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Original Research Article

Remaining Useful Life Prediction for a Multi-Component System with Degradation Interactions

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Abstract

Remaining useful life (RUL) prediction is crucial in prognostics and health management (PHM) systems. The primary objective is to forecast the time to failure (TTF) or anticipate the RUL of a system. In real industrial cases, systems typically consist of multiple components that can affect each other, and ignoring these dependencies when modeling PHM systems can lead to erroneous RUL predictions and ineffective maintenance planning. Recognizing this, the focus of this paper is on the prognostics of multi-component systems, where the degradation processes of the system are influenced by both internal factors specific to the components and external factors related to the environment.

Keywords: Multi-component system; Prognostic; Degradation modeling; Remaining useful life prediction.

1. Introduction

In many real industrial processes, systems often comprise components exhibiting failure dependencies. Failing to account for these component dependencies can result in inefficient remaining useful life (RUL) prediction. This paper addresses this issue by focusing on multi-component systems and presenting an algorithm for RUL prediction.

The presented algorithm provides a valuable tool for practitioners and researchers working with multi-component systems. It allows them to understand the system's behavior better and make informed decisions regarding maintenance and replacement strategies. Additionally, the conclusion and further work section highlight the significance of this research and suggest potential avenues for future exploration and improvement in the field of RUL prediction for multi-component systems.

The paper is organized as follows: Section 2 provides a comprehensive literature review on multi-component systems, encompassing three aspects: past review papers, an overview of multi-component models, and a review of mathematical models presented in the reviewed papers. In Section 3, a typical multi-component system is considered and subsequently modeled to capture the interdependencies among the components. The proposed model is solved in Section 4, where the algorithm for RUL prediction is described and implemented. Finally, Section 5 presents the study's conclusion and potential areas for further research.

2. Multi-component systems review

2.1 Past review paper

In the field of maintenance in multi-component systems, three notable review articles have been published to date, providing valuable insights into the topic.

The first review article, written by [1], focuses on multi-component maintenance models with economic dependency. The models are categorized into stationary and dynamic models. Stationary models assume a long-term stable situation with an infinite planning horizon, further divided into three subcategories: corrective maintenance, preventive maintenance, and opportunistic maintenance. Dynamic models, on the other hand, deal with short-term horizons and are divided into finite horizon and rolling horizon subcategories. Dekker et al.'s review classifies the reviewed papers into these four categories, providing a comprehensive overview of different problem classes and associated models in multi-component systems.

The second review paper, authored by [2], builds upon the subcategories proposed in the first review. However, it introduces a new understanding of the dependence and interaction that may exist between components in a system. The review identifies three types

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of dependencies: economic, structural, and stochastic. Accordingly, the reviewed papers are analyzed and discussed based on these types of dependencies, providing insights into various aspects of multi-component system maintenance.

The third review paper, conducted by [3], addresses determining maintenance plans for multi-component systems. This review paper can be considered the first systematic review focusing on multi-component system maintenance, considering system characteristics, maintenance characteristics, and mission profile characteristics. It offers a comprehensive literature analysis, guiding maintenance plan selection in the context of multi-component systems.

Together, these three review articles contribute to understanding maintenance in multi-component systems by categorizing and summarizing the existing literature. They offer insights into different aspects, including economic dependencies, component interactions, and maintenance plan selection, fostering a systematic approach to tackle the challenges of maintaining complex multi-component systems.

2.2 Reviewing multi-component models

Predicting the Remaining Useful Life (RUL) for multi-component systems requires an accurate estimation of degradation states, considering economic dependency, structural dependency, and stochastic dependency. Numerous research papers have been published in this field, which can be categorized based on the number of dependent factors considered:

a) One-factor systems:

- [4] employ stochastic dependency in modeling multi-component systems, using two Brownian motions for degradation. They recursively estimate degradation states and parameters using the Kalman filter and EM algorithm and predict RUL distribution using FHT with a known threshold.
- [5] consider a stochastic dependence in modeling a multi-component system. The Bayesian framework is used to update the predicted RULs.
- [6] investigate stochastic dependency in a multi-component system with two major units (A and B) subjected to two types of shocks. Type II shocks cause complete failures, while type I shocks cause minor failures. The probability of shock type is age-dependent.
- [7] study stochastic dependency in a two-component system with failure interactions, considering shock models and three maintenance policies for each model.
- [8] consider structural dependency and analyze the impact of disassembly operations on component degradation and reliability. They propose a connection matrix model considering components' properties, connection strength, and technician expertise. The proposed model is applied in a numerical example for reliability assessment and maintenance optimization.
- [9] consider stochastic dependence in the modeling of a multi-component system. They propose an integrated framework that combines real-time degradation models, mixed-integer optimization models, and solution algorithms for optimal wind farm maintenance and operations. The proposed framework, MC-CBOM, outperforms conventional methods regarding multiple wind farms with 300 turbines and 1200 components.
- [10] study stochastic dependency in modeling multi-component systems, focusing on its effects on the degradation process and remaining useful life. They estimate the dependent degradation state and unknown parameters using Kalman filtering and the expectation-maximization algorithm.
- [11] develop a dynamic Bayesian network-based maintenance decision framework for complex systems with stochastic dependence. They compare preventive and predictive maintenance strategies in six different thermal power plant scenarios, finding that the threshold-based maintenance strategy provides the lowest cost and maintenance number.
- [12] investigate condition-based maintenance of a two-component system considering stochastic dependence under imperfect inspection. They propose a dual periodical inspection policy, where Component 1 is repairable with two observable states (working and failed), and Component 2 degrades over time modeled by a Wiener process.
- [13] propose a probabilistic deep learning methodology for uncertainty quantification of multi-component systems' RUL. They combine a probabilistic model and a deep recurrent neural network to predict the components' RUL distributions. The proposed methodology is evaluated using benchmark data provided by NASA.
- [14] studies a multi-component system with hierarchical stochastic dependencies. They use the Nested Clayton Lévy copula to model a time-independent hierarchical dependence structure. A condition-based maintenance policy is proposed, dynamically planning inspection schemes and maintenance decisions based on collected information.

b) Two-factor systems:

