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Original Research Article

Analysis of Discrete Fix Up Limit Time of Two Systems Prediction

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Abstract

This paper studies a discrete fix-up limit policy for two systems. Because sometimes, a failed system cannot be completely fixed at the optimal fix-up limit time due to some logistic issues. This paper provides a chance to complete fixing up a failed system within a discrete fix-up limit time LT (L=1,2,3…) for a fixed T. The explicit expression of the expected long-term cost per unit time is derived for the two systems based on the assumptions of the systems. Finally, a numerical example is given to illustrate the theoretical results of the proposed model.

Keywords: Discrete-time; Fix up; Limit policy; Limit time; Subsystem.

1. Used notations and assumptions

1.1 Used notation

$CS_i(L)$	Fix up limit function of formation S_i, for $i = 1, 2$
$L^*_{S_1}$	Optimal discrete fix up limit time of the series formation S_1
$L^*_{S_2}$	Optimal discrete fix up limit time of the parallel formation S_2
μ_i	Mean failure time of formation S_i, for $i = 1, 2$
$H_{S_i}(t)$	Probability of fix up for failed formation S_i, for $i = 1, 2$
$\overline{H_{S_i}}(t)$	Probability of not fixing up failed formation S_i, for $i = 1, 2$
$H_i(t)$	Probability of fix up for failed component D_i, for $i = 1, 2, 3, \dots, 6$
$r_i(t)$	Fix up rate of a failed component D_i, for $i = 1, 2, 3, \dots, 6$
C_r	Cost of changing a failed formation S_i, for $i = 1, 2$, when the fix-up is not over within the discrete limit time LT, for a fixed T and $L = 1, 2, 3, \dots$
$C_x(t)$	Fix up cost of failed formation S_i, for $i = 1, 2$, during $(0, LT]$, for a fixed T and $L = 1, 2, 3, \dots$

1.2 Assumptions

1. Both the series-parallel (S_1) and parallel-series (S_2) formations are subjected to a failure, rectified by fix up.
2. The fix up rate for the failed series-parallel (S_1) and parallel-series (S_2) formations follows the non-homogeneous Poisson process, such that all fix up rates are increasing.
3. If the fix up for the failed series-parallel (S_1) and parallel-series (S_2) formations is not completed within the specified discrete limit time $LT(L = 1, 2, 3, \dots)$ for a fixed T, then it is replaced with a new one.
4. The cost for the fix-up of the two formations is proportional to time.

2. Introduction

The maintenance actions for multi-component systems have been more serious issues in the last few decades because the systems are becoming more complicated, having many relating or dependent components. Sometimes, a failed system cannot be fixed up entirely at the exact optimal fix-up limit due to some issues, but during idle periods, it can give the chance to finish fixing up the failed device completely. Many researchers have studied various repair limit problems in the maintenance literature. Bai and Hoang (2005) applied a quasi-renewal process to study a repair-limit risk free warranty with a threshold point on the number of repairs of a system, where replacement is deemed more cost-effective after that. Aven and Castro (2008) presented a minimal repair replacement model for a single unit system subjected to two types of failures under a safety constraint. Niwas and Garg (2018) built a mathematical model of a system based on the Markov process to examine the properties of an industrial plant under the charge-free warranty policy and also derived various reliability parameters. Xie et al. (2020) investigated the implications of cascading failures of a particular system and the effects of safety barriers on preventing failures. Maihula et al. (2021) studied some reliability measures such as reliability, mean time to failure availability, and profit function for a solar serial system with four subsystems to look for ways to improve the whole reliability of the solar system. Sanusi and Yusuf (2022)analyzed the reliability and profit of data center network topology. Also, they came up with a suitable maintenance technique to improve system performance, which is vital for reliability and maintenance managers. There are many maintenances, replacement, and inspection models, and recent research has attempted to unify some of them. Beichelt et al. (2006) proposed some replacement policies for a system based on two strategies. Strategy 1: after a failure, the repair cost is estimated. If the repair cost exceeds a given limit, the system is not repaired but replaced with a new one. Strategy 2: the system is replaced as soon as the total repair costs arising during its running time exceed a given limit. Kapur et al. (2007) proposed some aliment cost function of a unit subjected to two types of breakdown under the idea of a fix-up charge limit as listed : (i) the unit is replaced at the nth breakdown, or when the estimated moderate fix-up charge exceeds a particular limit *c*; (ii) a unit has two types of breakdown and is replaced at the nth type 1 breakdown, or type 2 breakdown, or when the estimated repair cost of type 1 breakdown exceeds a limit c; (iii) the unit is replaced at the nth type 1 breakdown, type 2 breakdown, or when the estimated fix up-charge due to type 1 breakdown exceeds a predetermined limit c. Chang et al. (2010) considered a replacement model with minimal repair based on a cumulative repair-cost limit policy, where the information of all repair costs is used to decide whether the system is repaired or replaced. Chen and Chang (2015) presented a charge function of a system involving two levels of alarms, such that the system undergoes precautionary care at a projected time T or immediately after the *n*th level-I alarm, and restorative care at the projected time T when the entire damage exceeds a catastrophic limit or immediately after any level-II alarm, whichever comes first. Lewaherilla et al. (2016) developed a minimal repair model for a fishing vessel such that the failure rate follows Weibull and non homogeneous Poisson process.

