

# Performance Analysis of K out of N Reverse Osmosis (RO) system in the treatment of wastewater using RAMD analysis

# AnasMaihulla<sup>1\*</sup>, Ibrahim Yusuf<sup>2</sup>, Saminu Bala<sup>3</sup>

- 1. Department of Mathematics, Sokoto State University, Sokoto, Nigeria
- 2,3. Department of Mathematical Sciences, Bayero University, Kano, Nigeria

#### Abstract

The recursive approach for assessing the reliability, availability, maintainability, and dependability (RAMD) of five subsystems in a reverse osmosis (RO) machine was investigated in this study. The components include the raw water tank, precision filter, carbonated filter, reverse osmosis membrane, and water producing tank. Furthermore, the efficacy of a reverse osmosis (RO) unit, mean time to failure (MTTF), mean time to repair (MTTR), and dependability ratio were also assessed. The major aim is to maximize efficiency and identify the critical subsystem or component of the reverse osmosis machine. And finding out how to fix the problem. We utilized data from the RO unit with a repair rate and a failure rate over a 100-day period to validate the technique. There were numerical numbers for each subsystem's dependability, as well as comparisons between subsystem dependability and total system dependability.

Keyword: Economy, Figures, Components, frequency, Repair rate

# 1. Introduction

RO is currently a critical approach in the production of clean water all over the world. As a result, downtimes for repairing operations (after failures, membrane blockage, pressure losses, and so on) or preventative maintenance (Membrane cleaning, component replacements, and so on) must be kept to a minimum in terms of duration and frequency to guarantee maximum availability. Indeed, enhancing the RO plant's availability (or dependability) as a whole system reduces operating and maintenance costs significantly. Water scarcity is expected to become a new developing concern as global freshwater demand grows. Shalana L. and colleagues [1]. Safder U. et al. [2] conducted a study on the availability and reliability analysis of integrated reverse osmosis. Reliability, Availability, and Maintainability were discussed (RAM Analysis). Hanumant P. et al. [3] used PSO to perform RAM analysis and availability optimization of a thermal power plant's water circulation system. The study used Combined Event Tree and Fuzzy Fault Tree Analyses to assess the reliability of hybrid systems of advanced treatment units for industrial wastewater reuse. Farzad P. et al. did the study [4]. M. Hajeeh and D. Chaudhuri [5] performed study on the dependability and availability of reverse osmosis desalination. Zhou, J. et al. [6] conducted a

study on the feasibility and reliability of the Life Cycle Assessment desalination. The Real-Time Implementation of an Expert Model Predictive Controller in a Pilot-Scale Reverse Osmosis Plant for Brackish and Seawater Desalination is investigated by Revas P. et al [7]. The most susceptible component of serial operations, such as evaporation systems in the sugar industry and water treatment facilities, was defined by Goyal et al. [8]. This study deconstructs the efficiency indicators of the power production system using STP. The power system was investigated using simple probability theory ideas and the Markovian birth-death process. A Markov process grows more complicated as it advances from one stage to the next. M.F. Idrees [9] also works with a Reverse Osmosis Plan's Performance Analysis and Treatment Technologies. S. Besarati, C. Li, and collaborators [10] performed some study on reverse osmosis desalination utilizing a supercritical organic Rankine cycle at low temperatures a few years ago. Any modern reverse osmosis system must have automation and dependability in order to fulfill environmental and economic criteria. Srivastava, S. [11]. The availability of freshwater resources is essential for human growth. Some places have limited freshwater resources, whereas others have enough. Over one billion people, according to the study, do not have access to safe drinking water. F. Calise

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<sup>\*</sup>anasmaihulla@gmail.com

and colleagues [12]. There has been a spike in interest in using the Reverse Osmosis phenomenon in saltwater desalination in recent years. It's being tested in the lab and on a small size, and it's shown to have a lot of benefits. T.Y. Cath and colleagues [13]. In reverse osmosis, the osmotic pressure differential between the two sides of the membrane causes water to flow through the semipermeable membrane. Other than the energy required to circulate solutions in the system, this technique has no energy cost for trans-membrane flow. Increasing the product's water quality while decreasing the energy consumption of seawater desalination. According to a research done by S.S. Kolluri et al., lower energy needs for pretreatment and RO operations, fewer pretreatment procedures, cheaper maintenance and chemical expenses, and lower RO membrane prices all contribute to cost reductions [14]. As a result of globalization, water shortage is worsening. The water cycle is being interrupted as a result of severe climate changes throughout the planet Muhammad F. I. et al. [15]. Groundwater is a less expensive and more predictable source of water since it is either exhausted or polluted. Polluted water might have biological or inorganic materials as residuals. Brown, S.L., and others [16]. M. Badruzzaman et al. [17] looked at the pretreatment options for seawater reverse osmosis plants. Biofouling behavior of zwitterionic silane-covered reverse osmosis membranes contaminated with marine microorganisms was investigated by F. Saffarimiandoab et al. [18]. Evita. et al. [19] produced a strategy plan for the reuse of treated municipal wastewater for agricultural irrigation on the island of Crete. Slvia C Oliveira and Marcos Von Sperling [20] created a wastewater treatment plant dependability study. Pressure loss, mass transfer, turbulence, and unsteadiness were investigated.

