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Original Research Article

Spherical Fuzzy Geometric Programming: A New Approach for System Reliability Assessment

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Abstract

The high risk of operational facilities and processes in strategic industries has made the continuous effort of operations managers to improve system reliability an undeniable necessity. The existing knowledge in fuzzy sets limits the sum of degrees of membership and non-membership of each element to more than one. However, in the real world, many ill-defined or highly complex situations require more consideration that is careful. Unlike normal fuzzy sets, the spherical fuzzy set pays attention to the degree of uncertainty of each element in decision-making situations and the degree of membership and non-membership. In addition, to help generalize the decision set, it considers the sum of squares of each membership function to be less than or equal to one. Achieving the success function is determined by maximizing the degree of membership and minimizing the degree of non-membership and uncertainty of each objective function in the spherical fuzzy set. Therefore, this paper develops a new algorithm based on the spherical fuzzy set called the spherical fuzzy geometric programming problem in system reliability. To evaluate the performance of the proposed algorithm, a descriptive example in the field of the rolling process of aluminum products is modeled in the form of a dual-objective problem, including maximization of reliability and minimization of cost.

Keywords: Reliability; Spherical fuzzy set; Geometric programming; Aluminum industry.

1. Introduction

The concept of reliability was born in the late 1940s and early 1950s. Reliability was first used in communication and transportation [1]. However, the aerospace industry and military applications were the first applications for reliability. However, reliability is particularly important for other industries, such as the nuclear, steel, and aluminum industries, due to the limited supply of electrical energy and the financial damage and environmental pollution caused by the cessation of their activities [2].

Reliability and durability are the first criteria for defining product quality [3]. Reliability is the extension of quality in the time domain [4] and is interpreted as the probability of no failure or break during a certain time interval [5]. Reliability can be seen as the successful operation of the system in a specified and predetermined period and conditions. Since the reliability of a system is defined in terms of probability, it directly means that uncertainty must be managed to assess the reliability of any system. More or less, all reliability assessment methods follow different types of probability distributions to account for failure and repair rates. However, real-world systems rarely follow these probability distributions to fail or be repaired. Therefore, to evaluate the reliability of a system, the fuzzy set theory approach is used.

Recently, in the theory of fuzzy sets, various types of uncertainty (degree of membership, non-membership, and uncertainty of elements) have been considered [6]. The basic theory of fuzzy sets was based on membership (or degree of belonging) and non-membership (or degree of non-belonging) with the sum of one. Degrees of membership and non-membership can be considered positive (success) and negative (failure) characteristics of a situation, respectively. Generally, the degree of membership is the same as the degree of success (acceptance/favorability), and the degree of non-membership is the same as the degree of failure (rejection/opposition).

Duffin and his colleagues introduced geometric programming in 1967 [7]. Various techniques have been developed to optimize geometric planning based on ordinary [8], intuitive [9], and neutrosophic [10] fuzzy

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sets [11-13]. Ahmad et al. (2020) developed a modified Neutrosophic approach for multi-objective planning problems [14]. Decision problems can be separated into two general states, discrete and continuous. Multi-indicator and multi-criteria decision-making problems are analyzed in a discrete mode. For the geometric programming problem in continuous mode, Kutlu Gündo˘gdu and Kahraman (2019) extended the ordinary, intuitive, and Pythagorean fuzzy set [15] to the spherical fuzzy set [16]. The naming of the spherical fuzzy set is based on the different degrees of membership of the elements of the set (i.e., positive, neutral, and negative). An example can explain intuitionistic fuzzy sets and Pythagorean fuzzy sets more clearly. For example, if in a multi-criteria decision problem, the degree of choosing an option is 0.6, rejecting an option is 0.7, and neutrality towards choosing/rejecting an option is 0.8, the uncertainty above sets cannot cover this situation. To deal with such cases, the spherical fuzzy set is a powerful decision-making tool (see [17]) and allows decision-makers to enter neutral thoughts into decision-making processes.

The biggest challenge facing system designers and employers is increasing system reliability without significantly increasing costs. In this article, a model has been developed that will have the ability to establish the desired balance of the system designer, employer, or contractor, and from the point of view that the improvement of system reliability by emphasizing the reliability of its components is considered at the same time as the system costs. It is a practical model.