- [15] develop a dynamic opportunistic condition based maintenance strategy for multi-component systems, considering stochastic and economic dependencies. They establish an optimization model to minimize long-term average maintenance costs by selecting the dynamic opportunistic maintenance zone and optimal group structure.
- [16] propose a flexible condition-based maintenance model using deep reinforcement learning for multi-component systems with dependent competing risks. The model maps degradation measurements to the maintenance decision space and leverages deep reinforcement learning for computational efficiency.
- [17] study the reliability of multi-component systems with stochastic and structural dependence, considering continuous degradation processes and categorized shocks. They derive the reliability of a series system and propose a simulation method to approximate the failure time of k-out-of-n systems.
- [18] propose a stochastic optimization model to reduce long-term total maintenance costs in complex systems with stochastic and economic dependence. They optimize preventive and corrective maintenance policies, focusing on periodic block-type and age-based maintenance policies.
- [19] propose a state-rate dependence approach for analyzing degradation and failure in a two-component repairable system, considering stochastic and economic dependencies.
- [20] propose a joint predictive maintenance and inventory strategy for complex systems with multiple non-identical components and stochastic and economic dependence. They optimize predictive maintenance and spare parts provisioning operations based on prognostic condition index and structural importance measures.
- [21] develop a condition-based maintenance policy for a two-component system with stochastic and economic dependencies. They propose adaptive preventive and opportunistic maintenance rules and find optimal decision variables using a cost model.
- [22] develop an analytic network process and cost-risk criticality analysis model for selecting a cost-effective, low-risk maintenance strategy for complex systems with stochastic and economic dependence. They consider four maintenance alternatives: failure-based, time-based, risk-based, and condition-based.
- [23] analyze a condition-based maintenance model for a multi-component system with stochastic and economic dependence. They use a Markov decision process and dynamic

C) Three-factor systems:

- [24] developed a bi-level approach to optimize condition-based maintenance (CBM) policy for multi-component systems with stochastic, economic, and structural dependencies. Based on system decisions, they identified the optimal group of components to be preventively maintained. The optimal inter-inspection interval, system thresholds, and component conditional reliability were jointly determined to minimize long-run average cost rates. The optimal maintenance policy was derived using a Monte Carlo simulation technique and a Particle Swarm Optimization (PSO)-based heuristic algorithm. The multi-component system considered in the paper has a k-out of- n: G structure.
- [25] propose a condition-based maintenance policy for continuously monitored multi-component systems with stochastic, economic, and structural dependencies. The policy incorporates a utility/reward function that minimizes system costs by choosing actions that minimize the total long-term penalty. The proposed policy outperforms alternative policies by reducing system life-cycle costs. Maintenance clustering is particularly beneficial for systems with strong economic dependence, while immediate replacements upon component failure are more advantageous for systems with high stochastic dependence.
- [26] studied maintenance optimization of a parallel-series system considering stochastic, economic, and structural dependencies. They jointly optimized maintenance strategies and modeled the system's degradation process to address stochastic dependence and limited capacity issues. The authors employed the factored Markov decision process (FMDP) and developed an improved approximate linear programming (ALP) algorithm. The study found that the current approach effectively handles decision optimization problems for moderate-sized multi-component systems with minimal maintenance decision-making errors.

The details of the reviewed papers are summarized in Table 1.

Table 1. Reviewing multi component models

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Zhang et al (2016) [4]	-	Stochastic	The RUL of every component can be derived using the FHT (first hitting time).	The degradations are affected by a common factor which is assumed to be public noise	Presents a methodology to predict the RUL of a class of multi-component systems with hidden degradation processes	The degradation states and model are unknown parameters are first identified recursively by the Kalman filter and EM algorithm. the RUL distribution of every component can be predicted by inferring the first hitting time (FHT)	*	PHM
Bian et al (2014) [5]	-	Stochastic	Brownian motion, Gamma process	Degradation-rate	Predict the residual lifetime of a multi-component system	A Bayesian framework is used to update the predicted RLDs, DRI modeling framework, and Base-Case DRI Model, and MLE is used to estimate the parameter	-	-
Shey et al. (2015) [6]	-	Stochastic	Non-homogeneous Poisson process	Shock Damage Interaction	cost minimization	Cumulative damage model	-	Replacement policy, shock model
Zhang et al. (2018) [7]	-	Stochastic	Weibull lifetime distribution, Exponential distribution used to describe the damages	Failure	Long-run average cost optimization	The failure processes of all components are modeled by either their failure rates or their degradation processes, imperfect maintenance (virtual age method)	-	Corrective maintenance, Preventive maintenance, Maintenance cost derivation (T policy, N policy, (N, T) policy)

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Dinh et al (2020) [8]	Series	Structural	The half-normal distribution is herein used to model the adjustment factor, B(t) is standard Brownian motion in the degradation model	-	This paper aims to investigate the impact of disassembly operations on the degradation processes and reliability of the components/system	Dependence modeling and formulation of disassembly impact, Degradation modeling with disassembly operations impact, Reliability assessment considering the disassembly operations impacts	-	Step 1: System structure analysis (component analysis, dependencies analysis…)- Step 2: Dependence modeling and formulation of disassembly impact (Directed graph, connection matrix, disassembly impact model)- Step 3: Degradation modeling with disassembly impact (degradation models, impacts model)- Step 4: Reliability assessment considering the disassembly operations impacts (maintenance policy, reliability model)
Bakir et al (2021) [9]	-	Stochastic	Bayesian approach	-	The proposed framework provides significant cost and reliability improvements. The proposed model adapts to a wide range of operational and maintenance scenarios.	The MC-CBOM policy outperforms all benchmark policies thanks to its ability to consider and adapt to the complex interactions between different decision layers.	-	CBM, Large-scale mixed integer optimization

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Niu et al (2022) [10]	Series structure, Parallel structure, Summation structure	Stochastic	B(t) indicates standard Brownian motion and observation noise and follows a normal distribution.	Degradation,	This study aimed to investigate the effects of the stochastic dependence between components on the degradation process and remaining useful life (RUL) of a system.	The PDF of the RUL was derived for a multi-component system using the FHT concept; the dependent degradation state and unknown parameters of the model were jointly estimated by Kalman filtering (KF), and the expectation maximization (EM) algorithm. The maximum likelihood estimation (MLE)	*	State-space model
Özgür-Ünlüakın et al (2021) [11]	The RAH system consists of two parallel motors groups	Stochastic	-	-	To reduce maintenance costs while increasing system reliability at the same time	Dynamic Bayesian networks, Tabu procedure, Generic algorithm for the proactive maintenance strategies	-	Multi-component hidden systems
Zhang et al. (2022) [12]	-	Stochastic	A Wiener process describes the degradation of component 2 with parameters depending on the state of component 1, the lifetime of component 1 follows a Weibull distribution	Component 2 fails if the degradation level of component 1 exceeds a predefined threshold; the whole system is renewed if the degradation of component 2 exceeds its preventive maintenance threshold	The expression of the maintenance cost rate, in the long run, is given to evaluate the maintenance policy	An algorithm is proposed to achieve maintenance optimization (The calculation of the maintenance cost in the long-run horizon); by taking the maintenance cost rate in the long run as the objective function, the Markov renewal process is implemented to solve the problem.	-	Condition-based maintenance of a two-component system under imperfect inspection,

