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Life extension for a coherent system through cold standby and minimal repair policies for their independent components

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Abstract

We consider life extension for a class of coherent system consisting of independent components with an increasing failure rate functions. The maintenance action is applied in a fixed component called the target component. To this end, the minimal repair and cold standby actions are provided. We also consider two alternative policies for the target component. A component following a new random variable, and another following the same distributions of the target component. These policies obviously increase the reliability and life of the target component and consequently, the life and reliability of a coherent system are also increased. In this regard, the life of the system is also extended. Some numerical results considering these life extensions are presented.

Keyword: Coherent System, ColdS, Minimal Repair, Preventive Maintenance.

Introduction

A system called by coherent if and only if ([1, 2, 3])

I : If all units are failed, the system does not work

: If all units work, the system work.

: If a unit is improved, the performance of the system does not degrade.

: For all units, there exists a unit state vector dictating the state of the system.

Let $T = \psi(X_1,X_2,...,X_n)$ denote the lifetime of a coherent system consisting of independent components whose lifetimes are denoted by $X_1,X_2,...,X_n$. Thus there exist a multinomial expression representing the reliability of such a system [4]. For more details about definitions, structure, relation, and ...of a coherent system, we refer the reader to [1, chapter 4], [5] and [6] . The applications of a coherent system are very vast. In fact, many engineering systems, all of the k-out-of-n, series, and parallel systems with independent and identically distributed lifetimes of their components are excellent examples of coherent systems. Some practical applications of coherent systems are as follows:

: Communications system [7].

: Data processing systems [8].

: Tires of a car.

It is obvious that all customers are interested in reliable systems. Increasing the reliability of coherent systems are a favorite topic of many scholars. These methods were introduced by [5], and developed recently by [1, 7, 9, 10, 11, 12] . Among them, redundancy or maintenance actions can be enumerated. There exist many papers dealing with maintenance or redundancy theories for a special case of coherent systems like as k-out-of-n, parallel, and series systems but in the case of just coherent systems, the study numbers are few. The active redundancies for coherent systems and their dependent components are studied in [6, 13] . Navarro et al. [4], utilizing the copula functions, investigate three different policies of the minimal repair of failed components for a coherent system with dependent components. Optimal age replacement of a coherent system consisting of independent, identical, and increasing failure rate components are provided in [3].

In this study, we consider a coherent system with independent and heterogeneous components. The results can obviously be used in the case of homogeneous components. The components lifetime considered following distributions with increasing failure rate properties. In this class, the maintenance activities are applied for a fixed component. These activities involve the minimal repair (replaced the component when it fails by the same component at the same age), the perfect

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repair (replaced the component during its work with another component), and the cold standby (replaced the component when it fails with another component). The aim is to increase the reliability of the system and consequently increasing the mean to failure (MTTF) of the coherent systems. The rest of the paper is organized as follows. In section 2, we present the formulas of minimal repair and cold standby activities. Furthermore, the MTTF of a coherent system is provided as our objective functions. Sections 3,4 and 5, deal respectively with life extension of coherent systems taking minimal repair, perfect repair, and cold standby maintenance. Finally, the conclusion of our study is presented in section 6.

Model descriptions

Assume that X and Y be two non-negative independent variables respectively following absolutely continuous cumulative distribution functions (CDF) F and G. Thus the CDF of $X+Y$ (convolution) is given by ([4, 12]):

$$F * G(t) = 1 - \bar{F}(t) - \int_0^t \bar{G}(t-x) f(x)dx, \tag{1}$$

where f indicates the probability distribution function (pdf) of the random variable X. The relation 1 can be used in the cold standby procedure of a component with CDF F by component with CDF G. Under a perfect repair in cold standby process, it is obvious that $F = G$, and consequently we have

$$F ** G(t) = 1 - \bar{F}(t) - \int_0^t \bar{F}(t-x) f(x)dx, \tag{2}$$

Regarding minimal repair policy, the failed component $X = x$ with CDF F is replaced by a worked component with the age of x following the CDF G. Hence the conditional revolution of $X+Y$ is as follows ([4, 12]):

$$F\#G(t) = 1 - \bar{F}(t) - \int_0^t \frac{\bar{G}(t)}{G(x)} f(x)dx, \tag{3}$$

and for the case of $F = G$ it is easy to see that:

$$F\#\# G(t) = 1 - \bar{F}(t) - \int_0^t \frac{\bar{F}(t)}{F(x)} f(x)dx, \quad = 1- F(t) + F(t)\log F(t). \tag{4}$$

Here, assume that T represents the lifetime of a coherent system consisting of independent component lifetimes $X_1,X_2,\ldots,X_n$ following CDFs $F_1,F_2,\ldots,F_n$ respectively. Thus $F_T(t)$ is a multinomial expression of F_is [4, 5] . The MTTF of such a system is given by

$$\mu_t = \int_0^\infty [1 - F_T(t)]dt \tag{5}$$

In this study, we are going to investigate the aforementioned policies on the MTTF of a coherent system. In continue without loss of generality, the number of components is considered as $n = 3$. Furthermore, some main situations are listed and their coefficients are derived.

system	Structure	CDF($F_T(t)$)
I	$max(X_1,min(X_2,X_3))$	$F_1(t)(1-(1-F_1(t))(1-F_2(t)))$
II	$min(X_1,max(X_2,X_3))$	$1-(1-F_1(t))(1-F_2(t)F_3(t))$
III	$min(X_1,X_2,X_3)$	$1-(1-F_1(t))(1-F_2(t))(1-F_3(t))$
IV	$max(X_1,X_2,X_3)$	$F_1(t)F_2(t)F_3(t)$

