

# Dynamic Reliability Analysis of Moment Resisting Concrete Frames

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## Abstract

A comprehensive reliability analysis of moment resisting frame is presented in this article in which uncertainties in loadings, material properties, and member cross section properties are included. Performance of designed structures and their reliability in real earthquakes is an important issue these days. In this research study, two moment resisting concrete frame structures, which is the most common type of structures in Iran, are designed based on Section 9 of the Iran National Building Regulations and its analytical model is constructed. Using OpenSEES software, the finite element simulations and reliability analyses are performed with different distribution functions for the mentioned parameters. By using the generated parameters, the ultimate limit functions for structural failure are examined. Results show that incorporating uncertainties in loading, cross section dimension, and material properties could result in a considerable impact on reliability of designed structures.

**Keyword:** Dynamic Analysis, Moment Resistant Frame, Reliability Analysis, Simulation Methods

## Nomenclature

$f_y$	Steel Yield Stress
$f'_c$	Concrete Characteristic Strength
$B_w$	Beam Width
$B_D$	Beam Depth
$C_w$	Column Width
$C_D$	Column Depth
DL	Dead Load
$LL_M$	Maximum Live Load
LL	Live Load
$Eq_{LS}$	Life Safety Level Earthquake
$Eq_{SL}$	Service Level Earthquake
$Eq_{IO}$	Immediate Occupancy Level Earthquake
$Eq_{CP}$	Collapse Prevention Level Earthquake
LS	Life Safety Level with probability of occurrence equal to 10% in 50 Years
SL	Service Level with probability of occurrence equal to 99.5% in 50 Years
IO	Immediate Occupancy Level with probability of occurrence equal to 50% in 50 Years
CP	Collapse Prevention Level with probability of occurrence equal to 2% in 50 Years
$VR_D$	Parameter which is variable based on structural design

## Introduction

The process of design and analysis of structures always has uncertainties that are usually examined deterministically for simplification [1, 2]. The impact of these uncertainties on these processes is normally shown by reliability index or probability of failure. In this paper, the reliability index of concrete frame structure under the effect of dynamic load and how the values of different variables of uncertainties such as material properties and structural geometry and changes in their coefficients of variation affect the reliability index are investigated.

The combination of dead load, live load and dynamic effect of earthquake load has been used in this research and structural analyses have been performed using OpenSEES software based on selected records and limit state functions of relative displacement failure modes [3, 4]. In this paper, among the simulation methods, Monte Carlo analysis and among the methods of reducing variance, Importance Sampling technique are presented [5, 6] and the results of failure probability and reliability index at different levels of reliability analysis are examined [6, 7].

In case of assessing the reliability of Iranian designed structures, there are researches that works on structures which designed by Iranian design codes. Yazdani et al. [8] work on assessing reliability index of steel moment frame structures. Jahani [9] work on beam elements which designed by Section 9 Iranian design code. Yazdani et al. [10] asses the reliability index of concrete

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moment frame by use of incremental dynamic analysis. In the last-mentioned research, the concrete structures designed by Section 9 of Iranian code. In this study, the numerical models are as in the Yazdani et al. [10].

### Structural Reliability

The possibility of structural rupture is one of the main issues in structural engineering. This can be examined by the reliability theory of structures. The performance of any structure can be expressed by a function of the main random variables of the structure which is called the limit state function. So that the positive value of the limit state function indicates safety and the negative value of the limit state function indicates rupture [2, 11]. Failure probability assessment and the possibility of structural rupture are key issues in structural reliability analysis. The concept of reliability has been used in various disciplines and has been interpreted in different ways. The most commonly accepted definition of reliability is that reliability is the probability that an item will perform its function in a given period of time under certain conditions for which it was designed and created). There are four different issues in the definition of reliability: probability, performance required, time, performance conditions. In the analysis and design of structures, reliability is defined as the probability that the structural failure does not exceed a certain limit during the useful life of the structure [1, 2].