Our interest in reverse osmosis machine systems originates from a major problem that the water purification industry is experiencing as a result of subsystem failure in machine systems. And the resultant sluggish progress in water filtration technology, as well as its importance in people's lives all around the world. In order to keep up with the rising complexity of machine systems, industries are putting in extra effort. The current work proposed a reliability modeling technique to analyze the overall performance of the RO system due to the non-availability of data for the RO system. We have presented a novel model of reverse osmosis subsystems in this article, which consists of five subsystems: The raw water tank, precision filter, carbonated filter, reverse osmosis membrane, and water generating tank are the components. The units in each subsystem are considered to have exponential failure and repair times, according to Ismail et al. [21].

The paper has been divided into six sections. Section 1 defines the introductory portion of the paper, which focuses on the relevant literature that was examined for the research of the suggested model. Section 2 displayed the result in tables. Section 3 discusses the system's block diagram and the transition diagrams for each subsystem, and the equations generated there in that will be utilized to analyze the proposed model. Section 4 discusses the system's materials and methods as well as the relevant definitions for the reliability models and certain specific instances. In part 5, RAMD Analysis for Subsystems as well as the entire system, numerical analysis for each subsystem, and the relationship between each subsystem with the entire RO system were analyzed. Section 6 capture definition of some terms used in the paper. Section 7 of the paper comes to a close with the results discussion. Section 8 was the conclusion part of the paper.

# 2. Transition diagram & equations

Failure and repair rates for the subsystems are presented in table 1. Table two pointed the Variation of reliability of subsystems with time. Variation in reliability of system due to variation in failure rate of raw water tank is detected in table three. Table four shows the Variation in reliability of system due to variation in failure rate of precision filter. Table five introduced the Variation in reliability of system due to variation in failure rate of carbonated filter. Table six contains the Variation in reliability of system due to variation in failure rate of Sand filter. Table seven shows the system's maintainability. Table eight depict Variation in reliability of system due to variation in failure rate of water producing tank. Lastly table nine shows RAMD indices for the R.O system.

**Table 1.** Failure and repair rates for the subsystems

Subsystems	Failure rate (Υ)	Repair rate (β)
$S_1$	0.0060	0.53
$S_2$	0.0093	0.82
S <sub>3</sub>	0.0020	1.16
S <sub>4</sub>	0.0050	0.42
S <sub>5</sub>	0.0032	0.27

Table 2. Variation of reliability of subsystems with time

Time in days	R <sub>S1</sub> (t)	R <sub>S2</sub> (t)	R <sub>S3</sub> (t)	R <sub>S4</sub> (t)	R <sub>S5</sub> (t)	R <sub>SYSTEM</sub> (t)
0	1	1	1	1	1	1
10	0.83527	0.86589	0.98020	0.86071	0.93800	0.57235
20	0.69768	0.74976	0.96079	0.74082	0.87985	0.32759
30	0.58275	0.64921	0.94176	0.63763	0.82531	0.18750
40	0.48675	0.56214	0.92312	0.54881	0.77414	0.10731
50	0.40657	0.48675	0.90484	0.47232	0.72615	0.06142
60	0.33960	0.42147	0.88692	0.40657	0.68113	0.03515
70	0.28365	0.36495	0.86936	0.34994	0.63890	0.02012
80	0.23693	0.31600	0.85214	0.30119	0.59930	0.01152
90	0.19790	0.27362	0.83527	0.25924	0.56214	0.00659
100	0.16530	0.23693	0.81873	0.22313	0.52729	0.00377

**Table 3.** Variation in reliability of system due to variation in failure rate of raw water tank

	RO system		Subsystem 1	
Time in days	γ= 0.0041	Y=0.0060	Y=0.0041	
0	1	1	1	1
10	0.65770	0.64533	0.95983	0.94176
20	0.43257	0.41645	0.92127	0.88692
30	0.28451	0.26874	0.88426	0.83527
40	0.18712	0.17342	0.84874	0.78663
50	0.12307	0.11192	0.81465	0.74082
60	0.08094	0.07222	0.78192	0.69768
70	0.05324	0.04661	0.75051	0.65705
80	0.03501	0.03008	0.72036	0.61878
90	0.02302	0.01941	0.69143	0.58274
100	0.01515	0.01252	0.66365	0.54881

**Table 4.** Variation in reliability of system due to variation in failure rate of precision filter

	RO system		Subsystem 2	
Time in days	$\begin{array}{c c} \gamma = & \\ 0.0080 & \gamma = 0.0052 \end{array}$		Y=0.0080	
0	1	1	1	1
10	0.61018	0.62750	0.92312	0.94933
20	0.37232	0.39377	0.85214	0.90122
30	0.22718	0.24709	0.78663	0.85556
40	0.13862	0.15505	0.72614	0.81221
50	0.08458	0.09730	0.67032	0.77105
60	0.05161	0.06105	0.61878	0.73198
70	0.03149	0.03831	0.57121	0.69489
80	0.01922	0.02404	0.52729	0.65968
90	0.01173	0.01509	0.48675	0.62625
100	0.00715	0.00947	0.44933	0.59452