The rest of the paper is organized as follows: Section 2 studies the spherical fuzzy geometric programming problem and the solution algorithm. Section 3 discusses the case study. Conclusion and future suggestions are also presented in Section 4.

2. Spherical fuzzy geometric programming model

The spherical fuzzy set results from developing and improving various classical fuzzy sets, such as ordinary fuzzy sets, intuitive fuzzy sets, and Pythagorean fuzzy sets. The difference between the spherical and other fuzzy sets is in introducing the degree of neutrality in decision-making processes to reflect reality. The objective functions are evaluated with three different membership functions, i.e., membership functions, indeterminate or neutral, and non-membership, respectively. Therefore, if it is necessary to adopt the degree of uncertainty, using a spherical fuzzy set is crucial for decision-making. The importance of developing geometric programming in the spherical fuzzy environment is quite noticeable because it is impossible to model many engineering and management problems without considering the uncertain aspects of the decision scenarios. Therefore, the formulation of geometric programming problems in the spherical fuzzy environment is necessary to achieve optimal answers in various optimization fields, such as the reliability of production systems.

2.1 The problem of geometric programming

The geometric programming problem is one of the nonlinear programming problems and a special case of them. The geometric programming problem's characteristic is related to how the decision variables are displayed or involved. The following sentences are provided to determine the geometric type of the objective functions. It is easy to show the problem of geometric programming with the help of these technical terms.

2.1.1 One-term geometric programming (Monomial)

Any algebraic expression that consists of only one term is defined as a monomial. The one-sentence expression used in geometric programming has the same meaning, with the difference that in algebraic calculations, the variable cannot have a negative or fractional power. However, in geometric programming, variable power can be any real number (including fractional and negative). If x_1, x_2, ..., x_n represent n real positive variables, equation 1 can be written to represent a monomial function with real value G of x.

$$G(x) = cx_1^{a_1}x_2^{a_2}\dots x_n^{a_n} \tag{1}$$

where c>0 and $a_n \in R$.

2.1.2 Polynomial geometric programming (polynomial)

Any algebraic expression consisting of several terms is called a multi-sentence or polynomial. Hence, the sum of one or more monomials, i.e., any real-valued function G of x, is written as a polynomial function in equation 2.

$$G(x) = \sum_{i=1}^{m} c_i x_1^{ai_1}x_2^{ai_2}\dots x_n^{a_{in}} \tag{2}$$

2.1.3 positive polynomial geometric programming (posynomial)

It is a real function of a posynomial's value G(x) if all coefficients c_i>zero. Therefore, the sum of one or more monomials with c_i>zero represents the posynomial (Equation 3).

$$G(x) = \sum_{i=1}^{m} c_i x_1^{ai_1}x_2^{ai_2}\dots x_n^{a_{in}} \tag{3}$$

The formulation of the geometric programming problem based on the previous definitions is presented in Equation 4.

$$\begin{aligned} &Minimize\ G_0(x) = \sum_{l=1}^{K_0} c_{0l}\prod_{j=1}^{J} x_j^{\lambda_{0lj}} \\ &Subject\ to: \\ &\quad g_r(x) = \sum_{l=1+K_{r-1}}^{K_r} c_{rl}\prod_{j=1}^{J} x_j^{\lambda_{rlj}} \le 1 \quad x_j > 0, \\ &j = 1,2,\dots,J \end{aligned} \tag{4}$$

Where crl>zero and λrlj are real numbers. The problem included in equation 4 is a constrained polynomial geometric programming problem. Each of the polynomial constraints includes a different number of conditions and is denoted by Kr for all r=0, 1, 2, ..., l.