Furthermore, they also made some comparative analyses of their proposed model with other related existing models. Laia et al. (2017) developed a bivariate *(n,k)* replacement policy with a cumulative repair cost limit for a two-unit system is studied, in which the system is subjected to a shock damage interaction between units. Each unit 1 failure causes random damage to unit 2, and these damages are additive. Unit 2 will fail when the total damage of unit 2 exceeds a failure level *K*, and such a failure makes unit 1 fail simultaneously, resulting in a total failure.

Several authors present various special preventive maintenance models. Safaei et al. (2018) explored a system's best precautionary aliment actions based on some stated terms. Wang et al. (2019) considered a repairable system with one repairman. When the system fails, the repairman fixes it immediately, and they derive an explicit expression of the long-run average cost rate function C(T,N) for the system based on some assumptions. Sheu et al.(2019) proposed precautionary replacement charge functions of a system prone to particular distress, in which the system is either replaced with the latest one or fixed up when distress occurs. Sudheeshet al. (2019) looked at the discontinuous replacement charge function before looking at the features of a system's mean time to failure (MTTF). Wang et al. (2019) obtained the charge function $C(T,N)$ for a fixable system with one repair worker, such that, as the system meets up a specified time T, the repairman will fix up the unit precautionary, and it will return to operation as soon as the fixing is completed. Mirjalili and Kazempoor (2020) presented three replacement plans for a system consisting of independent components with a rising failure rate. Safaei et al. (2020) used the copula framework to provide two optimal age replacement policies based on the expected cost and maximum availability functions. The challenge of adopting the best aliment strategy among three charge-effective aliment planning approaches was investigated by Rebaiaia and Ait-kadi (2020). Sanoubaret al. (2020) considered a time replacement strategy for a system, which is replaced at the breakdown or a specific replacement time, whichever comes first, and replacement charges are estimated to be non-decreasing. Wu et al. (2021) established corresponding replacement models for a deteriorating repairable system with multiple vacations of one repairman. Al-Chalabi (2022) developed a cost minimization model to optimize the lifetime of a drill rig used in the Tara underground mine in Ireland. The model can estimate the economic replacement time of fixable instruments applied in the mining and other production industries. In trying to optimize the repair plan for some systems, Bi et al. (2022) proposed a method for the enhancement of repair efficiency for systems such as gas and water networks system. Waziri (2021) offered a discontinuous projected replacement charge function for a unit subjected to three forms of breakdown involving fix-up. Also, Waziri and Yusuf (2021) came up with a discontinuous projected replacement charge model for a multi-component system involving two levels of breakdown.

Nakagawa (2005) explained that sometimes functional units could not be changed at the precise optimum times due to some issues: a shortage of spare units, lack of money or workers, or inconvenience of time required to complete the replacement, units may be instead replaced in idle times, *e.g.*, weekend, month-end, or year-end. However, the author of this paper did not come across any existing work presenting a discrete fix-up limit model. This reason motivated the author of this paper to convert the continuous fix-up a limit model of some two systems to a discrete one. Additionally, the paper will explore the characteristics of the model presented.

The subsequent sections of this paper are arranged in this order: Section 2 presents the used notations and assumptions. Section 3 presents the description of the system. Section 4 presents the formulation of the model. Section 5 presents the numerical example. Section 6 presents the general discussion of the results. Section 7 presents the significance of the results obtained. Finally, section 8presents the conclusion.