**Table 5.** Variation in reliability of system due to variation in failure rate of carbonated filter

	RO system		Subsystem 3	
Time in days	γ= 0.0080	Y=0.0052	Y=0.0080	
0	1	1	1	1
10	0.61018	0.62750	0.92312	0.94933
20	0.37232	0.39377	0.85214	0.90122
30	0.22718	0.24709	0.78663	0.85556
40	0.13862	0.15505	0.72614	0.81221
50	0.08458	0.09730	0.67032	0.77105
60	0.05161	0.06105	0.61878	0.73198
70	0.03149	0.03831	0.57121	0.69489
80	0.01922	0.02404	0.52729	0.65968
90	0.01173	0.01509	0.48675	0.62625
100	0.00715	0.00947	0.44933	0.59452

**Table 6.** Variation in reliability of system due to variation in failure rate of Sand filter

	RO system		Subsystem 4	
Time in days	γ= 0.0080	Y=0.0052	Y=0.0080	
0	1	1	1	1
10	0.61018	0.62750	0.92312	0.94933
20	0.37232	0.39377	0.85214	0.90122
30	0.22718	0.24709	0.78663	0.85556
40	0.13862	0.15505	0.72614	0.81221
50	0.08458	0.09730	0.67032	0.77105
60	0.05161	0.06105	0.61878	0.73198
70	0.03149	0.03831	0.57121	0.69489
80	0.01922	0.02404	0.52729	0.65968
90	0.01173	0.01509	0.48675	0.62625
100	0.00715	0.00947	0.44933	0.59452

**Table 6.** Variation in reliability of system due to variation in failure rate of water producing tank

	RO system		Subsystem 4	
Time in days	γ= 0.0080	Y=0.0052	Y=0.0080	
0	1	1	1	1
10	0.61018	0.62750	0.92312	0.94933
20	0.37232	0.39377	0.85214	0.90122
30	0.22718	0.24709	0.78663	0.85556
40	0.13862	0.15505	0.72614	0.81221
50	0.08458	0.09730	0.67032	0.77105
60	0.05161	0.06105	0.61878	0.73198
70	0.03149	0.03831	0.57121	0.69489
80	0.01922	0.02404	0.52729	0.65968
90	0.01173	0.01509	0.48675	0.62625
100	0.00715	0.00947	0.44933	0.59452

**Table 7.** RAMD indices for the R.O system

RAMD indices of subsyste ms	Subs ystem 1	Subsys tem 2	Subs ystem 3	Subs ystem 4	Subsyst em 5	RO system
Reliabili ty	$e^{-0.018i}$	$e^{-0.0144t}$	$e^{-0.002i}$	$e^{-0.015i}$	$e^{-0.0064t}$	$e^{-0.0558t}$
Maintai nability	$e^{-1.59t}$	$e^{-1.64t}$	$e^{-1.16t}$	$e^{-1.26t}$	$e^{-0.54t}$	$\frac{1}{-e^{-2.06(t)}}$
Availabi lity	0.975 47	0.9692 8	0.996 33	0.982 22	0.94425	0.35631
MTTF	152	108	500	200	313	1273
MTTR	2.00	1.00	1.00	2.00	4.00	10.00
Depend ability	0.991 66	0.9907 8	0.996 39	0.992 23	0.98252	0.95649
Depend ability ratio	76	108	500	100	78	

Time in days	M <sub>S1</sub> (t)	M <sub>S2</sub> (t)	M <sub>S3</sub> (t)	M <sub>S4</sub> (t)	M <sub>S5</sub> (t)	M <sub>SYSTE</sub> <sub>M</sub> (t)
0	0	0	0	0	0	0
10	0.99999	0.99999	0.99999	0.99999	0.99548	0.9954 5
20	1.00000	1.00000	0.99999	1.00000	0.99997	0.9999 6
30	1.00000	1.00000	1.00000	1.00000	0.99999	0.9999 9
40	1.00000	1.00000	1.00000	1.00000	0.99999	0.9999 9
50	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000
60	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000
70	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000
80	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000
90	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000
100	1.00000	1.00000	1.00000	1.00000	1.00000	1.0000

Table 8. System's Maintainability

# 3. System Block diagram and transition diagrams for individual subsystems

A<sub>1</sub>, A<sub>2</sub> and A<sub>3</sub> are representing first, second and third raw water tanks respectively. B<sub>1</sub> and B<sub>2</sub> are for the precision filters.  $C_1$  is for the carbonated filter.  $D_1$ ,  $D_2$ , and  $D_3$  stands for the first, second and third Reverse osmosis membranes respectively. Finally, E1 and E2 are for the water producing tanks.

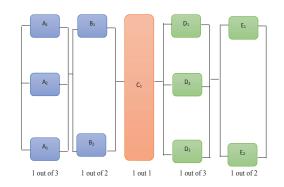


Figure 1. Block diagram for the system

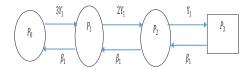


Figure 2. The raw water tank

$$\frac{d}{dt}P_0(t) = -3Y_1P_0 + \beta_1P_1 \tag{1}$$

$$\frac{d}{dt}P_1(t) = -(2\Upsilon_1 + \beta_1)P_1 + 3\Upsilon_1P_0 + \beta_1P_2 \tag{2}$$

$$\frac{d}{dt}P_2(t) = -(\Upsilon_1 + \beta_1)P_2 + 2\Upsilon_1P_1 + \beta_1P_3 \tag{3}$$