2.2 The multi-objective geometric programming problem

Real-world optimization problems rarely have a single objective. In fact, it is very common to have multiple goals in everyday life. For example, the transportation problem seeks to simultaneously optimize conflicting objectives such as cost, time, profit, etc. Therefore, multi-objective optimization techniques are one of the main concerns for solving this group of problems. Searching for a general solution that satisfies different objective functions is very challenging. However, a compromise solution is somewhat acceptable. However, the development of various multi-objective optimization techniques is a foundation for future research in this field. The nature of multi-objective geometric planning problems is often trivial. It exists in various real-world problems such as inventory control, system reliability, etc., with multiple objectives such as quality, time, cost, etc. Many researchers in MOGPP have made significant research contributions. Das et al. (2000) used the geometric programming technique to solve the multi-option inventory model [18]. Mahapatra and Roy (2009) used the multi-objective geometric programming technique to solve a single-container and multi-container maintenance model under uncertainty [19]. Islam and Roy (2006) used fuzzy multi-objective geometric programming techniques in the transportation problem [20].

The multi-objective geometric programming problem is mathematically formulated in Equation 5.

$$\begin{aligned} &Minimize\ G_k(x) = \textstyle\sum_{i=1}^{T_k^0} c_{ki}^0 \prod_{r=1}^{n} x_x^{\lambda_{kir}^0} \\ &Subject\ to: \\ &g_k(x) = \textstyle\sum_{s=1}^{T_k} c_{ks} \prod_{r=1}^{n} x_r^{\lambda_{ksr}} \le 1 \qquad x > 0,\ \ k = 1,2,\dots,m \end{aligned} \tag{5}$$

Where $c_{ji}^0 > zero$, $c_{ks} > zero$, a_{jir}^0, and a_{jir} are real numbers. G_k (x) is the kth objective function. The function g_k (x) is a real-valued function, and x= (x_1, x_2, ..., x_r) represents a set of decision variables.

It should be noted that a spherical fuzzy geometric planning problem is discussed in the spherical fuzzy decision-making environment. In the spherical fuzzy set with three membership functions of correctness, indeterminacy, and incorrectness, the sum of squares of all three degrees of membership should be between [0,1]. Nevertheless, the spherical fuzzy geometric programming problem is a more complete and convenient optimization tool than other geometric programming optimization techniques due to the presence of an indefinite membership degree when dealing with multi-objective decision problems.

According to Bellman and Zadeh (1970), a fuzzy decision-making set (D) consists of objectives (Z) and constraints (C) in a fuzzy environment [8]. The decision-making set is often used in various decision-making processes, and it can be expressed as equation 6:

$$D = (Z \cap C) \tag{6}$$

Therefore, the spherical fuzzy decision set can be adapted based on the fuzzy set theory for the spherical fuzzy environment. With this account, the spherical fuzzy decision set (D_{sf}), including spherical fuzzy objectives (Z_o) and spherical fuzzy constraints (C_o), is presented in equation 7.

$$\begin{aligned} &D_{sf} = (\cap_{o=1}^{O} Z_o)(\cap_{j=1}^{J} C_j) = \\ &(x, T_D(x), I_D(x), F_D(x)) \end{aligned} \tag{7}$$

Where T_D (x) is the correct membership function, I_D (x) is the indeterminate membership function, and F_D (x) is the incorrect membership function in the spherical fuzzy decision set D_{sf} (Equations 8 to 10).

$$T_D(x) = \begin{cases} T_{D1}(x), T_{D2}(x), T_{D3}(x), \dots, T_D(x) \\ T_{C1}(x), T_{C2}(x), T_{C3}(x), \dots, T_{CJ}(x) \end{cases} \quad \forall x \in X \tag{8}$$

$$I_D(x) = \begin{cases} I_{D1}(x), I_{D2}(x), I_{D3}(x), \dots, I_{Do}(x) \\ I_{C1}(x), I_{C2}(x), I_{C3}(x), \dots, I_{CJ}(x) \end{cases} \quad \forall x \in X \tag{9}$$

$$F_D(x) = \begin{cases} F_{D1}(x), F_{D2}(x), F_{D3}(x), \dots, F_{Do}(x) \\ F_{C1}(x), F_{C2}(x), F_{C3}(x), \dots, F_{CJ}(x) \end{cases} \quad \forall x \in X \tag{10}$$

For the marginal evaluation of each objective, first, the upper and lower bound are calculated separately for each objective function. If X^1, X^2, ..., and X^o are the decision variables obtained from solving the problem, the resulting matrix is according to Table 1.