### Importance Sampling Method

The probability of structural rupture is an integral function of the common probability density of all input random variables on the rupture domain. In the classical simple Monte Carlo simulations, random points are generated using the cumulative probability distribution functions. Because simple Monte Carlo generates sample points over all random variable spaces without any focus, this method requires a large number of sample points. Many methods have been proposed to reduce the number of sample points in Classical Monte Carlo method. Since these methods reduce the variance of the responses obtained, so they are also called "variance reduction methods". Importance Sampling is one of these methods of reducing variance [2, 5, 6]. In Importance Sampling method, the sampling process focuses on the failure area and this leads to faster convergence to the exact failure probability. For Importance Sampling, the probability of failure (rupture) can be written as follows [5, 6]:

$$P_f = \int_{g(x)<0} \frac{f_x(x)}{h_x(x)} h_x(x) d(x) \quad (1)$$

So that  $h_x(x)$  is a new sampling density function. The Equation (1) can be rewritten as in Equation (2):

$$P_f = \frac{1}{N} \sum_{j=1}^N I(x_j) \frac{f_x(x_j)}{h_x(x_j)} \quad (2)$$

As  $I_{(x)}$  is an indicator of the rupture or non-rupture of the structure during due to simulation process, as follows:

$$I(x_j) = \begin{cases} 0 & \text{if } G(x_j) > 0 \\ 1 & \text{if } G(x_j) \leq 0 \end{cases} \quad (3)$$

In Equation (3), when the simulation leads to rupture, the function  $I_{(x)}$  takes the value of one, and when the simulation leads to non-rupture, the function  $I_{(x)}$  takes the zero value. The main issue in the Important Sampling method is the selection of the important sampling density function that reduces the required sample points. The main idea of this technique is to obtain the sampling density function near the most probable design point [5, 6, 11]. For Importance Sampling, many different methods have been proposed that require the design point or the shape of the limit state function to obtain the appropriate sampling density function [2, 6, 11]. In most practical cases, the design point or shape of the limit state function is not clear. In this paper, by presenting a new method for Importance Sampling, the sampling density function is obtained by collecting information during the sampling process in an approach that directs the sampling to more important areas.

### Density Functions and Random Variables

In order to model the uncertainty of the design parameters in this study, different distribution functions have been used. The Random Variables (uncertainties) in this study are categorized such as the geometric shape of the sections and the characteristics of the materials as well as the loads applied to the structure. For each of these parameters, sample values are used using different distribution functions such as normal and log-normal distributions. All geometrical and material parameters, type of distribution, mean values and their coefficient of variations are shown in Table 1.

**Table 1.** Geometrical and Material Parameters

Parameter	Distribution Type	Mean	Coefficient of Variation
$f_y$	logNormal	4000	0.05
$f'_c$	logNormal	200	0.025
$B_w$	Normal	$VR_D$	0.10
$B_D$	Normal	$VR_D$	0.0667
$C_w$	Normal	$VR_D$	0.05
$C_D$	Normal	$VR_D$	0.05

As shown in Table 1, the steel yield strength,  $f_y$ , of each member is a lognormal random variable with mean 4000 kg/cm<sup>2</sup>, 5% coefficient of variation, and no correlation with  $f_y$  of the other members. The compressive strength of concrete,  $f'_c$ , for each member is a lognormal random variable same as the steel strength with mean 200 kg/cm<sup>2</sup>, 2.5% coefficient of variation, and and is not correlated with other parameters. The cross-sectional properties  $B_w$ ,  $B_D$ ,  $C_w$ ,

and  $C_D$  of the cross-section of each member are modelled as uncorrelated normal random variables with variable mean based on the designed shapes as depicted in Fig. 1, and have 10%, 6.67%, 5%, and 5% coefficient of variation respectively.

As mentioned above, in this study, the correlation between the parameters is not considered. The Turkstra's method has been used for the load combination [12]. Loading such as dead and live load and earthquake loading on structure and their distribution, mean values and their coefficient of variations are shown in Table 2.

