and the operation of the reverse osmosis filtration system in more detail.

4. Estimation of the parameters

Several methods can be used to estimate a Weibull distribution's parameters. These methods can generally be divided into two categories: graphical and statistical. Statistical techniques, however, rely more on the theory and logic found in mathematics and statistics. It is more adaptable and can be used with different model designs and data types. The estimators have well-developed asymptotic properties and are typically more accurate than graphical estimators. The majority of these methods, however, have trouble resolving more difficult equations and expressions. Fortunately, the statistical estimates were simple to derive thanks to the readily available, ever-more capable software programming tools, such as Maple 13. The commonly used statistical methods are Method of Moment Lawless [19], Method of Percentiles, Method of Maximum Likelihood Estimation, Lawless [19]; Nelson [20], Bayesian Method Soland, [21]; Papadopoulos and Tsokos, [22]; Dellaportas Wright, [23], and Interval Estimation Mann, [24]; Yusuf, [25]. Here, we limit our review to Maximum Likelihood Estimation (MLE), the most frequently used method. The likelihood function is maximized at the exact value of the parameters. Considered to be the most likely values are the parameter values, which are estimates of the parameters.

Suppose there are K_f components in a sample of K_0 Components failed in sample testing. Given the failure data following a Weibull distribution, K_1, K_2, ..., K_n are the lifetime of K_0 failed components; let K_n be the censoring time for the rest $K_0 - K_f$ components. The likelihood function of standard Weibull distribution has the form of:

$$(\alpha\beta) = \frac{K_0}{(K_0-K_f)!}\left(\frac{\beta}{\alpha^{\beta}}\right)^{K_f} \bigcap_{i=1}^{K_f} K_i^{\beta-1}\ Exp[-\frac{1}{\alpha^{\beta}}\sum_{i=0}^{K_f} K_i^{\beta} + (K_0-K_f)K_n^{\beta}] \quad (19)$$

The maximum likelihood estimation of the parameters is obtained by differentiating the log-likelihood function L concerning the parameters and setting the result equal to zero; we have the following normal equations (18). Thus, the Log-likelihood function will be

$$Ln(\alpha\beta) = Ln\frac{K_0}{(K_0-K_f)!} + K_f(Ln\beta - \beta Ln\alpha) + (\beta-1)\sum_{i=0}^{K_f} K_i - [-\frac{1}{\alpha^{\beta}}(\sum_{i=0}^{K_f} K_i^{\beta} + (K_0 - K_f)K_n^{\beta})] \quad (20)$$

Take the first derivative of (19) above concerning both parameters $(\alpha\beta)$ and set it to zero to get the maximum estimation. Hence,

$$\frac{\sum_{i=0}^{K_f} K_i^{\beta} LnK_i + (K_0-K_f)K_n^{\beta} LnK_n}{\sum_{i=0}^{K_f} K_i^{\beta} + (K_0-K_f)K_n^{\beta}} - \frac{1}{K_f\sum_{i=0}^{K_f} LnK_i} = 0 \quad (21)$$

And

$$\alpha = [\frac{1}{K_f}(\sum_{i=0}^{K_f} K_i^{\beta} + (K_0 - K_f)K_n^{\beta})]^{\frac{1}{\beta}} \quad (22)$$

Solving Equation (20), we can find the Maximum Likelihood Estimation of shape parameter, and the scale parameter estimation can be obtained from Equation (21), all using the Maple software. The following Figures were obtained for the cumulative probability function and probability density function of the modified Weibull distribution (For $= a, \beta = b$ and $\lambda =$ The failure rate):

5. Weibull Distribution for Reliability

The effect of failure and repair rates of various sub-systems on the reliability of the transient state has been investigated for a range of likely failure rates and repair rates of the subsystems. The Modified equations have been numerically solved using Maple Software. Calculating the monthly rates of subsystem repair and failure. The Markov technique was used to create Table 1 by Tabrizi (2021) [26].

$$MTBF = \frac{\sum(Start\ of\ down\ time - Start\ of\ uptime)}{Number\ of\ Failures} \quad (23)$$

$$MDT = \frac{\sum(Start\ of\ uptime - Start\ of\ downtime)}{Number\ of\ Failures} \quad (24)$$

$$MTTF = \frac{\sum(Total\ time\ for\ the\ experiment - Start\ of\ downti)}{Number\ of\ Failures} \quad (25)$$