Table 1. Payoff matrix

	Z_1	Z_2	...	Z_o
X^1	$Z_1(X^1)$	$Z_2(X^1)$	...	$Z_o(X^1)$
X^2	$Z_1(X^2)$	$Z_2(X^2)$	...	$Z_o(X^2)$
...	...	...	...	...
X^o	$Z_1(X^o)$	$Z_2(X^o)$	...	$Z_o(X^o)$

Equations 11 and 12 obtain the upper bound and lower bound mathematical expressions if these values are substituted in each objective function.

$$U_o = max\{Z_o(X^o)\} \ \ \forall o = 1, 2, \dots, O \tag{11}$$

$$L_o = min\{Z_o(X^o)\} \ \ \forall o = 1, 2, \dots, O \tag{12}$$

Where U_o and L_o represent the upper and lower bound of the Oth objective, respectively, the upper and lower bound for membership functions of correctness, non-determination, and incorrectness of each target in the spherical fuzzy environment is obtained using Equation 13.

$$\begin{aligned} &U_o^T = U_o \qquad\qquad L_o^T = L_o \\ &U_o^I = L_o^T + p_o(U_o^T - L_o^T) \qquad L_o^I = L_o^T \\ &L_o^F = L_o^T + q_o(U_o^T - L_o^T) U_o^F = U_o^T \end{aligned} \tag{13}$$

Where p_o and q_o are real numbers and are predetermined by the decision maker so that p_o, q_o ∈ (0, 1).

First case) Suppose the objective function is of the maximization type. In that case, the membership functions of correctness, indeterminacy, and incorrectness for each objective in the spherical fuzzy environment are obtained by equations 14, 15, and 16, respectively.

$$T_D(x)=\begin{cases}1 & if\ \ Z_o(x)\le L_o^T\\ 1-\frac{Z_o(x)-L_o^T}{U_o^T-L_o^T}\ , & if\ \ L_o^T\le Z_o(x)\le U_o^T\\ 0 & if\ \ Z_o^T\ge U_o^T\end{cases}\qquad(14)$$

$$I_D(x)=\begin{cases}1 & if\ \ Z_o(x)\le L_o^I\\ 1-\frac{Z_o(x)-L_o^I}{U_o^I-L_o^I}\ , & if\ \ L_o^I\le Z_o(x)\le U_o^I\\ 0 & if\ \ Z_o^I\ge U_o^I\end{cases}\qquad(15)$$

$$F_D(x)=\begin{cases}1 & if\ \ Z_o(x)\ge U_o^F\\ 1-\frac{U_o^F-Z_o(x)}{U_o^F-L_o^F}\ , & if\ \ L_o^F\le Z_o(x)\le U_o^F\\ 0 & if\ \ Z_o^T\le L_o^T\end{cases}\qquad(16)$$

Second case) If the objective function is of the minimization type, the membership functions of correctness, non-determination, and incorrectness for each objective in the spherical fuzzy environment are obtained by equations 17, 18, and 19, respectively.

$$T_D(x)=\begin{cases}0, & if\ \ Z_o(x)\le L_o^T\\ 1-\frac{U_o^T-Z_o(x)}{U_o^T-L_o^T}\ , & if\ \ L_o^T\le Z_o(x)\le U_o^T\\ 1, & if\ \ Z_o^T\ge U_o^T\end{cases}\qquad(17)$$

$$I_D(x)=\begin{cases}0, & if\ \ Z_o(x)\le L_o^I\\ 1-\frac{U_o^I-Z_o(x)}{U_o^I-L_o^I}\ , & if\ \ L_o^I\le Z_o(x)\le U_o^I\\ 1, & if\ \ Z_o^I\ge U_o^I\end{cases}\qquad(18)$$

$$F_D(x)=\begin{cases}0, & if\ \ Z_o(x)\le U_o^F\\ 1-\frac{Z_o(x)-L_o^F}{U_o^F-L_o^F}\ , & if\ \ L_o^F\le Z_o(x)\le U_o^F\\ 1, & if\ \ Z_o^T\le L_o^T\end{cases}\qquad(19)$$