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Nguyen et al (2022) [13]	Series, parallel, combined, bridge-type	Multi-independent-component systems, Structural	Lognormal distribution and a recurrent neural network	-	To develop an efficient approach able to provide the pdf of the RUL and outperform existing methods for both point-wise and probabilistic RUL predictions with a reasonable computational cost	A combination between a probabilistic model, i.e., lognormal distribution, and a recurrent neural network, i.e., LSTM model	-	PHM
Li et al. (2022) [14]	Each subsystem is parallel.	Hierarchical stochastic dependencies	Individual degradation is modeled by the Gamma process, the Inverse Gauss (IG) process with Gaussian the copula is used when modeling the deterioration process of heavy machine, Lévy copula permits the marginal to be Gamma process and the Wiener process	-	Reducing the inspection and maintenance cost, the optimal maintenance cost of policies	The Nested Lévy copula, Copula methods in dependent degradation modeling	-	Condition-based maintenance policy, Nested Lévy copula
Shi et al (2016) [15]	-	Stochastic and economic	The system's initial RUL is modeled as a Weibull distribution	The components are divided into three categories (A, B, and C) based on their interdependence characteristics	Minimization of the long-term average maintenance cost of the system	An approximate methodology for RUL prediction using a stochastic filter	-	Dynamic maintenance strategy, OM policy
Zhang et al. (2020) [16]	-	Stochastic and economic	Poisson Process (CPP) and Gamma Process (GP)	Agent-environment	Cost minimization objective	A new CBM model for a K-component system subject to dependent competing risks, which are general and different from existing models	*	CBM, a novel and flexible model based on a customized deep reinforcement learning for multi-component systems with dependent competing risks

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Shen et al (2018) [17]	Series, k-out of- n	Stochastic, Structural	The Gamma process governs degradation between two adjacent shocks,	Degradation (the degradation behavior of a particular component can influence that of another component), Moreover, categorized shocks are assumed to selectively affect one or more components by either causing a sudden jump in the degradation level or accelerating the degradation rate, or both.	The key contribution of this work is studying the reliability of a multi-component system with interacting components subject to continuous degradation processes and categorized shocks	Markov renewal process	-	A Markov renewal process
Martinod et al (2018) [18]	-	Stochastic and economic	The stochastic hazard rate of each component has a uniform distribution	State-rate	To reduce the long-term total maintenance cost of complex systems	Periodic block-type policy, Age-based policy, an ABAO corrective maintenance policy	-	Preventive and corrective maintenance policy
Xu et al. (2018) [19]	Component failure makes the system stop	Stochastic and economic	A proportional hazard model (PHM) with baseline Weibull hazard function and time-dependent stochastic covariates is used to describe the equipment deterioration process	State-rate	Effective degradation analysis and accurate condition assessment, cost minimization	A state discretization technique to model how the health state of one component affects the hazard rate of another, an extended proportional hazard model (PHM) to characterize the failure dependence and estimate the influence of the degradation state of one component on the hazard rate of another, an optimization model is developed to determine the optimal hazard-based threshold for a two-component repairable system	-	PHM (proportional hazard model), CBM

Author	**System structure**	**Dependencies**	**The distribution used in modeling**	**Interaction**	**Aim**	**Solution method**	**Environmental influence**	**Description**
Nguyen et al (2017) [20]	Combined	Economic, stochastic	Gamma distribution, Weibull distribution	Cost-rate	To reduce the total maintenance and inventory cost	present a joint predictive maintenance and inventory strategy for systems with complex structures and multiple non-identical components, opportunistic maintenance decision rules based on the criticality level of components and their spare parts availability are proposed an adaptive maintenance opportunity rule, Monte Carlo simulation techniques	-	Predictive maintenance, predictive inventory, prognostic, opportunistic maintenance, Joint predictive maintenance, and inventory strategy
Do et al. (2018) [21]	-	Economic, stochastic	Gamma distribution, Brownian motion process	Economic and degradation	Cost optimization	A particle filter is implemented to estimate the parameters of the proposed model, State dependence modeling, and the cost rate is evaluated using Monte Carlo simulation	-	Condition-based maintenance, an opportunistic maintenance policy, cost of preventive and corrective replacement, preventive and opportunistic maintenance
Shafiee et al. (2019) [22]	Combined	Stochastic and economic		Interactions between component failures	The model is proposed to select a cost-effective, low-risk maintenance strategy for different sets of components in a complex system	AHP, ANP, fuzzy logic	-	MCDM: CBM- TBM-RBM-FBM, the proposed model consists of two sets of criteria, namely, cost of maintenance and criticality of failure

Author	System structure	Dependencies	The distribution used in modeling	Interaction	Aim	Solution method	Environmental influence	Description
Liu et al (2021) [23]		Stochastic and economic	Degradation follows a bivariate gamma process	Degradation (correlation between the degradation processes)	The expected maintenance cost is minimized concerning the preventive replacement thresholds for the two components	Markov decision process (MDP), dynamic programming is used to compute the expected maintenance cost over a finite planning horizon	-	Preventive or corrective replacement, CBM
Wang et al (2022) [24]	k-out of- n	Stochastic and Economic, Structural	Wiener Processes, Brownian motion, inverse Gaussian	Rate–rate	Minimization of the long-run average cost rate	The Monte Carlo simulation technique combined with an improved Particle Swarm Optimization (PSO)-based heuristic algorithm	-	CBM policy, Perfect observations without including measurement error
Oakley et al. (2022) [25]	The series-parallel system combined	Stochastic and Economic, Structural	The workload at every time unit follows the truncated normal distribution; Gamma processes will be used to model the degradation of the components	Degradation-rate	Minimizes the overall cost of the system	To model component failure times using a random degradation threshold, maintenance clustering is especially beneficial for systems with a strong degree of economic dependence.	-	CBN policy
Zhou et al. (2016) [26]	Parallel-series	Stochastic and economic structure	The duration of both corrective and preventive maintenance follows the geometric distribution	-	To overcome the “curse of dimensionality” problem, the cost minimization	The factored Markov decision process (FMDP), An improved approximate linear programming (ALP) algorithm		The cost of preventive and corrective maintenance, Maintenance capacity

2.3 Mathematical models

The reviewed articles in the previous section have the mathematical models collected in Table 2.