$$\frac{d}{dt}P_{0}(t) = -3Y_{1}P_{0} + \beta_{1}P_{1}$$

$$\frac{d}{dt}P_{1}(t) = -(2Y_{1} + \beta_{1})P_{1} + 3Y_{1}P_{0} + \beta_{1}P_{2}$$

$$\frac{d}{dt}P_{2}(t) = -(Y_{1} + \beta_{1})P_{2} + 2Y_{1}P_{1} + \beta_{1}P_{3}$$

$$\frac{d}{dt}P_{2}(t) = -\beta_{1}P_{2} + Y_{1}P_{1}$$

$$(4)$$

Under steady state, equation (1) - (4) reduces to Substituting (5) into (2)

$$P_1 = \frac{\Upsilon_1}{\beta_1} P_0 \tag{5}$$

$$P_2 \frac{Y_1^2}{\beta_1^2} P_0 \tag{6}$$

Substituting (5) into (4)

$$P_3 = \frac{Y_1^3}{\beta_1^3} P_0 \tag{7}$$

Using normalization condition

$$P_0 + P_1 + P_2 + P_3 = 1 \tag{8}$$

Using normalization condition
$$P_{0} + P_{1} + P_{2} + P_{3} = 1$$
Substituting (5) and (6) (7) into (8) we have
$$P_{0} + = \frac{Y_{1}}{\beta_{1}} P_{0} + \frac{Y_{1}^{2}}{\beta_{1}^{2}} P_{0} + \frac{Y_{1}^{3}}{\beta_{1}^{3}} P_{0} = 1$$

$$P_{0} = \frac{\beta_{1}^{3}}{\beta_{1}^{3} + 6Y_{1}^{3} + 3\beta_{1}^{2} Y_{1} + 6\beta_{1} Y_{1}^{2}}$$
(9)

# The raw water tank

The delicate intake valve is specially designed to allow water in at the right time. Because the system is under pressure, if it was the only valve before the water arrived, it may be damaged by backflow, which could break the valve and reduce system pressure. As a result, it has a separate valve that only enables water to escape, preventing backflow from the pressurized system. This is known as a check valve, and it ensures that water may freely flow into the system but not back out.

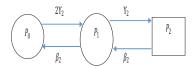


Figure3. Precision filter

$$\frac{d}{dt}P_0(t) = -2Y_2P_0 + \beta_2P_1$$
 (10)

$$\frac{d}{dt}P_1(t) = -(\Upsilon_2 + \beta_2)P_1 + 2\Upsilon_2P_0 + \beta_2P_2 \tag{11}$$

$$\frac{dt}{dt}P_{1}(t) = -(Y_{2} + \beta_{2})P_{1} + 2Y_{2}P_{0} + \beta_{2}P_{2}$$

$$\frac{d}{dt}P_{2}(t) = -\beta_{2}P_{2} + Y_{2}P_{1}$$
Under steady state, equation (10) - (12) reduces to
$$\frac{2Y_{2}}{dt}P_{2}(t) = -\frac{2Y_{2}}{dt}P_{2}(t) =$$

(13)

Substituting (13) into (12)
$$P_2 = \frac{2Y_2^2}{\vartheta_2^2}$$
(14)

$$P_2 = \frac{2\Upsilon_2^2}{\vartheta_2^2}$$

$$P_0$$

$$(14)$$

Using normalization condition

$$P_0 + P_1 + P_2 = 1$$
 (15)  
Substituting (13) and (14) into (15) we have

$$P_0 + \frac{2Y_2}{\vartheta_2} P_0 + \frac{2Y_2^2}{\vartheta_2^2} P_0 = 1$$
 (16)

$$P_0 = \frac{\beta_2^2}{\beta_2^2 + \gamma_2^2 + 2\beta_2 \gamma_2} \tag{17}$$

# Precision filter (pretreatment)

Precision filters can remove extremely small colloidal particles in water, such that the turbidity reached 1 degree, although this is still per milliliter of water for hundreds of thousands of colloidal particles with sizes ranging from 1 to 5 microns, the filter's pressure to extract water after To meet the following process of the water demands of the protection of the following long-running processes, small particles with a particle size of 100 microns or less are used to further reduce turbidity.

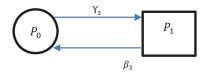


Figure 4. Carbonated filter

$$\frac{d}{dt}P_0(t) = -\Upsilon_3P_0 + \beta_3P_1 \tag{18}$$

$$\frac{d}{dt}P_1(t) = -\beta_3P_1 + \Upsilon_3P_0 \tag{19}$$
Under steady state, equation (18) and (19) reduces to

$$\frac{d}{dt}P_1(t) = -\beta_3 P_1 + Y_3 P_0 \tag{19}$$

$$-Y_3P_0 + \beta_3P_1 = -\beta_3P_1 + Y_3P_0$$
 (20)