Where the preservation of $\boldsymbol{U_o^{(0)}}\neq\boldsymbol{L_o^{(0)}}$ is required for all purposes. If the relationship $\boldsymbol{U_o^{(0)}}=\boldsymbol{L_o^{(0)}}$ holds for each objective, then the membership value will be one. Intuitively, our motivation is to maximize the degree of correct membership and not determine and minimize the degree of incorrect membership for spherical fuzzy objectives and constraints. The general formula of the spherical fuzzy geometric programming model for multi-objective problems is equation 20.

$$\begin{aligned}&Max\ \min_{o=1,2,\dots,O} T_o\big(Z_o(x)\big)^2\\&Min\ \max_{o=1,2,\dots,O} I_o\big(Z_o(x)\big)^2\\&Min\ \max_{o=1,2,\dots,O} F_o\big(Z_o(x)\big)^2\\&Subject\ to:\\&g_i(x)\le b_i,\ \forall\, i=1,2,\dots,I_1,\\&g_i(x)\ge b_i,\ \forall\, i=I_1+1,I_1+2,\dots,I_2,\\&g_i(x)=b_i,\ \forall\, i=I_2+1,I_2+2,\dots,I\\&x=(x_1,x_2,\dots,x_j)\ \in X,\quad x\ge 0\\&T_o\big(Z_o(x)\big)^2\ge I_o\big(Z_o(x)\big)^2,T_o\big(Z_o(x)\big)^2\ge\\&F_o\big(Z_o(x)\big)^2\\&0\le T_o\big(Z_o(x)\big)^2+I_o\big(Z_o(x)\big)^2+F_o\big(Z_o(x)\big)^2\le 1\end{aligned}\qquad(20)$$

The above mathematical model (equation 11) can be formulated as equation 21 by using auxiliary variables.

$$\begin{aligned}&Max\ \ \alpha^2\\&Min\ \ \beta^2\\&Min\ \ \gamma^2\\&Subject\ to:\\&T_o\big(Z_o(x)\big)^2\ge\alpha^2, I_o\big(Z_o(x)\big)^2\le\beta^2, F_o\big(Z_o(x)\big)^2\le\\&\gamma^2\\&g_i(x)\le b_i,\ \forall\, i=1,2,\dots,I_1,\\&g_i(x)\le b_i,\ \forall\, i=I_1+1,I_1+2,\dots,I_2,\\&g_i(x)=b_i,\ \forall\, i=I_2+1,I_2+2,\dots,I\\&x=(x_1,x_2,\dots,x_j)\ \in X,\ \ x\ge 0\\&\alpha^2\ge\beta^2,\alpha^2\ge\gamma^2, 0\le\alpha^2+\beta^2+\gamma^2\le 1\\&T_o\big(Z_o(x)\big)^2\ge I_o\big(Z_o(x)\big)^2, T_o\big(Z_o(x)\big)^2\ge F_o\big(Z_o(x)\big)^2\end{aligned}\qquad(21)$$

By solving the optimization model presented above (Equation 21), the optimal solution for the spherical fuzzy geometric programming problem is obtained.

The step-by-step algorithm designed to solve the spherical fuzzy geometric programming problem (SFGPP) is displayed in Figure 1.

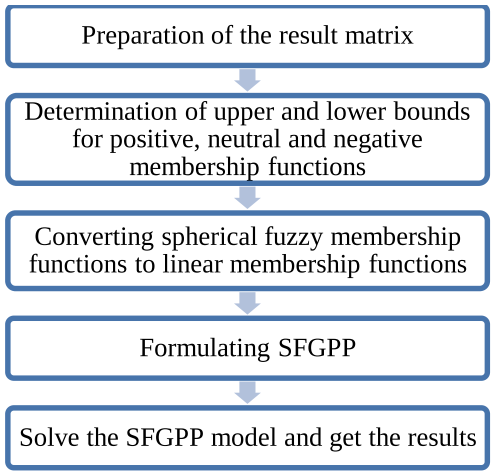

Figure 1. SFGP problem-solving process

3. A case study

The case study of this research is related to a company producing aluminum sheets and foil. This company operates in Hamadan province under the "Razan-Saaf Aluminum Company" brand name. Razan-Saaf Company was established in 2016. The company's products (aluminum foils) are produced in different alloys for industrial use and composite multi-layer wrappers for packing all kinds of food, medicine, cosmetics, and construction. As the only foiling company in the country's west, Razen-Saaf Group plays a key role in Iran's downstream aluminum industries. In 2022, Razan-Saaf had 28% of the market share (www.razan-saaf.com).