3. Systems description

Consider six components D_1,D_2,… andD_6, arranged in two different formations to form two systems, which are series-parallel formation(S_1)and parallel-series formation(S_2).S_1 formation has three subsystems (which are D_1D_2, D_3D_4 and D_5D_6), while S_2 formation is having two subsystems (which are $D_1D_2D_3$and $D_4D_5D_6$). It assumed that all the six components are subjected to a particular failure, rectified by minor fix up. The series formation fails due to the failure if at least one of the six component(s) fails due to the particular failure, and the failure is rectified by fix up the failed component(s). For the parallel formation, the formation fails due to the failure if all the components fail due to the particular failure, and such failure is rectified by fixing up all the six failed components. When a formation fails, its fix up is started immediately. When the fix up is not completed within the specified discrete limit time projected time LT $(L = 1, 2, 3, \dots)$ for a fixed T, it is replaced with a new one. Let C_r be the replacement cost of a failed formation that includes all costs caused by failure and replacement. Let $C_x(LT)$ be the expected charge of minor fix up during$(0, LT]$, for$L = 1, 2, 3, \dots$ and a fixed T, which includes all charges incurred due to fix up and downtime during and be bounded on a finite interval. Due to insufficient fund, fix up man or difficulty of time required to complete the fix up of the failed system, the failed system sometimes cannot be fix up completely within the exact optimum fix up limit times. The formation may be fix up in discrete limit time LT $(L = 1, 2, 3, \dots)$ for a fixed T.S_1 formation fails if at least one of the three subsystems fails, whileS_2 formation fails if at least one component fails from both the two subsystems fail.

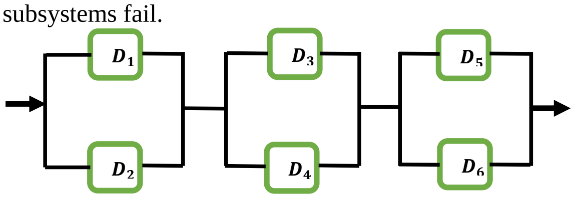

Figure 1. Reliability block diagram of S_1 formation

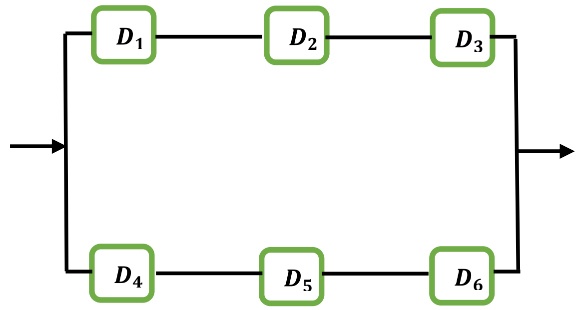

Figure 2. Reliability block diagram of S_2 formation.

4. Formulation of the model

The probability that series-parallel(S_1)formation will be fixed up within the discrete fix up limit time $LT(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$H_{S_1}(LT) = \left(1 - (1 - R_1^*(LT))(1 - R_2^*(LT))\right) \times \left(1 - (1 - R_3^*(LT))(1 - R_4^*(LT))\right) \times \left(1 - (1 - R_5^*(LT))(1 - R_6^*(LT))\right) \quad (1)$$

While, the probability that parallel-series(S_2)formation will be fixed up within the discrete fix up limit time $LT(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$H_{S_2}(LT) = 1 - (1 - R_1^*(LT)R_2^*(LT)R_3^*(LT)) \times (1 - R_4^*(LT)R_5^*(LT)R_6^*(LT)) \quad (2)$$

where

$$H_i(LT) = e^{-\int_0^{LT} r_i(t)dt}, \text{ for } i = 1, 2, 3, \dots, n. \quad (3)$$

The probability that series-parallel(S_1) and parallel-series (S_2) formations will be not fix up within the discrete fix up limit timeLT $(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$\bar{H}_{S_i}(LT) = 1 - H_{S_i}(LT), \text{ for} i = 1, 2. \quad (4)$$

The cost of replacement for the failed series-parallel (S_1) and parallel-series (S_2) formations that is not fix up within the discrete fix up limit time LT $(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$Costofreplacement = (C_r + C_x(LT))\bar{H}_{S_i}(LT), \text{ for } i = 1, 2 \quad (5)$$

The cost of fix up for the failed series-parallel (S_1) and parallel-series (S_2) formations within the periodic fix up limit timeLT $(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$Costofminorfixup = \int_0^{LT} C_x(t)dH_{S_i}(t), \text{ for} i = 1, 2. \quad (6)$$

Using equations (5) and (6), the cost for the series-parallel (S_1) and parallel-series (S_2) formations within the discrete fix up limit timeLT $(L = 1, 2, 3, \dots)$ for a fixed T in one cycle is

$$(C_r + C_x(LT))\bar{H}_{S_i}(LT) + \int_0^{LT} C_x(t)dH_{S_i}(t) = C_r\bar{H}_{S_i}(LT) + \int_0^{LT} \bar{H}_{S_i}(t)dC_x(t), \text{ for } i = 1, 2. \quad (7)$$