Table 2. Mathematical model of reviewing multi-component models

Author	Mathematical Model
Zhang et al. (2016) [4]	A stochastic process can describe every degradation $$X^{(i)}(t)=\eta_0^{(i)}+\eta^{(i)}t+\xi^{(i)}B(t)+\sigma^{(i)}B^{(i)}(t), i=1,2,\ldots,n \text{ .}$$
Bian et al (2014) [5]	The degradation signal of component i $$S_i(t)=S_i(0)+\int_0^t r_i[v;\kappa_i,\boldsymbol{h}(\boldsymbol{S}(v))]\mathrm{d}v+\epsilon_i(t),$$ $$\boldsymbol{r}(t)=\begin{cases}\boldsymbol{\mu}_1 & t_0\le t<p_1\\ \boldsymbol{\mu}_2 & p_1\le t<p_2\\ \ldots & \ldots\\ \boldsymbol{\mu}_L & p_{L-1}\le t<p_L\\ \boldsymbol{\mu}_{L+1} & p_L\le t\le t_q.\end{cases} \quad \hat{h}_i(s_i(t))=\begin{cases}0 & s_i(t)<\hat{g}_{i,1}\\ 1 & \hat{g}_{i,1}\le s_i(t)<\hat{g}_{i,2}\\ \ldots & \ldots\\ \widehat{M}_i & s_i(t)\ge\hat{g}_{i,\widehat{M}_i-1}\end{cases}$$
Shey et al. (2015) [6]	The probability that type I shocks occur exactly k times during [0, t] $$P_k(t)=P\{M(t)=k\}=\frac{\left(\int_0^t q(x)r(x)\mathrm{d}x\right)^k\exp\left\{-\int_0^t q(x)r(x)\mathrm{d}x\right\}}{k!}$$ A corrective replacement is carried out at the first type II shock. The probability of this event is given by $$P_{II}=\sum_{j=0}^{N-1}P(Z_j<K)\int_0^T P\{M(t)=j\}\,\mathrm{d}F_p(t)=\sum_{j=0}^{N-1}G^{(j)}(K)\int_0^T\left[H^{(j)}(t)-H^{(j+1)}(t)\right]\mathrm{d}F_p(t)$$
Zhang et al. (2018) [7]	The average long-run cost $$C_{\infty I}(T)=\frac{c_3-(c_3-c_2)\sum_{n=0}^{\infty}r^np_n(T)\bar{G}_{\sigma_L}(T)+\sum_{n=1}^{\infty}(n-1)c_1r^{n-1}\bar{r}\int_0^T\bar{G}_{\sigma_L}(t)dV_n(at)}{\int_0^T\bar{F}_{sI}(t)dt}+\frac{\sum_{n=0}^{\infty}nr^nc_1(\int_0^T p_n(t)dG_{\sigma_L}(t)+p_n(T)\bar{G}_{\sigma_L}(T))}{\int_0^T\bar{F}_{sI}(t)dt}$$

Author	Mathematical Model
Dinh et al. (2020) [8]	The degradation process of component i $$X_{Hi}(t) = X_i(t) + \sum_{k=0}^{N(t)} H_{iG^k}$$ $$H_{iG^k} = \sum_{j=1}^{n} \delta_{ij} \cdot I_{ij}^{G^k}$$
Bakir et al. (2021) [9]	Degradation model for component k in turbine i $$D_{i,k}(t) = \varphi_{i,k}(t;\kappa,\theta_{i,k}) + \varepsilon_{i,k}(t;\sigma)$$ The underlying base degradation function for component k of turbine i $$\varphi_{i,k}(t;\kappa,\theta_{i,k})$$
Niu et al. (2022) [10]	The intrinsic degradation process of component i $$X^{(ii)}(t) = X^{(ii)}(0) + \int_0^t \mu_i(\gamma,\theta_i)d\gamma + \sigma_B^{(i)}B(t),$$ The drift term $$\mu_i(t,\theta_i) = \lambda_i r t^{r-1}$$
Özgür-Ünlüakın et al. (2021) [11]	The joint probabilities of the variables in a Dynamic Bayesian Network $$P(X_{1:T}) = \prod_{t=1}^{T}\prod_{i=1}^{N} P(X_t^i \mid Pa(X_t^i)).$$
Zhang et al. (2022) [12]	The degradation process of component 2 given that component 1 failure occurs once at θ $$Y_{\theta,t} \sim \mathcal{N}\left(\mu_1\theta + \mu_2(t-\theta), \sigma_1^2\theta + \sigma_2^2(t-\theta)\right)$$
Nguyen et al. (2022) [13]	The RUL of component *i* $$\prod_{i=1}^{n_s}\prod_{t=1}^{n_t} L(\mu_i^t, \sigma_i^t \mid RUL_i^{*^{(0:t)}}) = \prod_{i=1}^{n_s}\prod_{t=1}^{n_t}\prod_{j=0}^{t} \frac{1}{RUL_i^{*^j} \cdot \sigma_i^t\sqrt{2\pi}} \times \exp\left(-\frac{(\ln RUL_i^{*^j} - \mu_i^t)^2}{2(\sigma_i^t)^2}\right)$$ $$RUL_i^{*^{(0:t)}} = [RUL_i^{*^0}, RUL_i^{*^1}, \ldots, RUL_i^{*^t}]$$
Li et al. (2022) [14]	The deterioration stochastic process of component j in subsystem i can be modeled as follows. $$X_t^{ij} = \sum_{n=1}^{\infty} U_{ij}^{-1}(\Gamma_n^{ij}/T)\mathbb{1}_{[0,t]}(v_n),$$ $(\Gamma_n^{ij})_{n\in\mathbb{N}}$are coupled by a copula function C in such a way that $$U(U_{11}^{-1}(\Gamma_n^{11}), U_{12}^{-1}(\Gamma_n^{12}), \ldots, U_{MN_M}^{-1}(\Gamma_n^{MN_M})) = C(\Gamma_n^{11}, \Gamma_n^{12}, \ldots, \Gamma_n^{MN_M})$$