Aluminum foil is a type of flat-rolled aluminum. The thickness of the aluminum foil is from 5 to 150 micrometers (from 0.005 to 0.15 mm). Flat-rolled aluminum products more than 0.15 mm thick include aluminum tape, sheets, and plates. The production of aluminum foil can be done in four main stages: (1) foundry, (2) hot rolled, (3) cold rolled, and (4) cut and rolling, summarized.

The usual technology for producing flat aluminum rolled products (such as sheet, strip and foil) begins with casting molten aluminum into large trapezoidal molds (aluminum slabs) by special vertical casting machines. Aluminum ingots are cooled directly from cold casting to room temperature. The ingot is then transferred to the pressure furnace to be heated above the recrystallization temperature (about 500°C) and ready for the hot rolling stage. This process is called annealing, and it is done to distribute and distribute the compounds of different alloys evenly on the surface of the ingot so that the ingot has a homogeneous structure. After finishing the hot rolling stage, the coils enter the cold rolling stage to reduce the thickness. Cold aluminum strips can be rolled in all kinds of rolling factories.

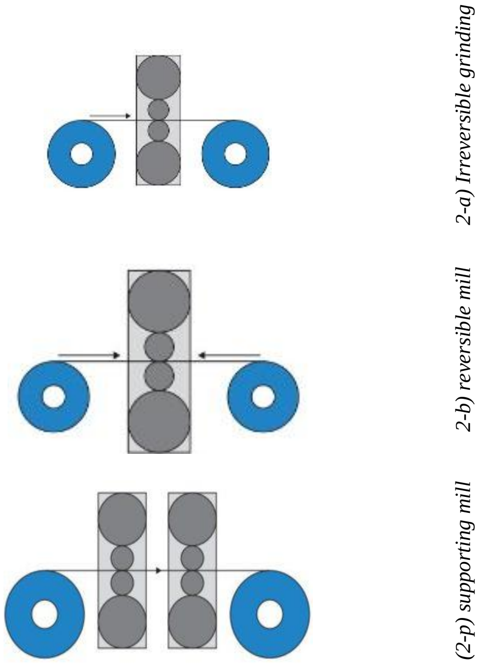

Figure 2. Types of mills for aluminum strip rolling

For rolls weighing 10 to 15 tons, one-way (non-reversible) rolling rollers are usually used (Figure 2-a). A single-cage reversible roller is used for small coils weighing up to 5 tons (Figure 2-b). For large rolls weighing more than 25 tons and for a large volume of production, several rollers are used in series (back to back) in the rolling mill (Figure 2-p).

Razan-Saaf Company is engaged in a continuous improvement program targeting its aluminum cold rolling performance to achieve maximum customer satisfaction and compete in the market. In this regard, it is implementing the backup mill model and has targeted an increase in market share by 35% for the 1404 horizon by increasing production reliability. Therefore, upgrading the cold rolling process to increase the production volume and reduce the production time is on the agenda of the company's managers. Creating a corporate competitive advantage can happen with the right support mill. Management was interested in this as part of their mission.

In the following, the proposed spherical fuzzy geometric programming model is used in solving the reliability optimization problem of the cold rolling system. First, the problem is written in the AMPL language and solved using Knitro 0.5.0 global optimization solvers "online facility provided by the University of Wisconsin" [21].

The aluminum cold rolling process in the case study with three support cages in series is considered a system reliability optimization problem. In fact, the aluminum cold rolling unit follows a reliability series system with three components. Suppose that R_i (i=1,2,3) represents the individual reliability of mill i of the series system. Similarly, R_s (R_1, R_2, R_3) and C_s (C_1, C_2, C_3) are the reliability and costs of all three mills in the series system. The company's management intends to maximize the reliability of the aluminum cold rolling system and minimize the total cost associated with all three components. Degree of membership (a_R), degree of uncertainty (d_R), and degree of non-membership (r_R) are conditions of system reliability. At the same time, the degree of membership (a_c), degree of indeterminacy (d_c), and degree of non-membership (r_c) are the constraints of the system cost, respectively. The satisfaction objective value for system reliability and cost is denoted by R_0 and C_0, respectively. Table 2 presents the data related to the problem.