For loadings uncertainties same as other parameters distribution and means are considered. DL is modelled as uncorrelated normal random variable with mean 450 kg/m<sup>2</sup> and coefficient of variation of 10%. Gamma distribution is used for LL with variable mean based on loading code and coefficient of variation of 31%.  $LL_M$ ,  $Eq_{LS}$ , and  $Eq_{SL}$  are modeled as Extreme Type I random variables with means 200kg/m<sup>2</sup> for  $LL_M$  and variable mean for the two latter based on Table. 3 and 12%, 20%, and 20% coefficient of variation for each one respectively. Finally, for  $Eq_{IO}$  and  $Eq_{CP}$  lognormal random variable are used with mean based on Table. 3 and they have 60% coefficient of variation.

**Table 2.** Loading Parameters

Parameter	Distribution Type	Mean	Coefficient of Variation
DL	Normal	450	0.10
$LL_M$	Extreme Type I	200	0.12
LL	Gamma	Variable	0.31
$Eq_{LS}$	Extreme Type I	Table 3	0.20
$Eq_{SL}$	Extreme Type I	Table 3	0.20
$Eq_{IO}$	logNormal	Table 3	0.60
$Eq_{CP}$	logNormal	Table 3	0.60

Besides, for seismic analysis based on distribution functions, the mean acceleration values were used for each distribution according to Table 3 [13, 14, 10].

**Table 3.** Spectral Acceleration in Earthquake Level

Structure	Earthquake Level			
	SL	IO	LS	CP
5 Floors	0.245	0.414	0.827	0.943
8 Floors	0.154	0.259	0.436	0.553

Probability Distribution of Earthquake loading in both Collapse Prevention and Immediate Occupancy Cases are usually modelled by Log-Normal Distribution [10]. When the structural behaviour is changed from Collapse to Operational Mode, the uncertainty of Load Parameters is

often modelled by Extreme Type Distributions as well as Type I [19]. Furthermore, due to the higher uncertainties of Collapse behaviour compared to Operational Cases, it is recommended to use higher values of COV (about three times) rather than to Normal (Operational) earthquake loading.

Uncertainties of Strength parameters are selected based on common practice, referred in reliability textbooks [1, 19].

### Limit State Function

To model the failure of structures under different load conditions, the displacement limit function is defined in which according to each level of operation, the limit state function for that level is formulated as stated in Equation (4).

$$g(x, t) = \theta_{all} - \theta(x, t) \text{ for } 0 \leq t \leq t_{end} \quad (4)$$

Where  $\theta_{all}$  is the maximum allowable interclass displacement at the desired performance level,  $\theta$  is the interclass displacement during analysis, and  $t_{end}$  is the time interval of the analysis. Different values of  $\theta_{all}$  at different performance levels of the structures [7, 10] are presented in Table 4, which their values are defined according to the 2800 Iranian regulations [15] and FAMA instructions [7].

**Table 4.** Relative Inter-story Drifts

Structure	Earthquake Level			
	SL	IO	LS	CP
5 Floors	0.005	0.010	0.025	0.0297
8 Floors	0.005	0.010	0.020	0.0280

### Numerical Model Specifications

In this study, two moment resisting frame structures of 5 and 8 stories, 5 bays, with medium ductility designed according to the rules of national building regulations of Iran and regulations 2800 [10, 15, 16, 17] have been selected. The height of the stories of the structures is equal to 3 meters, the loading bay width of each level of the frames is equal to 4 meters, the middle bay width of the frames is equal to 5 meters, the side bay(s) width is equal to 4 meters, and structures are located at Tehran (with very high seismicity) and the soil classification is type II based on Iran and regulations 2800. The main natural periods of the structures are 0.69 and 0.89 seconds respectively. Geometric details and cross sections of two designed structures can be seen in Figure 1 [10]. Cross sections designed reinforcement details for beams and columns of structures are as in Table 5 and Table 6 for columns and beams respectively.