The failure rate;

$$\alpha_i = \frac{1}{MTTF} \quad (26)$$

Table 1. Values of failure rate, repair rate, and probability of states for calculation of reliability in a Reverse Osmosis system by Markov technique (Using the number of hours in a year as 8760)

Subsystems	No. of Occurrence	Total Repair time	MTTR in Hrs	MTBF in Hrs	MTTF	Repair rate	Failure rate
Pre-treatment	6	190	31.667	1460	1428.33	0.03158	0.0007001
H. P. Pump	5	105	21	1731	1714	0.04762	0.0005834
R.O Membrane	7	63	5.1	1242.429	1224.286	0.1961	0.000817

Table 2. Effect of failure rate (α_1) of the subsystem Pre-treatment on the system's reliability by Weibull

Time↓ α_1→	**0.0007001**	**0.0008751**	**0.00010501**	**0.0012251**
30	0.9498	0.9384	0.8384	0.7655
60	0.9253	0.9139	0.8135	0.7421
90	0.9192	0.9078	0.8075	0.7151
120	0.9177	0.8963	0.8000	0.7001
150	0.9173	0.8759	0.7905	0.6811
180	0.9162	0.8458	0.7605	0.6422
210	0.9121	0.8158	0.7355	0.6154
240	0.9091	0.8008	0.7155	0.5923
270	0.8961	0.7858	0.7021	0.5721
300	0.8883	0.7652	0.6895	0.5486
330	0.8662	0.7354	0.6705	0.5199
360	0.8534	0.7140	0.6554	0.5010

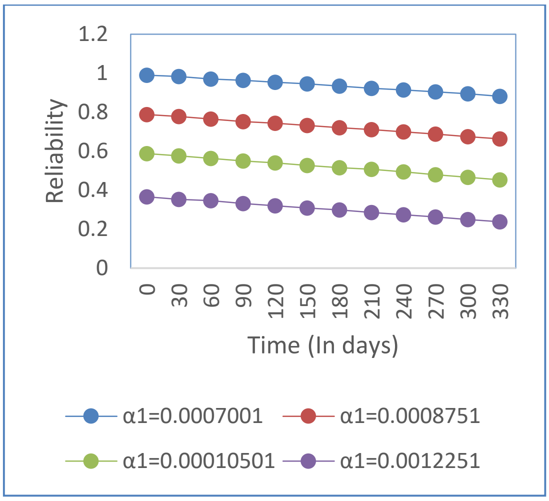

Figure 3. Variation of reliability of Pre-treatment subsystem with time

Table 3. Effect of failure rate (α_2) of the subsystem High-pressure pump on system's reliability by Weibull

Time↓ α_2→	**0.0005834**	**0.0007294**	**0.0008754**	**0.0010214**
30	0.9936	0.7056	0.5771	0.3778
60	0.9405	0.6892	0.5532	0.3551
90	0.9045	0.6675	0.5321	0.3209
120	0.8944	0.6502	0.5145	0.3055
150	0.8541	0.6355	0.5003	0.2870
180	0.8106	0.6178	0.4871	0.2605
210	0.7899	0.6001	0.4682	0.2467
240	0.7461	0.5867	0.4404	0.2221
270	0.7099	0.5641	0.4266	0.2033
300	0.6904	0.5396	0.4096	0.1899
330	0.6500	0.5155	0.3845	0.1669
360	0.6152	0.4962	0.3653	0.1448

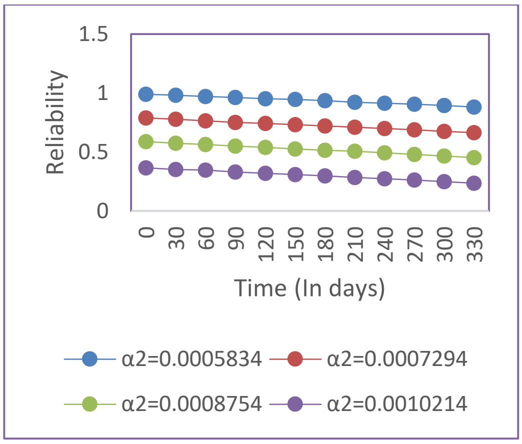

Figure. 4. Variation of reliability of High-pressure pump subsystem with time

Table 4. Effect of failure rate (α_3) of the subsystem RO-Membrane on the system's reliability by Weibull