The mean failure time for the series-parallel (S_1) and parallel-series (S_2) formations within the discrete fix up limit time LT ($L = 1, 2, 3, \ldots$) for a fixed T in one cycle is

$$Meantime = \mu + \int_0^{LT} \bar{H}_{S_i}(t)dt, \text{ for } i = 1, 2. \quad (8)$$

Using equations (7) and (8), the fix up limit function for the series-parallel (S_1) and parallel-series (S_2) formations within the discrete fix up limit time LT ($L = 1, 2, 3, \ldots$) for a fixed T in one cycle, is

$$CS_i(L) = \frac{C_r \bar{H}_{S_i}(LT) + \int_0^{LT} \bar{H}_{S_i}(t) dC_x(t)}{\mu + \int_0^{LT} \bar{H}_{S_i}(t)dt}, \text{ for } i = 1, 2. \quad (9)$$

Note following observations:

1. Observed that, as L approaches zero, we have

$$CS_i(0) \equiv \lim_{L\to 0} CS_i(L) = \frac{C_r}{\mu}, \text{ for } i = 1, 2. \quad (10)$$

2. Observed that, as L approaches infinity, we have

$$CS_i(\infty) \equiv \lim_{L\to\infty} CS_i(L) = \frac{\int_0^{LT} \bar{H}_{S_i}(t) dC_x(t)}{\mu + \int_0^{\infty} \bar{H}_{S_i}(t)dt}, \text{for } i = 1, 2. \quad (11)$$

5. Numerical example

In this section, two numerical examples were provided to illustrate the proposed replacement cost model's characteristics. Let the fix up rate of the fixable failure for the six components obeys the Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \quad (12)$$

Where $\propto_i > 1$ and $t \geq 0$.

For illustration purposes, let us consider a simple numerical example based on the assumptions. Suppose

1. $\propto_1 = 4$, $\propto_2 = 2$, $\propto_3 = 3$, $\propto_4 = 2$, $\propto_5 = 2$, and $\propto_6 = 3$.
2. $\lambda_1 = 0.03$, $\lambda_2 = 0.002$, $\lambda_3 = 0.03$, $\lambda_4 = 0.001$, $\lambda_5 = 0.02$ and $\lambda_6 = 0.01$.
3. $C_r = 30$, $\mu = 2$ and $C_x = 2t^2$.

By substituting the parameters for the fix up rate in equation (12), the following equations below are obtained as follows:

$$r_1(t) = 0.12t^3 \ ; \quad (13)$$
$$r_2(t) = 0.06t \ ; \quad (14)$$
$$r_3(t) = 0.09t^2 \ ; \quad (15)$$
$$r_4(t) = 0.002t^2 \ ; \quad (16)$$
$$r_5(t) = 0.02t^3; \quad (17)$$
$$r_6(t) = 0.03t^2 \ . \quad (18)$$

Table 1 and Table 2 below are obtained by substituting the assumed costs of replacement and fix up ($C_r = 30$, $\mu = 2$ and $C_x = 2t^2$) and fix up rate for the six components(equations (13), (14), (15), (16), (17) and (18)) in equation (9). Noting that in the continuous case, the optimal repair limit time for parallel formation is larger than that of the series formation, that is why the choice of values of T for $CS_2(L)$ is larger than that of $CS_1(L)$.

Table 1. Values of CS_1(L) with various values of T versus L.