Author	Mathematical Model
Shi et al. (2016) [15]	The CM history of component j in D_k until t_i $$S_i^{(k)} = \left\{ z_i^{(k)} \leqslant Z_i^{(k)} \leqslant z_i^{(k)} + \Delta z_i^{(k)}, S_{i-1}^{(k)} \right\} = \left\{ z_1^{(k)} \leqslant Z_1^{(k)} \leqslant z_1^{(k)} + \Delta z_1^{(k)}, z_2^{(k)} \leqslant Z_2^{(k)} \leqslant z_2^{(k)} + \Delta z_2^{(k)}, \ldots, z_i^{(k)} \leqslant Z_i^{(k)} \leqslant z_i^{(k)} + \Delta z_i^{(k)} \right\}$$ The RUL of component k $$f_i^{(k)}\left(t \middle\| s_i^{(k)}, s_i^{(j)}\right) = P\left(t \leqslant T_i^{(k)} \leqslant t + \Delta t \middle\| T_i^{(k)} > 0, S_i^{(k)}, S_i^{(j)}\right) \Big/ \Delta t, \quad \Delta t \to 0$$ The system's initial RUL is modeled as a Weibull distribution. $$f_0(t) = \alpha\beta(\alpha t)^{\beta-1} \exp(-(\alpha t)^{\beta})$$
Zhang et al. (2020) [16]	Based on the CPP (Compound Poisson Process) model, the degradation of component i $$d_i(t) = \sum_{t_{i,j} \leq t} m_{i,j} + \sum_{t_{s,j} \leq t} m_{s,j}$$ The PDF of the degradation increment from time t_1 to t_2 based on GP (Gamma Process) $$f_{(t_2,t_1)}(u) = \begin{cases} \frac{(t_2,t_1) u (t_2,t_1) e^{u}}{(t_2,t_1)} & if \quad u \geq 0 \\ 0 & , if \quad u < 0 \end{cases}$$
Shen et al (2018) [17]	The accumulated degradation of component i just after the jth shock arrives. $$W_j^{(i)} = W_{j-1}^{(i)} + Z(T_j - T_{j-1}) + D_{im}\mathbf{1}\{i \in U_m\},$$
Martinod et al (2018) [18]	The weighted average cost for the jth component in the periodic block-type actions is expressed as $$\Gamma p_{j,\eta} = Cp_j T_j \left(1 - \frac{\alpha_{p1} + p\ \alpha_{p2}}{1 + p}\right)$$ The weighted average cost for the jth component in the age-based policy actions are the following. $$\Gamma p_{j,\eta} = \frac{Cp_j}{A_j}\left(1 - \frac{\alpha_{p1} + p\ \alpha_{p2}}{p + 1}\right)$$
Xu et al. (2018) [19]	As to identify the optimal number of change points V*, the contrast function, which measures the quality of the number of segments, is given below: $$U(V) = \sum_{l=1}^{N} \left[\frac{1}{n^{(l)}} \sum_{v=1}^{V} n^{(l)}{}_v \log\|\hat{\Sigma}_{\tau^{(l)}{}_v}\| + V\left[\left(V + 1\right) c^{(l)}{}_1 + Vc^{(l)}{}_2 + \ldots + c^{(l)}{}_{v+1}\right]\right]$$ The total cost C cycle in association with repair and replacement for each regenerative cycle is: $$C_{cycle} = N_1 C_{m1} + N_2 C_{m2} + C_p = C_m + C_p$$ The average cost per unit of time C_{AC} is expressed as follows $$C_{AC} = \frac{C_{cycle}}{E[T_r]} = \frac{N_1(D)C_{m1} + N_2(D)C_{m2} + C_p}{W(D)} = \frac{C_m(D) + C_p}{W(D)}$$

Author	Mathematical Model
Nguyen et al (2017) [20]	The total cost rate can be rewritten as $$C_{\infty}^{T}(T,\varpi_p,\varpi_o)=\frac{E[\sum_{k=1}^{N}C_k^T]}{E[T_{rep}-D(T_{rep})]},$$ $$C_k^T=\underbrace{C_{T_k}^{insp}+C_{(T_{k-1},T_k)}^{remp}+C_{T_k}^{remp}}_{C_k^M}+\underbrace{C_{(T_{k-1},T_k]}^{order}+C_{(T_{k-1},T_k)}^{hold}}_{C_k^I},$$
Do et al. (2018) [21]	The cost rate is generally defined as $$C^{\infty}(\Delta T,m_p^1,m_o^2,m_p^2,m_o^2)=\frac{\mathbb{E}[C^{Tre}(\Delta T,m_p^1,m_o^2,m_p^2,m_o^2)]}{\mathbb{E}[T_{re}]},$$ $$C^{Tre}(\Delta T,m_p^1,m_o^2,m_p^2,m_o^2)=\frac{\sum_{k=1}^{m}(C_{ins}^k+C_{main}^k)+T_{down}.c_d}{m.\Delta T}$$
Shafiee et al. (2019) [22]	The desirability index for alternative i, D_i, is defined as the following equation $$D_i=\sum_{j=1}^{J}\sum_{k=1}^{K_j}C_jM_{kj}A_{ikj}$$ Where J is the index set for criterion j, K_j is the index set of sub-criteria for criterion j, C_j represents the relative importance of criterion j, M_{kj} is the relative importance of sub-criterion k of criterion j and $A_{ikj}$is the rating of alternative i on sub-criterion k of criterion j.
Liu et al. (2021) [23]	In a univariate gamma process, degradation at time t, Y (t), follows a Bivariate gamma distribution. $$f_{\alpha t,\beta}(y)=\frac{\beta^{\alpha t}}{\Gamma(\alpha t)}e^{-\beta y}y^{\alpha t-1}.$$ $$Y(t)\sim Ga(t;\alpha,\beta)$$
Wang et al. (2022) [24]	Degradation level of the unmaintained component *j* modeled by a basic Wiener process $$X_j(t)=\mu_jt+\sigma_jB(t)$$ The RUL of the *j*th component is governed by an IG distribution with the PDF as $$f_{r_j(u\mid\tilde{x}_j)}(v)=\frac{L_j-\tilde{x}_j}{\sqrt{2\pi\tilde{\sigma}_j^2v^3}}\exp\left(-\frac{(L_j-\tilde{x}_j-\mu_jv)^2}{2\tilde{\sigma}_j^2v}\right).$$
Oakley et al. (2022) [25]	The degradation of Component ij at t_k $$D_{ij}(t_k)=\int_{t_{ij}^*}^{t_k}\Delta D_{ij}(t;\theta_{D_{ij}},x_i(t),\Pi_i(t)=\pi_i(t))dt$$ $t_{ij}^*$is the time of the most recent replacement of Component ij