Table 2. Input data

a_R	d_R	r_R	R_0
0.3	0.24	0.5	0.3
C_3	C_2	C_1	C_0
45	40	40	100
	a_c	r_c	d_c
	24	40	18

Therefore, the geometric programming problem can be formulated as equation 22.

$$\begin{aligned} &Maximize\ G_1 = \prod_{i=1}^{3} R_i = R_1 R_2 R_3 \\ &Minimize\ G_2 = \prod_{i=1}^{3} C_i R_i^{a_i} \\ &Subject\ to: \\ &0 < R_i \leq 1\ ; i = 1,2,3 \end{aligned} \quad (22)$$

The correct membership functions for reliability objective functions and spherical fuzzy cost are obtained as equations 23 and 24, respectively.

$$T_D(G_1(x)) = \begin{cases} 1, & if\ \ G_1(x) \leq 0.3 \\ 1 - \frac{G_1(x) - 0.3}{0.3}\ , & if\ 0.3 \leq G_1(x) \leq 0.3 + 0.3 \\ 0, & if\ \ G_1(x) \geq 0.3 + 0.3 \end{cases} \quad (23)$$

$$T_D(G_2(x)) = \begin{cases} 1, & if\ \ G_2(x) \leq 100 \\ 1 - \frac{G_2(x) - 100}{24}\ , & if\ 100 \leq G_2(x) \leq 100 + 24 \\ 0, & if\ \ G_2(x) \geq 100 + 24 \end{cases} \quad (24)$$

The uncertainty membership functions are obtained as Equations 25 and 26 for the objective functions of reliability and spherical fuzzy cost, respectively.

$$I_D(G_1(x)) = \begin{cases} 0, & if \ \ G_1(x) \le 0.24 \\ 1 - \frac{0.3-G_1(x)}{0.24}, & if \ \ 0.3 \le G_1(x) \le 0.3+0.24 \\ 1, & if \ \ G_1(x) \ge 0.3+0.24 \end{cases} \quad (25)$$

$$I_D(G_2(x)) = \begin{cases} 0, & if \ \ G_2(x) \le 100 \\ 1 - \frac{100-G_2(x)}{18}, & if \ \ 100 \le G_2(x) \le 100+18 \\ 1, & if \ \ G_2(x) \ge 100+18 \end{cases} \quad (26)$$

The incorrect membership functions for reliability objective functions and spherical fuzzy cost are obtained as equations 27 and 28, respectively.

$$F_D(G_1(x)) = \begin{cases} 1, & if \ \ G_1(x) \ge 0.3 \\ 1 - \frac{0.3-G_1(x)}{0.5}, & if \ \ 0.3 \le G_1(x) \le 0.3+0.5 \\ 0, & if \ \ G_1(x) \le 0.3+0.5 \end{cases} \quad (27)$$

$$F_D(G_2(x)) = \begin{cases} 1, & if \ \ G_2(x) \ge 100 \\ 1 - \frac{100-G_2(x)}{40}, & if \ \ 100 \le G_2(x) \le 100+40 \\ 0, & if \ \ G_2(x) \le 100+40 \end{cases} \quad (28)$$

Now the spherical fuzzy geometric optimization model for the research problem is formulated as equation 29.

$$\begin{aligned} & Max(\alpha^2 - \beta^2 - \gamma^2) \\ & Subject\ to: \\ & T_1(G_1(x))^2 \ge \alpha^2, I_1(G_1(x))^2 \le \beta^2, F_1(G_1(x))^2 \le \gamma^2 \\ & T_2(G_2(x))^2 \ge \alpha^2, I_2(G_2(x))^2 \le \beta^2, F_2(G_2(x))^2 \le \gamma^2 \\ & 0 < R_i \le 1; i = 1,2,3 \\ & \alpha^2 \ge \beta^2, \alpha^2 \ge \gamma^2, 0 \le \alpha^2 + \beta^2 + \gamma^2 \le 1 \end{aligned} \quad (29)$$

In addition to the spherical fuzzy geometric programming problem technique, the present problem was also solved with two other methods, including (1) the intuitive fuzzy geometric programming problem and (2) the neutrosophic fuzzy geometric programming problem. The problem-solving results using three techniques, SFGPP, IFGPP, and NFGPP, are summarized in Table 3. The results presented in Table 3 show that the proposed SFGPP approach's performance is better than the previous two approaches.