L	CS_1(L)as T=0.2	CS_1(L)as T=0.4	CS_1(L)as T=0.6	CS_1(L) as T=0.8	CS_1(L)as T=1	CS_1(L)as T=1.5	CS_1(L)as T=2
1	9.94	9.82	9.61	9.28	8.78	6.71	3.84
2	9.82	9.28	8.10	6.17	3.84	0.57	0.47
3	9.61	8.10	5.01	1.88	0.57	0.52	0.67
4	9.28	6.17	1.88	0.46	0.47	0.67	0.84
5	8.78	3.84	0.57	0.47	0.57	0.80	1.00
6	8.10	1.88	0.44	0.55	0.67	0.92	1.14
7	7.22	0.81	0.49	0.63	0.76	1.04	1.27
8	6.17	0.46	0.55	0.70	0.84	1.14	1.39
9	5.01	0.44	0.61	0.77	0.92	1.24	1.50
10	3.84	0.47	0.67	0.84	1.00	1.33	1.60
11	2.77	0.51	0.72	0.91	1.07	1.42	1.69
12	1.88	0.55	0.77	0.97	1.14	1.50	1.78
13	1.23	0.59	0.83	1.03	1.21	1.58	1.86
14	0.81	0.63	0.88	1.09	1.27	1.65	1.93
15	0.57	0.67	0.92	1.14	1.33	1.71	2.00
16	0.46	0.70	0.97	1.20	1.39	1.78	2.06
17	0.43	0.74	1.01	1.25	1.45	1.84	2.13
18	0.44	0.77	1.06	1.30	1.50	1.89	2.18
19	0.45	0.81	1.10	1.35	1.55	1.95	2.24
20	0.47	0.84	1.14	1.39	1.60	2.00	2.29
21	0.49	0.88	1.18	1.44	1.65	2.05	2.33
22	0.51	0.91	1.22	1.48	1.69	2.10	2.38
23	0.53	0.94	1.26	1.52	1.74	2.14	2.42
24	0.55	0.97	1.30	1.56	1.78	2.18	2.46
25	0.57	1.00	1.33	1.60	1.82	2.22	2.50