Author	Mathematical Model
Zhou et al. (2016) [26]	The conditional transition probability of the system is simplified as: $$\Pr(\mathbf{X}'_s \mid \mathbf{X}_s, \mathbf{A}_s) = \prod_{n=1}^{N_s} \Pr(X'_{un}, X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn})$$ $$\Pr(X'_{un}, X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = \Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) \cdot \Pr(X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn})$$ Situation one: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = (\mathbf{P}_{\mathbf{un},X_{dn}})_{X_{un}X'_{un}}$$ Situation two: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un} = X_{un})$$ Situation three: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un} = 1)P_{p,un} + I(X'_{un} = S_{un} - 1)(1 - P_{p,un})$$ Situation four: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un} = 1)P_{c,un} + I(X'_{un} = S_{un})(1 - P_{c,un})$$

3. Developing a new multi-component model

This section considers a system with n identical components arranged in a series configuration. To illustrate this concept, we will use the example of a gearbox. A gearbox typically consists of multiple gears connected consecutively, each meshing with the preceding and succeeding gears. This configuration allows for the transmission of motion and torque from the input to the output gear(s).

In a gearbox system, the failure or fault in one gear can potentially affect the adjacent gears within the gear train. When a gear fails or experiences a fault, such as a tooth breakage or chipping, it can lead to an imbalance in load distribution, increased friction, and abnormal wear patterns on the teeth of the neighboring gears. Consequently, the neighboring gears may experience higher loads than normal, accelerating wear, increasing stress levels, and potentially causing failures in their teeth.

The transmission of faults or failures between gears in a gear train can occur through various mechanisms, and it is crucial to address these issues promptly to prevent further cascading failures. The failure of a gear in a gear train can result in downtime, costly repairs, and potential safety risks, particularly if the gear train is a critical component in an industrial or transportation application.

Our research aims to predict the time of system failure before it occurs, allowing for timely actions to be taken. In the context of this study, we focus on a system with n=3 identical components, as illustrated in Figure 1.

By considering the specific case of n=3 components, we aim to develop a predictive model for the system's remaining useful life (RUL). This model will enable us to estimate the time remaining until the system reaches failure, thereby providing an opportunity for proactive maintenance and mitigation measures.

Our research seeks to enhance the understanding of gearbox systems and their failure dynamics, enabling more effective maintenance strategies and improved system reliability.

Now, let's consider n=3 as in Figure 1.

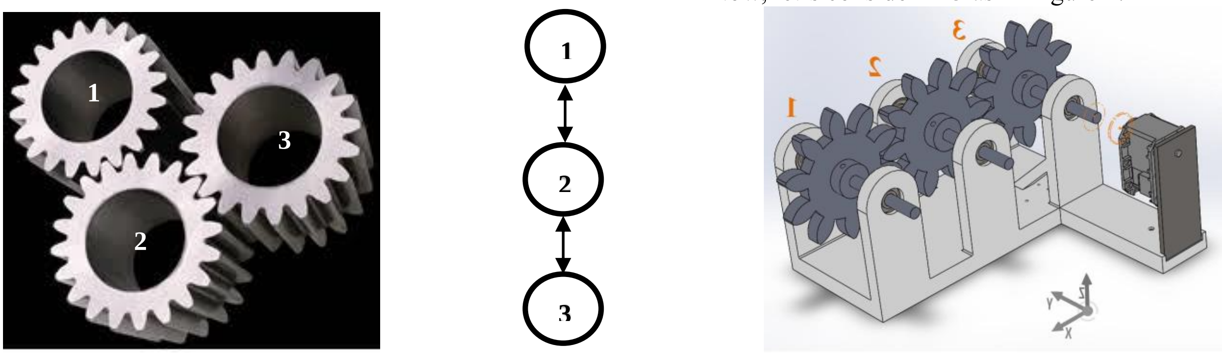


Figure 1. The assumed multi-component system, n=3

Now, let's consider the degradation function as in the following:

$$x_k^{(i)} = x_{k-1}^{(i)} + \eta^{(i)}\Delta t_{k-1} + \alpha_1 x_k^{(i-1)} + \alpha_2 x_k^{(i+1)} + \alpha_3 x_k^{(i-1)} x_k^{(i+1)} + w_{k-1}^{(i)} \qquad i = 1, \dots, n \qquad (2)$$

In which:

$w_{k-1}^{(i)} = \sigma_B^{2\,(i)} B(t)\,, \qquad \Delta t_{k-1} = 1$

$\eta^{(i)}$= the Constant inherent degradation rate of each component

$x_k^{(i)}$= the degradation value of component i in at t_k

$\alpha_1, \alpha_2, \alpha_3$ = the Constant coefficients

$x_k^{(i-1)} x_k^{(i+1)}$= the mutual effect of destroying the components $(i-1)$ and $(i+1)$ on component i

$w_{k-1}^{(i)}$ = the state transition noise of component i, $w_{k-1}^{(i)} \sim N\left(0, \sigma_B^{2\,(i)} \Delta t_k\right)$

$\sigma_B^{2\,(i)}$ = diffusion coefficients of component i

$B(t)$= standard Brownian motion

Figure 2 shows that each component, based on its position in the system, is influenced by other neighboring components. Components 1 and 2 are only affected by Component 2. However, Component 2 is influenced by both Component 1 and Component 3. Therefore, the mutual effect of these two components on Component 2 should be considered. The mutual effect is denoted as $x_k^{(1)} x_k^{(3)}$. The model parameters include η, α_1 , α_2 , α_3 , σ_B^2. These parameters can be estimated using the least square method.

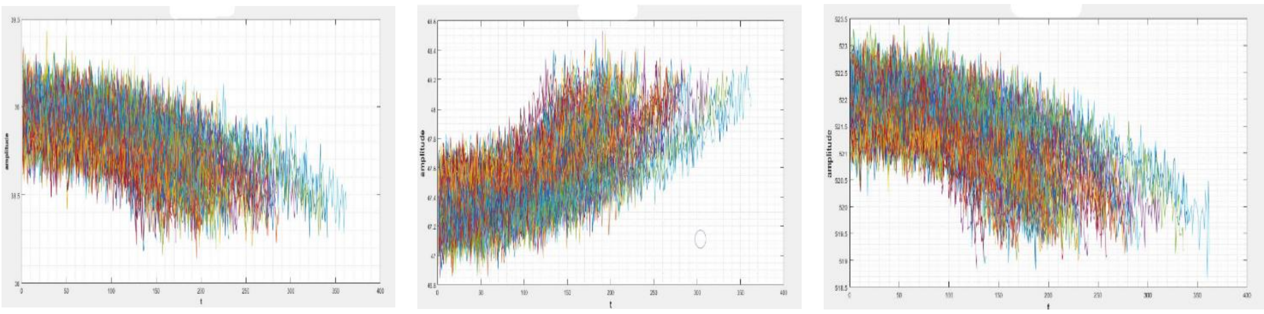


Figure 2. Degradation trajectories for components 1, 2, and 3 (left to right) for 100 units (smoothed curves presented)

4. Numerical studies

This section uses a simulated case study to demonstrate how our developed model work. Our case system has three components; a sensor is installed to monitor its degradation trajectory. Hence, we have three simulated degradation datasets. Each data set constitutes 100 degradation trajectories that generally amount to 20631 data points for each component. Figure 2 shows these degradation trajectories for components 1 to 3.