Table 3. Optimal solution based on IFGPP, NFGPP, and SFGPP methods

		Method		
		SFGPP	**NFGPP**	**IFGPP**
Objective functions	R_s	*0.618454*	*0.613664*	*0.523472*
	C_s	*81.254*	*80.372*	*78.766*
Decision variables	R_1	*0.835402*	*0.824809*	*0.771292*
	R_2	*0.862134*	*0.863345*	*0.875617*
	R_3	*0.858712*	*0.861773*	*0.775105*

The results of this study show that reliability and system costs have a direct relationship with each other. This means that improving the reliability of the system requires more money. The lowest cost is related to the IFGPP approach, with a value of 78.766, which has lower reliability (0.523) than other approaches. However, the highest cost (81.254) has been made for the SFGPP approach, which has reached a reliability of 0.618. To explain the obtained results further, it can be said that various types of incorrect human tasks in the process-based system can lead to the failure of production machinery, interruption of the production chain, and even catastrophic events. Therefore, for decision-makers in system safety and reliability analysis, it is vital to evaluate human system reliability concerning the interaction between humans, the environment, and machines; meanwhile, minimizing the related uncertainty is an important task. Therefore, the advanced SFGPP method can sufficiently identify the reliable index of a complex system (human-environment-machine) and help decision-makers to prioritize critical items. Figure 3 displays the results of the solution approaches for easier understanding.

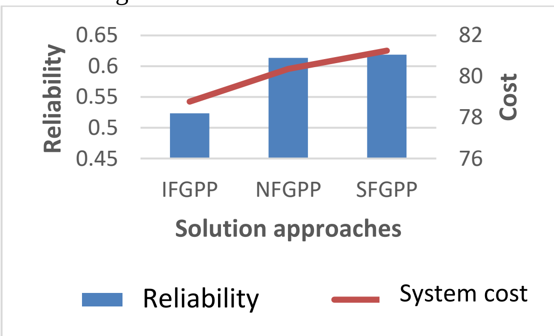

Figure 3. Comparison of system reliability and cost in solution approaches

4. Conclusion

There are different methods to solve mathematical optimization problems. However, for non-linear mathematical problems, the geometric programming method is generally more efficient compared to other mathematical programming methods, such as non-linear programming. The geometric programming problem's structure differs from other general mathematical programming problems, and they include polynomial expressions in their objective functions. In an uncertain environment, a different formulation of geometric planning in decision-making processes can be beneficial due to considering the degree of uncertainty for the set elements. Recently, the spherical fuzzy set has significantly contributed to decision-making problems by creating enough opportunities to obtain false and contradictory information. Therefore, this paper examines the spherical fuzzy geometric programming problem under a spherical fuzzy environment, which includes maximizing the positive (correct) membership function and minimizing the neutral (indeterminate) and negative (incorrect) membership functions in the spherical fuzzy decision set. The sum of squares less than or equal to one is a constraint that applies to all membership functions. The cold rolling process in producing aluminum sheets and foil was studied to demonstrate the application of the spherical fuzzy geometric programming problem. In optimizing the reliability of the aluminum cold rolling system, the performance of the SFGPP technique was better than IFGPP and NFGPP techniques. This result is related to the greater adaptation of the proposed approach to the actual decision-making conditions.

Many real-world problems, such as product pricing, inventory control, system reliability, etc., are put into the geometric programming model. In addition, the proposed algorithm based on the spherical fuzzy set provides special flexibility while solving the geometric programming problem with polynomial objectives. In addition, the designed optimization framework can help decision-makers handle the degree of uncertainty while solving various real-life problems. Therefore, many engineering and management problems can be formulated as spherical fuzzy geometric programming problems.

5. References

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