Under steady state, equation (18) and (19) reduces to
$$-Y_3P_0 + \beta_3P_1 = -\beta_3P_1 + Y_3P_0 \qquad (20)$$

$$2\beta_3P_1 = 2Y_3P_0 \qquad (21)$$

$$P_1 = \frac{Y_3}{\beta_3}P_0 \qquad (21)$$
Now using permulication condition

Now using normalization condition

$$P_0 + P_1 = 1 (22)$$

Substituting (21) into (22) we have

$$P_0 + \frac{\hat{Y}_3}{\beta_3} P_0 = 1 \tag{23}$$

$$P_0 = \frac{\beta_3}{\beta_3 + \Upsilon_3} \tag{24}$$

# Carbon filter (pretreatment)

Activated carbon filters are used to remove water colors, odors, and a wide range of chemical and biological species, lowering the residual value of water and pesticide pollution, as well as other potentially hazardous pollutants. The construction of activated carbon filters and quartz sand filters differs in that the activated carbon's strong adsorption capacity is placed within the quartz sand filter for removal without the need to filter out organic material, as well as adsorption of residual chlorine in the water, using water containing less than or the chlorine equivalent There are various forms of membrane degradation, and reverse osmosis membranes are especially susceptible to chlorine at 0.1ML/M3 and

SDI less than or equal to 4. Furthermore, the surface of activated carbon generates non-crystalline portions of certain oxygen-containing functional groups during the activation process, and these functional groups can have chemical properties. Adsorption of activated carbon catalytic oxidation of bad news, which may efficiently remove a number of metal ions in water.

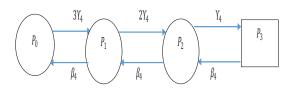


Figure 5. Reverse osmosis membrane

$$\frac{d}{dt}P_0(t) = -3Y_4P_0 + \beta_4P_1 \tag{25}$$

$$\frac{d}{dt}P_1(t) = -(2\Upsilon_4 + \beta_4)P_1 + 3\Upsilon_4P_0 + \beta_4P_2 \tag{26}$$

$$\frac{dt}{dt}P_2(t) = -(Y_4 + \beta_4)P_2 + 2Y_4P_1 + \beta_4P_3 \tag{27}$$

$$\frac{d}{dt}P_{0}(t) = -3Y_{4}P_{0} + \beta_{4}P_{1}$$
(25)
$$\frac{d}{dt}P_{1}(t) = -(2Y_{4} + \beta_{4})P_{1} + 3Y_{4}P_{0} + \beta_{4}P_{2}$$
(26)
$$\frac{d}{dt}P_{2}(t) = -(Y_{4} + \beta_{4})P_{2} + 2Y_{4}P_{1} + \beta_{4}P_{3}$$
(27)
$$\frac{d}{dt}P_{3}(t) = -\beta_{4}P_{3} + Y_{4}P_{2}$$
(28)
Under steady state, equation (25) - (28) reduces to
$$P_{1} = \frac{Y_{4}}{\beta_{4}}P_{0}$$
(29)

$$P_1 = \frac{\Upsilon_4}{\beta_*} P_0 \tag{29}$$

Substituting (29) into (26)

$$P_2 = \frac{\Upsilon_4^2}{\beta_4^2} P_0 \tag{30}$$

Substituting (29) into (26)
$$P_2 = \frac{\Upsilon_4^2}{\beta_4^2} P_0 \tag{30}$$
Substituting (30) into (28)
$$P_3 = \frac{\Upsilon_4^3}{\beta_4^3} P_0 \tag{31}$$
Using normalization condition

Using normalization condition

$$P_0 + P_1 + P_2 + P_3 = 1 (32)$$

Using normalization condition
$$P_0 + P_1 + P_2 + P_3 = 1$$
Substituting (29), (30) and (31) into (32) we have
$$P_0 + \frac{\gamma_4}{\beta_4} P_0 + \frac{\gamma_4^2}{\beta_4^2} P_0 + \frac{\gamma_4^3}{\beta_4^3} P_0 = 1$$

$$P_0 = \frac{\beta_4^3}{\beta_4^3 + 6\gamma_4^3 + 3\beta_4^2 \gamma_4 + 6\beta_4 \gamma_4^2}$$
(33)

# **RO** Membrane

Desalination using reverse osmosis technology, reverse osmosis membrane had only 0.001 micron pore size, may be detrimental to remove dissolved sediments and bacteria, viruses, and such forth, desalination rate greater than 99.6%, In line with national standards to generate clean water, the host section comprises the security filter, high-pressure pump, and reverse osmosis membrane. The high-pressure pump is the host of one of the main equipment for the reverse osmosis membrane elements to offer appropriate protection. Apply pressure to overcome the penetration resistance to pressure and run to achieve the rating water.

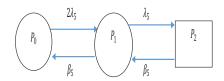


Figure 6. Water generating tank

$$\frac{d}{dt}P_{0}(t) = -Y_{5}P_{0} + \beta_{5}P_{1}$$
(34)
$$\frac{d}{dt}P_{1}(t) = -(2Y_{5} + \beta_{5})P_{1} + Y_{5}P_{0} + \beta_{5}P_{2}$$
(35)
$$\frac{d}{dt}P_{2}(t) = -\beta_{5}P_{2} + Y_{5}P_{1}$$
(36)
Under steady state, equation (34) - (36) reduces to
$$P_{t} = \frac{2Y_{5}}{2}P_{5}.$$
(37)

$$\frac{d}{dt}P_1(t) = -(2Y_5 + \beta_5)P_1 + Y_5P_0 + \beta_5P_2 \tag{35}$$

$$\frac{d}{dt}P_2(t) = -\beta_5 P_2 + \Upsilon_5 P_1 \tag{36}$$

$$P_1 = \frac{2Y_5}{g_-} P_0 \tag{37}$$

Substituting (37) into (36)