Now, we estimate the parameters of the model using the least square method:

$$\rightarrow \quad \Delta x_k^{\,(1)} = x_k^{(i)} - x_{k-1}^{(i)} = \eta^{(i)}\Delta t_{k-1} + \alpha_1 x_k^{(i-1)} + \alpha_2 x_k^{(i+1)} + \alpha_3 x_k^{(i-1)} x_k^{(i+1)} + w_{k-1}^{(i)} \qquad (3)$$

The obtained models are:

For Component 1:

$\Delta x_k^{\,(1)} = \eta + \alpha x_k^{(2)} + w_{k-1}^{(1)} \quad \rightarrow \quad \Delta x_k^{\,(1)} = 1.6827 - 0.035 x_k^{(2)} + w_{k-1}^{(1)}$

$w_{k-1}^{(1)} \sim N\left(0, \sigma_B^{2\,(1)} \Delta t_k\right) \quad , \quad w_{k-1}^{(1)} \sim N(0, 0.0202)$

For component 2:

$\Delta x_k^{\,(2)} = \eta + \alpha_1 x_k^{(1)} + \alpha_2 x_k^{(3)} + \alpha_3 x_k^{(1)} x_k^{(3)} + w_{k-1}^{(2)} \rightarrow$

$\Delta x_k^{\,(2)} = 91.7988 - 1.8533 x_k^{(1)} - 0.1887 x_k^{(3)} + 0.0039 x_k^{(1)} x_k^{(3)} + w_{k-1}^{(2)}$

$w_{k-1}^{(2)} \sim N\left(0, \sigma_B^{2\,(2)} \Delta t_k\right) \quad , \quad w_{k-1}^{(2)} \sim N(0, 0.024)$

For component 3:

$\Delta x_k^{\,(3)} = \eta + \alpha x_k^{(2)} + w_{k-1}^{(3)} \quad \rightarrow \quad \Delta x_k^{\,(3)} = -0.4892 + 0.0101 x_k^{(2)} + w_{k-1}^{(3)}$

$w_{k-1}^{(3)} \sim N\left(0, \sigma_B^{2\,(3)} \Delta t_k\right) \quad , \quad w_{k-1}^{(3)} \sim N(0, 0.1736)$

According to the developed model, the estimated remaining useful life (RUL) curves for all three components are shown in Table 3. In fact, Table 3 consists of three columns, each assigned to one component. Starting from the left column, a degradation trajectory for Component 1 is shown in the first row using the data presented in Figure 2. Since Component 1 is influenced by Component 2, in the second row, a degradation trajectory for Component 2 is shown using the data presented in Figure 2 again. In the third row, the degradation trajectory of Component 1 is shown. Finally, in the fourth row, the estimated value of RUL for Component 1 is illustrated. The same process has been carried out for Component 2 and 3 in columns 2 and 3, respectively. Note that the second column, which corresponds to Component 2, is influenced by both Component 1 and Component 3.