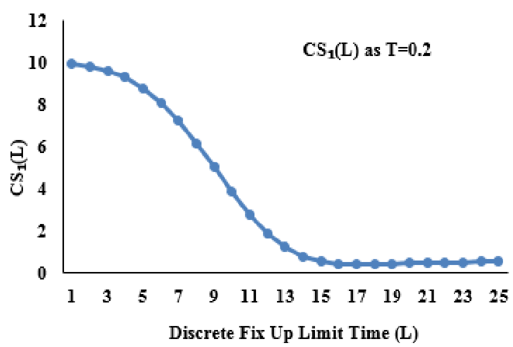

Figure 1. The plot of $CS_1(L)$ versus L as the value of T = 0.2

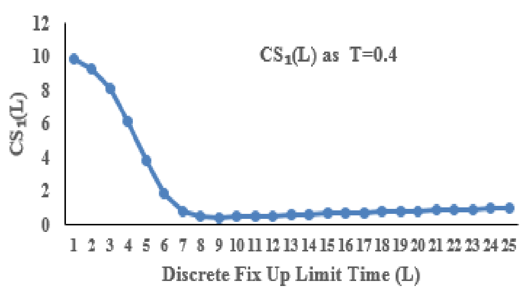

Figure 2. The plot of $CS_1(L)$ versus L as the value of T = 0.4

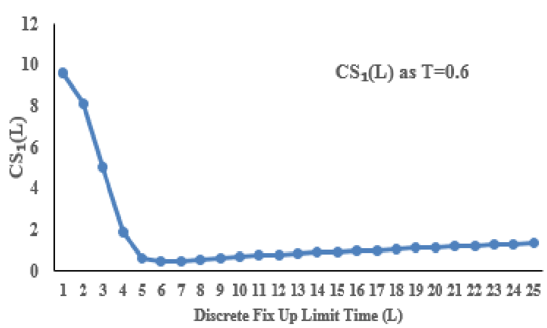

Figure 3. The plot of CS_1 (L)versus L as the value of T = 0.6

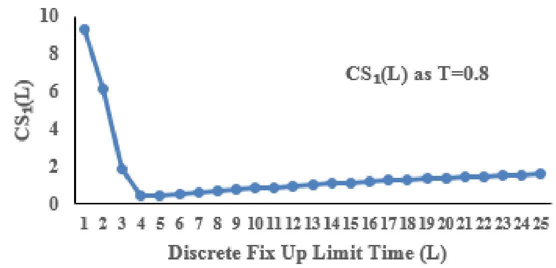

Figure 4. The plot of $CS_1(L)$ versus L as the value of T = 0.8

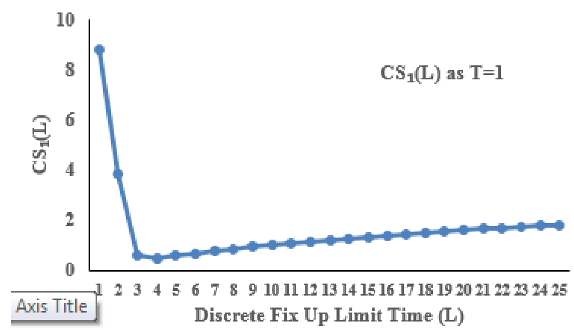

Figure 5. The plot of $CS_1(L)$versus L as the value of T = 1

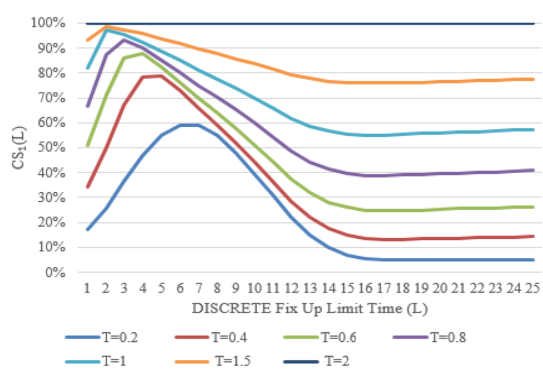

Figure 6. Comparing the values of $CS_1(L)$as T=0.2, 0.4, 0.6, 0.8, 1, 1.5 and 2.

Table 2. Values of $CS_2(L)$with various values of T versus L .

L	$CS_2(L)$as T=1	$CS_2(L)$as T=10	$CS_2(L)$as T=20	$CS_2(L)$as T=30	$CS_2(L)$as T=40	$CS_2(L)$as T=50	$CS_2(L)$as T=60
1	10.000	9.982	9.892	9.738	9.513	9.224	8.883
2	10.000	9.892	9.513	8.883	8.101	7.273	6.477
3	10.000	9.738	8.883	7.687	6.477	5.432	4.596
4	10.000	9.513	8.101	6.477	5.130	4.146	3.460
5	9.999	9.224	7.273	5.432	4.146	3.325	2.814
6	9.998	8.883	6.477	4.596	3.460	2.814	2.456
7	9.995	8.504	5.757	3.950	2.989	2.501	2.268
8	9.992	8.101	5.130	3.460	2.670	2.317	2.182
9	9.988	7.687	4.596	3.090	2.456	2.215	2.158
10	9.982	7.273	4.146	2.814	2.317	2.168	2.172
11	9.976	6.867	3.771	2.608	2.230	2.158	2.211

L	CS_2(L)as T=1	CS_2(L)as T=10	CS_2(L)as T=20	CS_2(L)as T=30	CS_2(L)as T=40	CS_2(L)as T=50	CS_2(L)as T=60
12	9.969	6.477	3.460	2.456	2.182	2.172	2.264
13	9.961	6.106	3.202	2.346	2.160	2.203	2.327
14	9.953	5.757	2.989	2.268	2.159	2.245	2.394
15	9.944	5.432	2.814	2.215	2.172	2.295	2.464
16	9.935	5.130	2.670	2.182	2.196	2.349	2.534
17	9.925	4.852	2.552	2.164	2.227	2.406	2.604
18	9.915	4.596	2.456	2.158	2.264	2.464	2.672
19	9.904	4.361	2.379	2.161	2.305	2.522	2.739
20	9.892	4.146	2.317	2.172	2.349	2.581	2.803
21	9.880	3.950	2.268	2.189	2.394	2.638	2.864
22	9.867	3.771	2.230	2.211	2.440	2.694	2.924
23	9.853	3.608	2.202	2.236	2.487	2.749	2.980
24	9.839	3.460	2.182	2.264	2.534	2.803	3.034
25	9.824	3.325	2.168	2.295	2.581	2.854	3.085
26	9.808	3.202	2.160	2.327	2.627	2.904	3.134
27	9.791	3.090	2.158	2.360	2.672	2.952	3.181
28	9.774	2.989	2.159	2.394	2.717	2.998	3.225
29	9.756	2.897	2.164	2.429	2.760	3.043	3.266
30	9.738	2.814	2.172	2.464	2.803	3.085	3.306