Time↓ α_3→	**0.000817**	**0.001021**	**0.001225**	**0.001429**
30	0.9901	0.7881	0.5882	0.3661
60	0.9827	0.7776	0.5761	0.3535
90	0.9704	0.7642	0.5630	0.3466
120	0.9631	0.7521	0.5502	0.3317
150	0.9529	0.7433	0.5398	0.3202
180	0.9450	0.7322	0.5270	0.3091
210	0.9343	0.7211	0.5161	0.2994
240	0.9221	0.7104	0.5067	0.2861
270	0.9139	0.69994	0.4944	0.2732
300	0.9048	0.6881	0.4797	0.2619
330	0.8951	0.6754	0.4653	0.2494
360	0.8819	0.6633	0.4522	0.2374

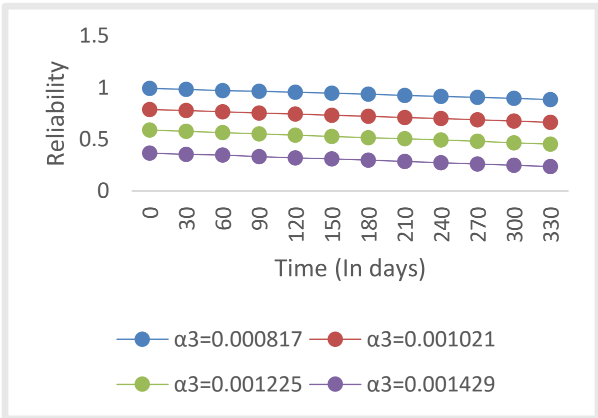

Figure. 5. Variation of reliability of RO-Membrane subsystem with time

6. Analysis and Results

The study's conclusions state that Weibull distribution analysis was used to evaluate the reverse osmosis system's strength, effectiveness, and performance improvement. Suppose the strength, efficiency, and performance of the reverse osmosis system are evaluated using the proposed modified technique systems' level of complexity. In that case, users can save money on medical care because of tainted water. The effect of the rates of the reverse osmosis sub-systems on the reliability of the transient state has been investigated for various likely failure rates and repair rates of the subsystems. The Markov method has been used to generate NMW and numerically solve the differential equations using the Maple software. Collecting information on daily subsystem failure rates. Tables 2 and corresponding Figure 3 demonstrated that, as failure rate (α_1) for the subsystem Pre-treatment was adjusted from 0.0007001 to 0.0012251 to compute the system's reliability while maintaining other metrics like α_2 = 0.0005834 and α_3 = 0.000817. The outcomes are displayed in Table 2. It demonstrates that as the failure rate of the Pre-treatment increases from 0.0007001 to 0.0012251, the system's reliability reduces roughly from 0.14% to 0.12%. In contrast, as the period increases from 30 to 360 days, it decreases roughly from 3.3% to 3.2%.

Table 3 and corresponding Figure 4 show the second subsystem's failure rate. The reliability of the system is computed by varying failure rate α_2 of High-Pressure pump from 0.0005834 to 0.0010214 and keeping other parameters such as α_1 = 0.0007001 and α_3 = 0.000817, fixed. It is noticed that with an increase in the failure rate α_2 of High-Pressure pump 0.0005834 to 0.0010214, the system's reliability decreases approximately by 0.61% to 0.58%. However, it decreases approximately from 3.3% to 5.2%, with an increase from 30 days to 360 days.

Next, we studied the effect of the failure rate of the subsystem, namely the RO-Membrane, on the system's reliability by varying its failure rate from 0.000817 to 0.001429. Fixing the other failure and repair rates of the subsystems, such as α_1 = 0.0007001 and α_2 = 0.0005834, the system's reliability is calculated using the data shown in Table 4 corresponding to Figure 5. We observed that, with the increase in the failure rate of RO-Membrane from 0.000817 to 0.001429, the system's reliability decreases by approximately 17.7% to 19.6%. In contrast, it decreases by approximately 3.3% to 3.2%, increasing from 30 to 360 days.

Table 1 summarizes MTTF, MTTR, and generation of the failure rate by using the Markov chain and repair rate in a total of 8760 hours a year.

7. Conclusion

According to the comparison of the tables above 2 to 4, the subsystem RO Membrane has a greater impact on the reliability of the entire system than any of the other subsystems. Other subsystems hardly impact pre-treatment reliability in the reverse osmosis water purification system. To increase overall reliability, we deduce that management should give the subsystem RO-Membrane the utmost attention.

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