Table 1. A coherent system with 3components

The most used statistical distribution in reliability analysis is Weibull distribution. This distribution has so important properties that are extensively discussed in the literature. Based on the Weibull distribution, there were constructed so many flexible modified distribution that can be utilized in reliability modeling. For a comprehensive discussion on Weibull distribution and it's modified see [2, 14] . The Weibull distribution with shape parameter α and scale parameter λ denoted by $W(\alpha,\lambda)$ is given by

$$f_X(x) = \frac{\alpha x^{\alpha-1}}{\lambda^\alpha} \exp\left(-\left(\frac{x}{\lambda}\right)^\alpha\right), \tag{6}$$

If $\alpha > 1$ the distribution has an increasing failure rate feature and can be used for the modeling of components lifetime. In this study, we consider the scale parameter 1 and the shape parameter 1.5,2,2.5 respectively for components 1,2,3.

Minimal repair

Under a minimal repair policy, the failed component is replaced by another one having the same age. The alternative unit could have the same reliability with the failed component (4) or not (3). If T and μ_t denote the lifetime and MTTF of a coherent system, thus $F_T(t) = q(F_1(t),F_2(t),\ldots,F_n(t))$ where q is multinomial expression of CDFs of lifetime components $X_1,X_2,\ldots,X_n$. Moreover, it is directly seen that $\mu_t = \int_0^\infty [1 - F_T(t)]dt$. Now, consider a minimal repair action on the i–th component. The extended lifetime and extendedMTTF (EMTTF) of the new version of this system is given by

$$F_T\#^i G(t) = q\big(F_1(t), F_2(t), \ldots, F_t(t), \ldots, F_0(t)\big),$$

and

$$\mu_t\#^i G = \int_0^\infty [1 - F_T\#^i G(t)\,]dt.$$

and similarly

$$F_T\#\#^i F(t) = q\big(F_1(t), F_2(t), \ldots, F_t\#G(t), \ldots, F_0(t)\big),$$

and

$$\mu_t\#\#^iF_i = \int_0^\infty [1 - F\#^iG_T(t)\,]dt.$$

Minimal repair	Component 1	Component 2	Component 3
μ_t	1.050	1.050	1.050
Same	1.540	1.123	1.118
W(1.5,2)	2.314	1.155	1.165
W(2,2)	2.109	1.158	1.166
W(2.5,2)	2.019	1.160	1.169
W(3,2)	1.974	1.163	1.171

Table 2. The EMTTF of the system I under minimal repair policy

Minimal repair	Component 1	Component 2	Component 3
μ_t	0.707	0.707	0.707
Same	0.968	0.781	0.775
W(1.5,2)	1.053	0.846	0.848
W(2,2)	1.063	0.846	0.848
W(2.5,2)	1.072	0.848	0.849
W(3,2)	1.080	0.850	0.851

Table 3. The EMTTF of the system II under minimal repair policy

The numerical results due to the minimal repair policies for systems I and II are tabulated in Tables 2 and 3. The optimal EMTTF of corresponding systems is also represented. These values are obviously present the good performances of the minimal policy.

Cold standby

Under a cold standby process, the failed component is replaced by a new one. The alternative unit could have the same reliability with the failed component (2) or not (1). If T and μ_t denote the lifetime and MTTF of a coherent system, thus $F_T(t) = q(F_1(t),F_2(t),...,F_n(t))$ where q is multinomial expression of CDFs of lifetime components $X_1,X_2,...,X_n$. Moreover, it is directly seen that $\mu_t=\int_0^\infty[1 - F_T(t)]dt$. Now, consider a minimal repair action on the i–th component. The extended lifetime and extended MTTF (EMTTF) of the new version of this system is given by is system is given by

$$F_T *^i G(t) = q\big(F_1(t), F_2(t), \dots, F_1 * G(t), \dots, F_0(t)\big),$$

and

$$\mu_t *^i G = \int_0^\infty [1 - F_T *^i G_T(t)\,]dt.$$

and similarly

$$F_T **^i F(t) = q\big(F_1(t), F_2(t), \dots, F_t ** G(t), \dots, F_0(t)\big),$$

and

$$\mu_t **^i F_i = \int_0^\infty [1 - F **^i G_T(t)\,]dt.$$

Minimal repair	Component 1	Component 2	Component 3
μ_t	0.500	0.500	0.500
Same	0.624	0.600	0.586
W(1.5,2)	0.638	0.606	0.589
W(2,2)	0.643	0.609	0.591
W(2.5,2)	0.645	0.611	0.593
W(3,2)	0.646	0.611	0.593

Table 4. The EMTTF of the system III under cold standby process

The cold standby policy performing for systems III and IV in the numerical form are tabulated in Tables 4 and 5. The MTTF of these systems is extended in optimal form. The corresponding values are also provided.

Conclusion

A coherent system consisting of independent and repairable components are considered. The lifetime of their components is considered with increasing failure rate features. Three replacement policies including cold standby and minimal repair policies applying to these components are investigated. The aim of performing these actions is to extend the life of the system. The main question is when the operator acts

Minimal repair	Component 1	Component 2	Component 3
μ_t	1.320	1.320	1.320
Same	1.914	1.882	1.874
W(1.5,2)	2.756	2.749	2.751
W(2,2)	2.706	2.699	2.700
W(2.5,2)	2.698	2.690	2.692
W(3,2)	2.703	2.695	2.696

Table 5. The EMTTF of the system IV under cold standby process

preventive maintenance to achieve optimal EMTTF of the systems. The corresponding relations of these policies have been studied and the comparison between these policies is tabulated through some simulation studies.

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