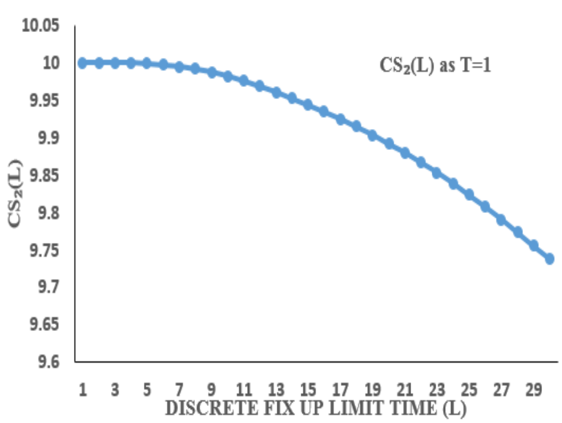

Figure 7. The plot of CS_2(L) versus L as the value of T = 1

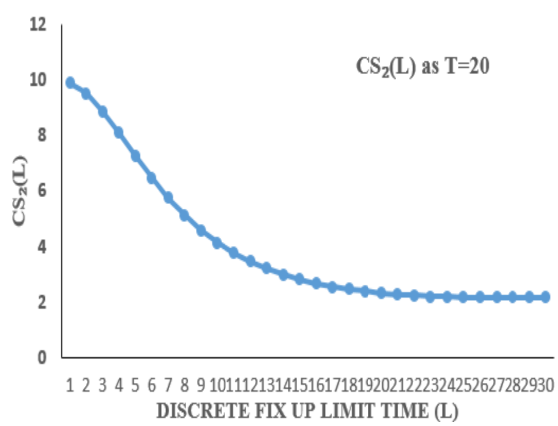

Figure 9. The plot of CS_2(L) versus L as the value of T = 20

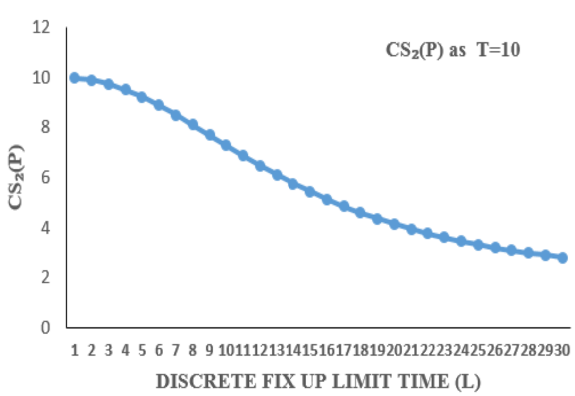

Figure 8. The plot of CS_2(L) versus L as the value of T = 10

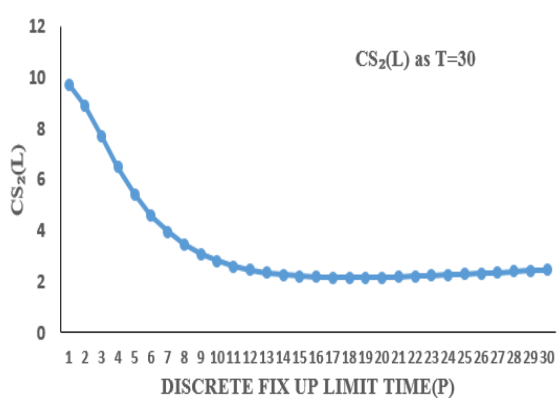

Figure 10. The plot of CS_2(L) versus L as the value of T = 30

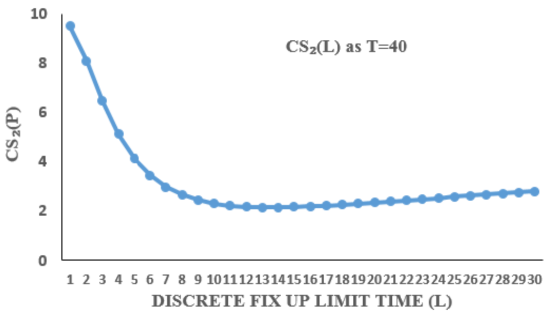

Figure 11. The plot of $CS_2(L)$ versus L as the value of T = 40

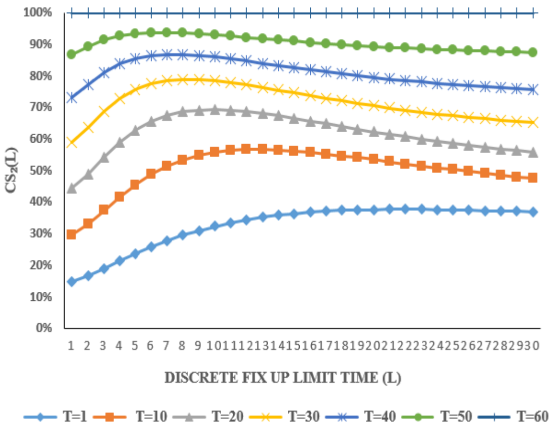

Figure 12. Comparing the values of $CS_2(L)$ as T=1, 10, 20, 30, 40, 50 and 60

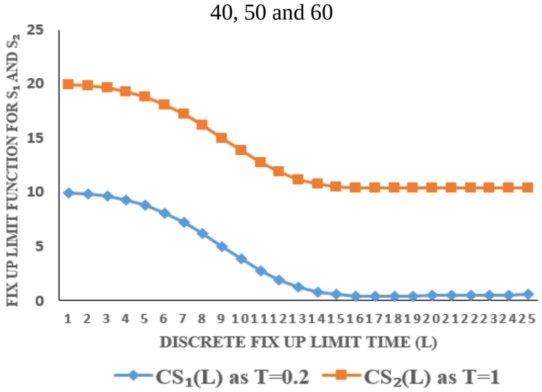

Figure 13. Comparing the values of $CS_1(L)$ and $CS_2(L)$ as T=1

Some observations from the results obtained are as follows

1. Table 3 below presents the optimal discrete fix up limit times of the series-parallel and parallel-series formations with various values of T, extracted from Tables 1 and 2.