$$P_2 = \frac{2Y_5^2}{\beta_5^2} P_0 \tag{38}$$

Using normalization condition

$$P_0 + P_1 + P_2 = 1 (39)$$

Substituting (37) and (38) into (39) we have

$$P_{0} + \frac{Y_{2}}{\beta_{5}} P_{0} + \frac{Y_{5}^{2}}{\beta_{5}^{2}} P_{0} = 1$$

$$P_{0} = \frac{\beta_{5}^{2}}{\beta_{5}^{2} + Y_{5}^{2} + 2\beta_{5}Y_{5}}$$
(40)

$$P_0 = \frac{\beta_5^2}{\beta_5^2 + \gamma_5^2 + 2\beta_5\gamma_5} \tag{41}$$

# Water generating tank

The water is now clean, and it is pumped into a tank, where it is held under pressure until the faucet is switched on. The tank is equipped with two bladders that pressurize the water and allow it to enter and exit as needed. The tank is under continuous pressure, and water only fills it to around two-thirds of the input pressure. A pressurized air bladder lies at the bottom of the tank, and a butyl water bladder, a thick material similar to the inside lining of a steel food can, rests on top. When you turn on the faucet, the air pressure delivers a steady stream of water out while the intake valve opens to allow more water in, keeping a constant level of pressure operating the pump.

# 4. Materials and methods

All of the metrics mentioned in this paper are only applicable in the steady-state era, when all failure and repair rates are exponentially distributed.

# 4.1 List of notations and definitions

P<sub>0</sub>: Represent the initial state of the system working in full capacity state.

 $P_1$ : Represent the state in which one parallel unit is failed  $P_2$ : Represent the state in which two consecutive parallel unit is failed

 $P_3$ : Represent the state in which three consecutive parallel unit is failed

 $P_{\Delta}$ : Represent the state in which four consecutive parallel unit is failed

Represent the failure rates subsystems  $\beta_{j=1,2,...,4}$ Represent the repair rates subsystems  $P_{x}(t)$ Probability to remain at xth state at time t  $\frac{d}{dt}P_x(t)$ , x=0,1,2,3. Represent the derivative with respect

R(t): Reliability function M(t): Maintainability function MTTF: Mean time to failure MTTR: Mean time to repairs

d: Dependability

 $D_{min}$ : Dependability ratio  $A_{sys}$ : System availability A<sub>i: i=1,2,3</sub> Units in Subsystem 1 B<sub>j: j=1,2</sub> Units in Subsystem 2 C: Units in Subsystem 3  $D_{k: k=1,2,3}$  Units in Subsystem 4

E<sub>n: n=1,2</sub> Units in Subsystem 5

#### 4.2 Reliability

The chance that a device will run without failure for a particular period of time is referred to as reliability under the operational conditions indicated.

$$R(t) = \int_{t}^{\infty} f(x)dx$$
 Reliability function (42)

# **4.3 MTBF**

Mean Time between Failures (MTBF): The mean time between failures refers to the average length of satisfactory system functioning. The MTBF is the reciprocal of the constant failure rate or the ratio of the test time to the number of failures when the failure rate is relatively stable across the operational duration.

MTBF = 
$$\int_0^\infty R(t)dt = \int_0^\infty e^{-\theta t} = \frac{1}{\theta}$$
 Mean Time between Failure (43)

# **4.4 MTTR**

Mean Time between repairs (MTTR): is the reciprocal of the system repair rate.

$$MTTR = \frac{1}{\beta} \tag{44}$$

Mean Time to Repair

# 4.5 Availability

Availability: Availability is a performance criterion for repairable systems that takes into account the system's reliability as well as its maintainability. It is defined as the possibility that the system will work properly when required.

Availability
$$= \frac{\text{Life time}}{\text{Total time}} = \frac{\text{Life time}}{\text{Life time+Repair time}} = \frac{\text{MTTF}}{\text{MTTF+MTTR}}$$
(45)

#### 4.6 Maintainability

Maintainability: is a design, installation, and operation feature defined as the chance that a machine can be kept in, or returned to, a particular operational condition within a certain time interval when maintenance is required.

$$M(t) = 1 - e^{(-\frac{-t}{MTTR})}$$
Maintainability (46)

**4.7 Dependability:** Dependability was mentioned as a design feature. It evaluates performance based on average failure and repair rates, as well as reliability and availability. The advantage of dependability is that it enables for cost, reliability, and maintainability comparisons. For random variables with exponential distributions, the dependability ratio is stated in table 4:

# 5. RAMD Analysis for Subsystems as well as the entire system

# Dependability of subsystem 1

Dependability of subsystem 1
$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{lnd}{d-1}} - e^{-\frac{d \ln d}{d-1}})$$

$$d = \frac{\beta}{\lambda} = \frac{MTBF}{MTTR}$$

$$\beta_{s} = 0.53$$
(47)

$$d = \frac{\beta}{\lambda} = \frac{MTBF}{MTTP} \tag{48}$$

$$d_1 = \frac{\beta_1}{\lambda_1} = \frac{0.53}{0.006} = 88 \tag{49}$$

$$D_{\min(s1)} = 1 - (\frac{1}{87})(e^{-0.05146} - e^{-4.52880})$$
 (50)

$$D_{\min(s1)} = 0.98921 \tag{51}$$

# Dependability of subsystem 2

Dependability of subsystem 2
$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{lnd}{d-1}} - e^{-\frac{d \ln d}{d-1}})$$

$$d = \frac{\beta}{\gamma} = \frac{MTBF}{MTTR}$$

$$\beta_{2} = 0.82$$
(52)

$$d = \frac{\beta}{\gamma} = \frac{MTBF}{MTTR} \tag{53}$$

$$d_2 = \frac{\beta_2}{\gamma_2} = \frac{0.82}{0.0093} = 88 \tag{54}$$

$$d_{2} = \frac{\beta_{2}}{\gamma_{2}} = \frac{0.82}{0.0093} = 88$$

$$D_{\min(s2)} = 1 - (\frac{1}{105})(e^{-0.05146} - e^{-4.52880})$$
(55)

$$D_{\min(s2)} = 0.98921 \tag{56}$$

Dependability of subsystem 3
$$D_{min} = 1 - \left(\frac{1}{d-1}\right) \left(e^{-\frac{lnd}{d-1}} - e^{-\frac{d \ln d}{d-1}}\right)$$

$$d = \frac{\beta}{\gamma} = \frac{MTBF}{MTTR}$$

$$\beta = \frac{1}{16}$$
(58)

$$d = \frac{\rho}{\gamma} = \frac{MTBF}{MTTR} \tag{58}$$

$$d_3 = \frac{\beta_3}{\gamma_3} = \frac{1.16}{0.002} = 580 \tag{59}$$

$$D_{\min(s3)} = 1 - (\frac{1}{579})(e^{-0.01099} - e^{-6.37402})$$

$$D_{\min(s3)} = 0.99829$$
(60)

$$D_{\min(s3)} = 0.99829 \tag{61}$$

$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{\ln d}{d-1}} - e^{-\frac{d \ln d}{d-1}})$$
 (62)

$$d = \frac{\beta}{\lambda} = \frac{MTBF}{MTTR} \tag{63}$$

Dependability of subsystem 4
$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{lnd}{d-1}} - e^{-\frac{d \ln d}{d-1}}) \qquad (62)$$

$$d = \frac{\beta}{\lambda} = \frac{MTBF}{MTTR} \qquad (63)$$

$$d_4 = \frac{\beta_4}{\gamma_4} = \frac{0.42}{0.005} = 84 \qquad (64)$$

$$D_{\min(s4)} = 1 - \left(\frac{1}{83}\right) (e^{-0.05338} - e^{-4.48420})$$
(65)

$$D_{\min(s4)} = 0.98871 \tag{66}$$

# Dependability of subsystem 5

Dependability of subsystem 5
$$D_{min} = 1 - (\frac{1}{d-1})(e^{-\frac{lnd}{d-1}} - e^{-\frac{d \ln d}{d-1}}) \qquad (67)$$

$$d = \frac{\beta}{\gamma} = \frac{MTBF}{MTTR} \qquad (68)$$

$$d = \frac{\beta}{\gamma} = \frac{MTBF}{MTTR} \tag{68}$$

$$d_5 = \frac{\beta_5}{\gamma_5} = \frac{0.27}{0.0032} = 84 \tag{69}$$

$$D_{\min(s5)} = 1 - (\frac{1}{83})(e^{-0.05338} - e^{-4.48420})$$
 (70)

$$D_{\min(s5)} = 0.98871 \tag{71}$$

# System dependability

$$D_{\min(sys)} = D_{\min(s1)} \times D_{\min(s2)} \times D_{\min(s3)} \times D_{\min(s4)}$$

$$\times D_{\min(s5)} \tag{72}$$

$$D_{\min(sys)} = 0.99166 \times 0.99098 \times 0.99639$$

$$\times 0.99423 \times 0.98252$$

$$D_{\min(sys)} = 0.95649 \tag{73}$$

### **System Reliability**

$$R_{sys}(t) = R_{s1}(t) \times R_{s2}(t) \times R_{s3}(t) \times R_{s4}(t)$$
 (74)

# **System Availability**

Arranged in series, failure of one cause the complete failure of the system.

$$A_{svs} = A_{s1} \times A_{s2} \times A_{s3} \times A_{s4} \times A_{s5} \tag{76}$$

$$= (\frac{\beta_1^3}{\beta_1^3 + 6Y_1^3 + 3\beta_1^2 Y_1 + 6\beta_1 Y_1^2})$$

$$\times (\frac{\beta_2^2}{\beta_2^2 + Y_2^2 + 2\beta_2 Y_2}) \times (\frac{\beta_3}{\beta_3 + Y_3})$$

$$\times (\frac{\beta_3^3}{\beta_4^3 + 6Y_4^3 + 3\beta_4^2 Y_4 + 6\beta_4 Y_4^2})$$

$$\times (\frac{\beta_5^2}{\beta_5^2 + Y_5^2 + 2\beta_5 Y_5})$$
Agree = 0.97547 \times 0.96928 \times 0.99633 \times

$$\times \qquad (\frac{\beta_5^2}{\beta_5^2 + \gamma_5^2 + 2\beta_5 \gamma_5})$$

$$A_{SYS} = 0.97547 \times 0.96928 \times 0.99633 \times 0.98222 \times 0.94425 = 0.35631$$
 (78)