**Table 5.** Columns total reinforcement area (cm<sup>2</sup>)

Story	Column 3	Column 2	Column 1
5 story			
1	20.35	20.35	25.13
2	20.35	20.35	13.57
3	20.35	20.35	12.31
4	12.31	13.57	12.31
5	12.31	12.31	9.04
8 story			
1	20.35	25.13	30.41
2	20.35	20.35	20.35
3	20.35	20.35	20.35
4	20.35	20.35	16.08
5	25.13	20.35	12.31
6	20.35	16.08	12.31
7	9.04	13.57	12.31
8	10.17	9.04	9.04

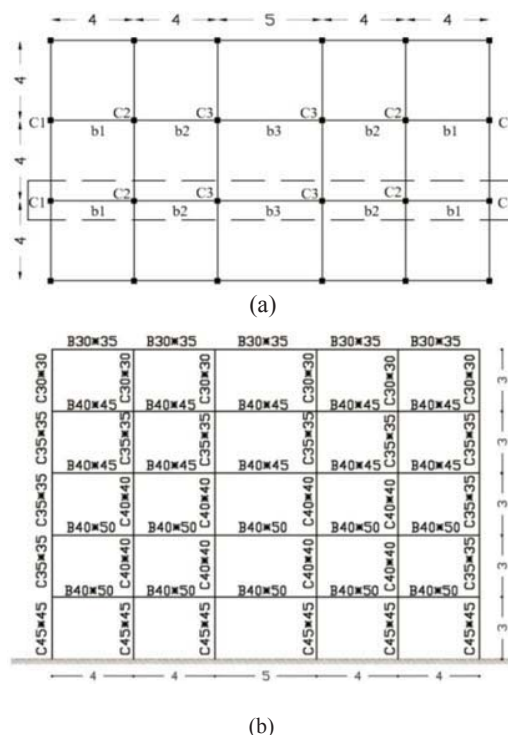
**Table 6.** Beams reinforcement area (cm<sup>2</sup>)

Story	Beam 3		Beam 2		Beam 1	
	Top	Bottom	Top	Bottom	Top	Bottom
5 story						
1	11.96	5.09	11.96	8.04	11.96	8.04
2	11.96	5.09	11.96	8.04	11.96	8.04
3	11.43	4.62	10.96	6.63	10.96	6.63
4	11.43	4.62	10.96	6.63	10.96	6.63
5	7.1	2.67	6.03	2.67	6.03	2.67
8 story						
1	10.96	6.63	10.96	8.83	10.96	8.83
2	10.96	6.63	10.96	8.83	10.96	8.83
3	10.96	6.63	10.96	8.83	10.96	8.83
4	10.96	6.63	10.96	8.83	10.96	8.83
5	9.58	6.16	9.58	6.16	9.58	6.16
6	9.17	4.62	8.57	5.09	8.57	5.09
7	6.94	3.08	6.94	4.62	6.94	4.62
8	6.94	2.36	5.81	2.36	5.81	2.36

### Structural Reliability Analysis

OpenSEES software has been used to perform both finite element analysis and structural reliability analysis [3, 4]. OpenSEES is an open-source structural analysis software which was created in 1990 and used widely by researchers because its simplicity and robustness. This software also has reliability analysis capability to perform structural reliability analysis. Structural and reliability analysis were performed with OpenSees 2.5.0 including nonlinearity and P-Delta effect. All members were subdivided into four displacement-based beam-column elements to capture the variation in curvature. Steel reinforcement models with Steel01 materials. Concrete01 material is used for concrete material, and a 30 percent

increase is used for core concrete compression strength. Gravity loading is applied as distributed uniform loading and seismic load is applied as time history of acceleration. The analyses performed in this research is Importance Sampling [5, 6], which were all performed by OpenSEES. For each structure, 10,000 analyses runs were performed to simulate the Importance Sampling method. For each sample, random variables were evaluated based on their distribution and the constructed structural model was analysed. It should be mentioned that each member had a unique random parameter for all parameters under consideration with no correlation with others. The procedure to do the reliability analysis is shown in Fig. 2. To verify the numerical model and validate the results of the created models in OpenSees, the first modal period of the constructed models compared with the IDARC2D-v7 [16] models in [10]. In [10], for IDARC2D-v7 the first modal periods are 0.72 and 0.87 sec for 5 and 8 story frames respectively. In this study, the modal periods are 0.71 and 0.89 sec for the OpenSees models which have good agreement with Yazdani [10].