Table 3. RUL for each component

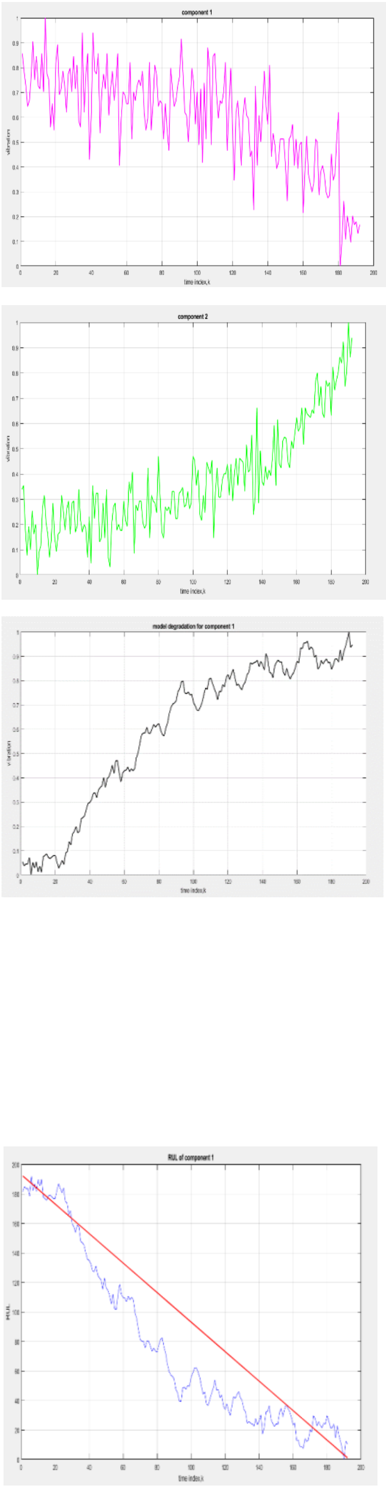
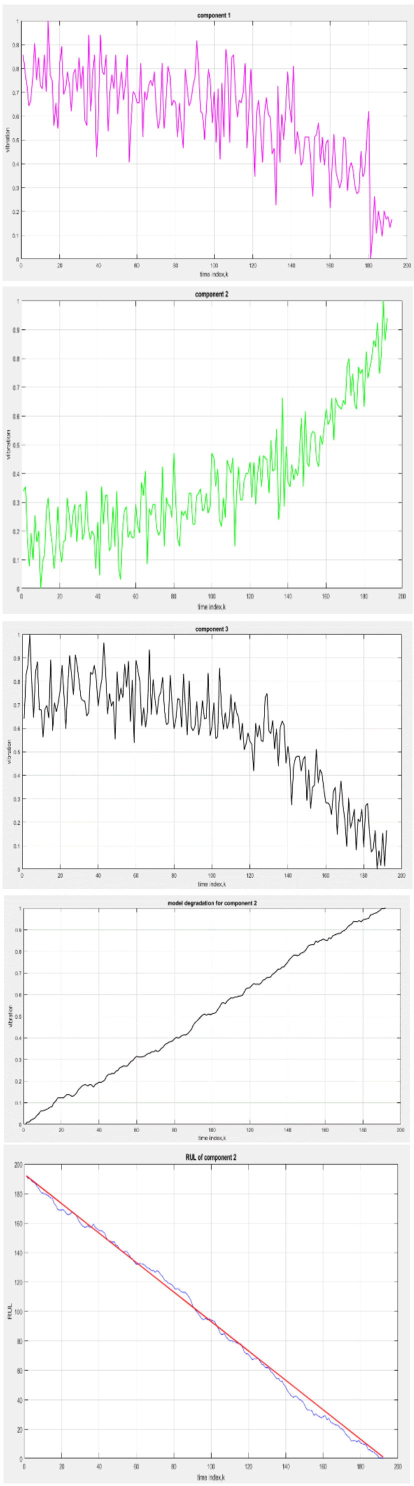
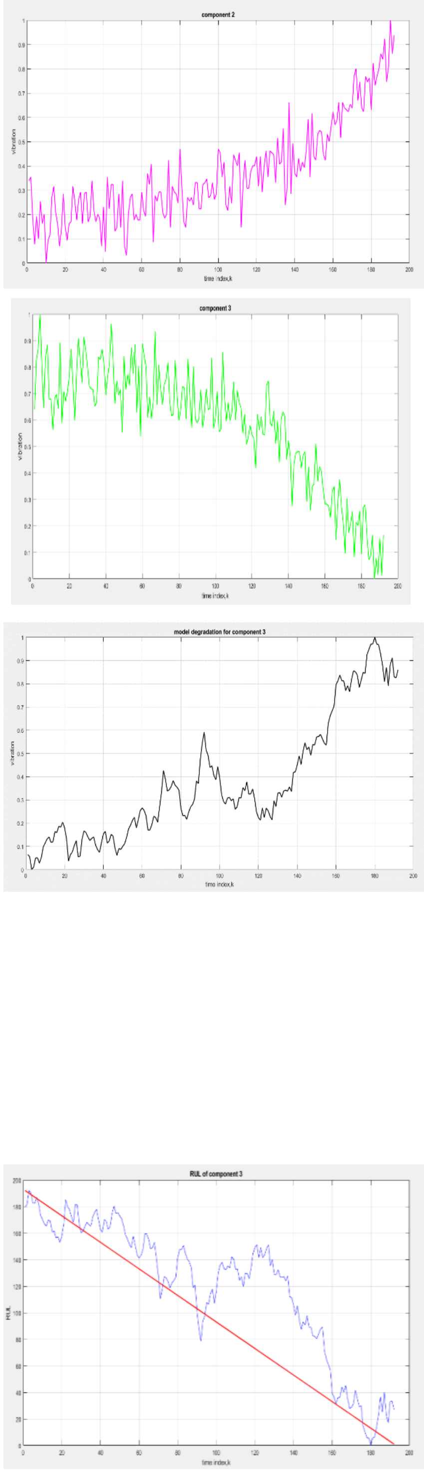
Component 1	**Component 2**	**Component 3**
		

Figure 1 illustrates the system under consideration, which follows the reliability theory and is characterized as a series system. In a series system, the system fails if its components fail. Consequently, this system's predicted Remaining Useful Life (RUL) is determined by identifying the minimum RUL prediction among the three components, as presented in Table 3.

The RUL predictions for the individual components are compared, and the minimum value among them is selected as the RUL prediction for the system. The obtained RUL values for the system are represented by the blue dots in Figure 3. Additionally, the yellow dots depict a fitted line, representing the trend and pattern of the RUL values.

The graphical representation in Figure 3 provides a visual representation of the RUL predictions for the system over time. The blue dots signify the RUL values for the system at different time points, while the yellow dots represent the fitted line, which approximates the overall trend in the RUL values.

By examining the RUL predictions for the system and visualizing them in Figure 3, one can gain insights into the system's projected lifetime and components. This analysis helps understand the potential failure points and plan maintenance strategies accordingly.

Overall, Figure 1 demonstrates the series system structure, while Figure 3 presents the RUL predictions and the fitted line, enabling a comprehensive understanding of the RUL estimation for the system.

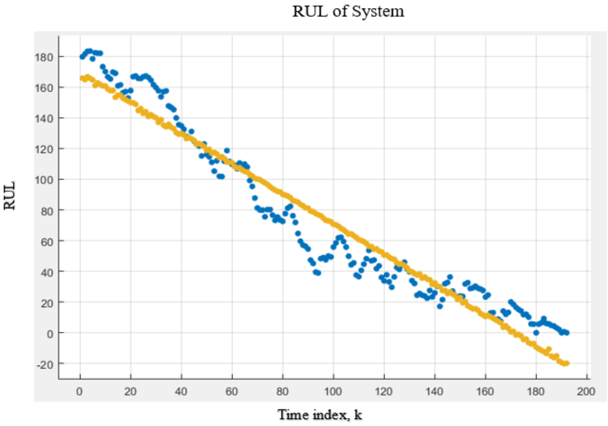


Figure 3. RUL of the system

5. Conclusion

This paper presents the development of a multi-component degradation model that specifically focuses on identical components. The components in the system are interconnected in a series configuration, meaning that the failure of one component impacts the functioning of the others. The model considers this characteristic and incorporates it into the degradation modeling process. To account for the influence of environmental factors, the model utilizes Brownian motion, which allows for the integration of environmental effects on the degradation process.

The model's unknown parameters are estimated using the least squares method, which helps optimize the model's accuracy. The overall RUL of the system is subsequently determined by estimating the Remaining Useful Life (RUL) of each component.

A specific numerical example is presented to validate the developed model's effectiveness. The results demonstrate that the model effectively estimates the degradation process of each component. As a result, an acceptable estimation of the RUL for the entire system is achieved.

In summary, this paper contributes to the field of PHM systems by introducing a multi-component degradation model with identical components, considering the interdependency among the components in a series configuration. Incorporating environmental factors using Brownian motion enhances the model's realism. By applying the model to a numerical example, it is demonstrated that the model effectively estimates the degradation process of each component and provides a reliable estimation of the overall RUL for the system.

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