# System Maintainability

$$M_{sys}(t) = M_{s1}(t) \times M_{s2}(t) M_{s3}(t) \times M_{s4}(t)$$

$$\times M_{s5}(t)$$

$$- (1 - e^{-\beta_1(t)}) \times (1 - e^{-\beta_2(t)})$$
(79)

$$\begin{array}{l}
\times \left(1 - e^{-\beta_3(t)}\right) \\
\times \left(1 - e^{-\beta_4(t)}\right) \times (1 \\
- e^{-\beta_5(t)})
\end{array} (80)$$

$$1 - \rho^{-}(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5)t$$

$$M_{sys}(t) = M_{s1}(t) \times M_{s2}(t)M_{s3}(t) \times M_{s4}(t)) \times M_{s5}(t)$$

$$= (1 - e^{-\beta_1(t)}) \times (1 - e^{-\beta_2(t)}) \times (1 - e^{-\beta_3(t)}) \times (1 - e^{-\beta_3(t)}) \times (1 - e^{-\beta_5(t)})$$

$$1 - e^{-(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5)t}$$

$$= (1 - e^{-1.59(t)}) \times (1 - e^{-1.64(t)}) \times (1 - e^{-1.16(t)}) \times (1 - e^{-1.26(t)})) \times (1 - e^{-0.54(t)})$$

$$(81)$$

=1 - 
$$e^{-2.06(t)}$$
 (82)  
MTBF =  $\int_0^\infty R(t)dt = \int_0^\infty e^{-\theta t} = \frac{1}{\theta}$  Mean Time between Failure

#### 6. Definition of some terms

#### **6.1 MTTR**

**Mean Time between repairs (MTTR):** is the reciprocal of the system repair rate.

$$MTTR = \frac{1}{\beta}$$

Mean Time to Repair

# 6.2 Reliability

The chance that a device will run without failure for a particular period of time is referred to as reliability under the operational conditions indicated.

$$R(t) = \int_{t}^{\infty} f(x)dx$$
Reliability function

**6.3 Mean Time between Failures (MTBF):** The average period of good system functioning is referred to as the mean time between failures. When the failure rate is reasonably consistent over the operating period, the MTBF is the reciprocal of the constant failure rate or the ratio of the test time to the number of failure

**6.4 Availability:** Availability is a performance criterion for repairable systems that takes into consideration both the system's dependability and maintainability. It is defined as the likelihood that the system will function properly when it is needed.

Availability = 
$$\frac{\text{Life time}}{\text{Total time}} = \frac{\text{Life time}}{\text{Life time+Repair time}} = \frac{\text{MTTF}}{\text{MTTF+MTTR}}$$

**6.5 Maintainability**: Is a design, installation, and operation feature that is generally stated as the likelihood that a machine can be kept in, or returned to, a given operational condition within a specified time interval when maintenance is necessary.

$$M(t) = 1 - e^{(-\frac{-t}{MTTR})}$$
  
Maintainability function

**6.6 Dependability:** dependability was stated as a design element It assesses performance by utilizing average failure and repair rates, as well as dependability and availability. The benefit of dependability is that it allows for the comparison of cost, reliability, and maintainability. The dependability ratio for random variables with exponential distribution is as follows:

$$eta = Repair\ rate$$
 ,  $Y=$  Failure rate  $d = rac{eta}{Y} = rac{MTBF}{MTTR}$ 

# 7. Discussion

Tables 2 and 3 illustrate the reliability and maintenance characteristics of all subsystems. Table 4 lists all of the additional RAMD measures. The system's reliability after 40 days of operation is only 0.10731, according to the numerical analysis in table 2 and the corresponding

values in figure7. It occurred as a result of the least dependable subsystems one and four, namely the raw water tank and RO membrane, both of which had corresponding values of just 0.48675 and 0.54881, respectively. It is advised in this situation that poor performance be given more attention and that appropriate maintenance procedures be created to improve their reliability. After 50 days, the maintainability of subsystem 5 (water producing tank) is only 0.99548 which is the least probability of the subsystem to be maintained or repair after fault. As a result of the subsystem's maintainability, the system's maintainability is low. Although the maintainability of the remaining subsystems appears to be excellent, greater attention to the subsystem is required by providing additional redundancy and potential replacement of the afflicted subsystem. The time-dependent reliability behavior of numerous subsystems, as well as the variability in their failure rates, were depicted in Tables 5, 6, 7, 8, and 9. The most important, extremely sensitive component is the R.O Membrane, require specific care in order to increase system dependability. Regular maintenance plans that correctly monitor the failure rates of the R.O Membrane subsystem, as discussed previously, will undoubtedly improve the efficacy and operating time of the reverse osmosis water treatment system.

# 8. Conclusion

RO is an essential technique for desalinating seawater to produce drinking water. The failure behavior of a desalination system's components determines its performance. Because the RO system is designed to be low-power, the dependability of its subsystems must be maintained at a high level through good design and material selection in order for the plant to operate continuously. This research looked into the system's availability and reliability, as well as other aspects including MTTF, MTTR, dependability analysis, and maintainability. More investigation into the case of redundancy in precision filters and carbonated filters is needed.

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