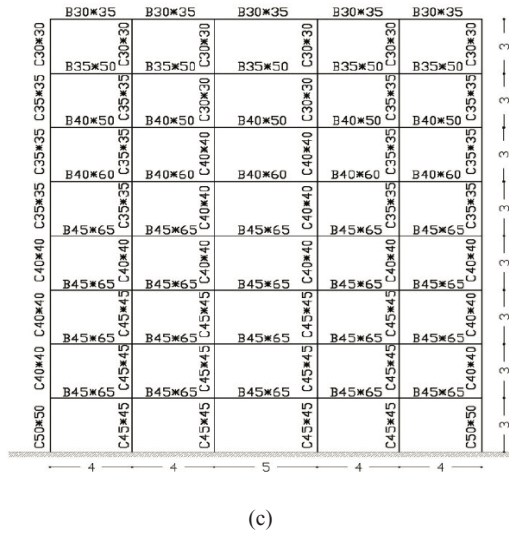


Figure 1. Details of Geometry and Cross Sections of Structures, a) plan, b) 5 story, and c) 8 story frame cross section details

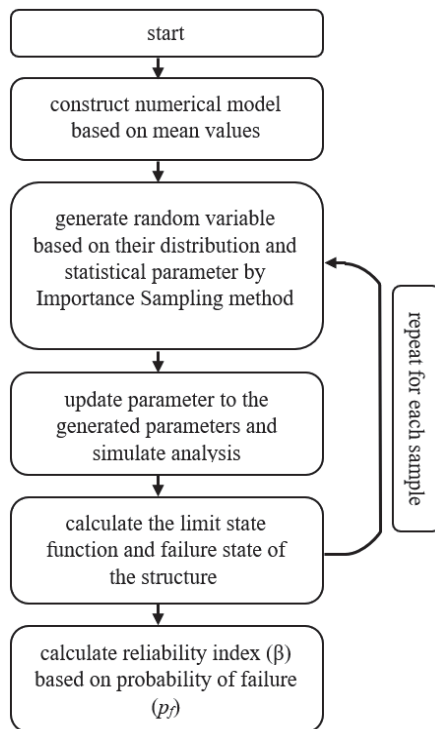


Figure 2. Reliability analysis flow diagram

### Results and Discussion

Structural and reliability analyses were performed using different distribution functions of parameters, by generating 10,000 set of samples for each structure. These sets of samples consist of parameters of beams, columns, and loading uncertainties in the Importance Sampling technique. The response of structures, which was designed based on design codes, evaluated at different levels of performance. The limit function, as mentioned before is defined in terms of relative inter story drift. Reliability index and failure probability of desired performance level are obtained as in Tables 7 and 8 respectively.

Table 7. Reliability index ( $\beta$ )

Structure	Earthquake Level			
	SL	IO	LS	CP
5 Floors	2.303	2.394	2.499	2.585
8 Floors	2.333	2.344	2.624	2.667

Table 8. Failure Probabilities ( $P_f$ )

Structure	Earthquake Level			
	SL	IO	LS	CP
5 Floors	0.0107	0.0083	0.0062	0.0049
8 Floors	0.0098	0.0095	0.0044	0.0038

### Conclusions

In this study, the effects of different uncertainties of concrete frame structures were investigated by OPENSEES software. By examining the effect of uncertainties and their different distribution functions used on these types of structures designed with Iranian regulations, the following results were obtained [15, 17, 18]:

- Considering the uncertainties and randomness of variables under study including loading and cross sections, with the desired limit state functions increases the probability of failure.
- Uncertainties in loading variables, according to the desired limit state function, have a greater impact than other structural variables.
- In all cases, inclusion of uncertainties increases the probability of failure, but there is still a safe margin in using Iranian National Building Regulations.
- Considering the effect of correlation between different strength and loading parameters is planned as future concept of this research.

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