Table 3. Optimal discrete fix up limit times for the S_1 and S_2 formations

Values of T for S_1	0.2	0.4	0.6	0.8	1	1.5	2
$L^*_{S_1}$	17	9	6	4	4	3	2
Values of T for S_2	1	10	20	30	40	50	60
$L^*_{S_2}$	Very large	Large	27	18	14	11	9

2. From Figure 6, observe that: $(CS_1(L), T = 1) < (CS_1(L), T = 10) < (CS_1(L),\ T = 20) < (CS_1(L), T = 30) < (CS_1(L), T = 40) < (CS_1(L), T = 50) < (CS_1(L), T = 60)$.
3. From Table 3, the optimal discrete fix up limit times of the series-parallel and parallel-series formations decreases as the value of T decreases.
4. From Figure 12, observe that: $(CS_2(L), T = 1) < (CS_2(L), T = 10) < (CS_2(L),\ T = 20) < (CS_2(L), T = 30) < (CS_2(L), T = 40) < (CS_2(L), T = 50) < (CS_2(L), T = 60)$.
5. Figures 1, 2, 3, 4 and 5 are the sketches of$CS_1(L)$ versus the discrete fix up limit time (L) for the series-parallel formation.
6. Figures 7, 8, 9,10 and 11 are the sketches of$CS_2(L)$ versus the discrete fix up limit time(L) for the parallel-series formation.
7. From Figure 13, observe that :$CS_1(L) < CS_2(L)$, as T=1.

6. General discussion of result

In search of the properties of the discrete fix up limit model constructed for the series-parallel (S_1) and parallel-series(S_2) formations, the results obtained showed that the optimal discrete fix up limit time of the series-parallel formation (S_1) is lower than that of the parallel-series formation (S_2). This situation occurs as the result of the arrangement of the components. It can be seen that, as the value of T increases, the optimal discrete fix up limit times of series-parallel (S_1) and parallel-series (S_2) formations decrease. As the value of T increases, the values of the discrete fix up limit function for both the series-parallel(S_1) and parallel-series(S_2) formations also increase. Furthermore, the values of the discrete fix up limit function for the series-parallel formation are less than that of the parallel-series formation.

7. Significance of results

Failed component(s) or subsystem(s) of various systems such as commercial vehicles, hydropower plants, and chemical plants sometimes requires a repair at ideal times to get the full attention of a professional repairman or to avoid scarcity of product on the normal working days. For the significance of this paper, the results obtained in this paper provided a vital theoretical strategy or policy for maintaining multi-components systems at a projected discrete fix up limit ideal times, such as weekend, month-end, or year-end. The results' advantages provide a chance of obtaining the optimal discrete fix up limit time of multi-components. Furthermore, sometimes transportation systems are less busy on some day(s) of the week of the year, which can plan or selected for all repairs.

8. Conclusion

This paper developed a discrete fix up limit model for series-parallel(S_1) and parallel-series(S_2) formations exposed to a fixable failure to provide a chance of completing fixing up a failed system within a discrete fix up limit time LT $(L = 1, 2, 3, \dots)$ for a fixed T. It is assumed that, if a formation fails, the fix up is started immediately. When the fix up is not completed within the discrete fix up limit time, it is replaced with a new one. A numerical example was provided to investigate the characteristics of the constructed discrete fix up limit function for the series-parallel(S_1) and parallel-series(S_2) formations. From the results obtained, one can see that the value of T affects the discrete fix up limit model because of the two reasons as follows (i) as the value of T increases, the optimal discrete fix up a limit time for the series-parallel(S_1) and parallel-series(S_2) formations decreases; (ii) and, as the value of T increases, the value of the discrete fix up limit function for both the series-parallel(S_1) and parallel-series(S_2) formations also increases. To finalize the discussion of the results obtained, the results showed that, the optimal fix up limit time of the parallel-series formation is higher than that of the series-parallel formation.

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