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A Reliable Fracture Mechanics

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Abstract

A branch of human knowledge, which treats the behavior of cracked structures, is called fracture mechanics. Since there is no intact structure in the world, then the paramount importance of fracture mechanics in human life is accentuated. The main parameter of fracture mechanics is called crack compliance, which is the amount of flexibility added to the flexibility of the intact structure due to the presence of a crack with specified size. The compliance, similar to flexibility, is the sole characteristics of the cracked structure. In this way for a given structure with a specified crack, there should be a single compliance. Unfortunately, in classical fracture mechanics that is not the case! The number of crack compliances for a clacked structure is equal to the number of researchers who treated the case! This diversity in the results stems from the presence of epistemic uncertainty in the mathematical basis of classical fracture mechanics. In view of the need for remedy, the Abdolrasoul Ranjbaran Team (ART), investigated the case and proposed a reliable fracture mechanics, which is based on sound logical reasoning. The proposed reliable fracture mechanics is described in the presented paper. The paper is managed via fourteen titles as, introduction, the mathematical basis of the classical fracture mechanics, birthplace of the state based philosophy, strong form of governing equation, analytical solution by Laplace transform, the weak form equation, the finite element equation, logical basis of the state based philosophy, state functions, Persian curves, reliable crack compliance, energy release rate, stress intensity factor, and weight function for the stress intensity factor in sections one to fourteen respectively. The paper concludes with a list of cited references.

Keywords: Classical fracture mechanics, Reliable fracture mechanics, State based philosophy, Persian Curves, Flexibility, Crack compliance, Stress intensity factor, Energy release rate.

Introduction

A branch of human knowledge, which treats the behavior of cracked structures, is called fracture mechanics. Since there is no intact structure in the world, then the paramount importance of fracture mechanics in human life is accentuated. The main parameter of fracture mechanics is called crack compliance, which is the amount of flexibility added to the flexibility of the intact structure due to the presence of a crack with a specified size [1-38]. A brief review of the current state of classical fracture mechanics is as follows.

The development of classical fracture mechanics theory dates back to the studies of Inglis [1], Griffith [2], Westergaard [3] and Irwin (1957) [4] based on the concepts of linear elasticity. The stress solution for an elliptical hole in a semi-infinite plate subjected to uniform tension at far end is developed by Inglis [1]. The Inglis solution is used by Griffith [2] to introduce the energy criteria for fracture analysis based on the concept of energy release rate. Westergaard [3] derived the linear elastic solution for the stress field around a crack tip. The stress field ahead of a crack tip is described by means of only one parameter, the so called stress intensity factor (SIF), by Irwin [4]. He shown that the (SIF) is uniquely related to the energy release rate. The (SIF) has become the fundamental concept in classical fracture mechanics. Various models and criteria of fracture and crack growth have been formulated in terms of the (SIF) by Anderson et al. [5]. The crack problem is formulated as a boundary value problem of elasticity, and solved in terms of the displacement field and/or the stress field and the (SIF) is obtained from the expansion of the stress field in the vicinity of a crack tip. The weight function concept proposed by Bueckner [6] and Rice [7] has proven to be the most efficient method for calculating the (SIF) (as noted by Atroschenko [8]). The weight function (Green's function) represents the (SIF) for a crack subjected to a unit force applied to the crack surface. The (SIF) due to an arbitrary stress field can be determined by integrating the product of the weight function and the applied stress field over the crack domain. Because of singularities in the weight functions and nonlinearity of the stress field the classical process for computation of (SIF) is very difficult. During the last four decades, great attention has been paid by several authors to the diagnosis of cracks in rotating machinery. The review papers by Wauer [9], Gasch [10], Dimarogonas [11], and Papadopoulos [12] give valuable information and

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knowledge. The strain energy release rate method is used to determine the compliance of cracked circular shaft. The (SIF) for circular cross section is not available (This argument was perhaps correct up to 10 years ago, but now that is not the case, interested reader may refer to Ranjbaran [54] and related references!) consequently the shaft is considered to be a sum of elementary independent rectangular strips which are used in calculation of the compliance by integrating along the crack tip. If the crack depth exceeds the radius of the shaft the energy release rate method has singularity near the edges of the crack tip, Abraham et al. [13]. Dimarogonas [14] stated that this problem does not reflect reality, but it is due to the method of formulation. The difficulty in computation of the (SIF), and the fact that most of the problems stem from the methods of calculation, led the present authors to launch a new investigation to remedy the problems as described in this paper.

The compliance, similar to flexibility, is the sole characteristics of the cracked structure. In this way for a given structure with a specified crack, there should be a single compliance. But, in classical fracture mechanics that is not the case [1-38]! The number of crack compliances for a clacked structure is equal to the number of researchers who treated the case! This diversity in the results stems from the presence of epistemic uncertainty in the mathematical basis of classical fracture mechanics. In view of the need for remedy, the Abdolrasoul Ranjbaran Team (ART), investigated the case and proposed a reliable fracture mechanics, which is based on sound logical reasoning.

Mathematical Basis of Classical Fracture Mechanics

First, Mathematical basis of classical fracture mechanics is digested in determination of three parameters, i.e. the stress intensity factor (K), the energy released due to creation of a unit surface area of crack or the energy release rate (G), and the flexibility added to the flexibility of the structure or the crack compliance(c_S). The stress intensity factor, at the forefront of classical fracture mechanics, is defined in Eq. (1), in which (W) is the weight function [6, 7] and (σ) is the effective stress.

$$K = \int_S W \sigma dx \tag{1}$$

The second parameter, i.e. the energy release rate, is defined in terms of the stress intensity factor and the elastic modulus (E) as in Eq. (2).

$$G = K^2 / E \tag{2}$$

The crack compliance is finally determined as the second derivative of released energy with respect to the load at far end (P) as in Eq. (3).

$$c_S = \frac{d^2}{dP^2} \int_S G dx \tag{3}$$

A close insight into these equations revealed that all have epistemic uncertainty which stems from the assumptions and the principles used in their derivation. For example it is not possible to determine the released energy of a specific crack because the affected volume is not known! Moreover the crack compliance which is a characteristic of the structure, similar to the flexibility of the structure, is incorrectly related to the load at far end! These two and several other wrong bases, concluded in unreliable results in the classical fracture mechanics. In support of this detection, the crack compliance for several flexural members are compared as follows.

Rizos et al [16] defined their flexural crack compliance ($FRIZ$) as in Eq. (4):

$$\frac{FRIZ}{5.346} = \begin{pmatrix} 1.86\xi^2 - 3.95\xi^3 + 16.375\xi^4 - 37.226\xi^5 + 76.81\xi^6 \\ -126.9\xi^7 + 172\xi^8 - 143.97\xi^9 + 66.56\xi^{10} \end{pmatrix} \tag{4}$$

The flexural crack compliance ($FOST$)proposed by Ostachowics and Krawczuk [17] is defined as in Eq. (5):

$$\frac{FOST}{6\pi\xi^2} = \begin{pmatrix} 0.6384 - 1.035\xi + 3.7201\xi^2 - 5.1773\xi^3 \\ + 7.5535\xi^4 - 7.332\xi^5 + 2.4909\xi^6 \end{pmatrix} \tag{5}$$

Chondros et al. [18] defined the flexural crack compliance ($FCHO$) as in Eq. (6):

$$\frac{FCHO}{6\pi\left(1-\nu^2\right)} = \begin{pmatrix} 0.6272\xi^2 - 1.04533\xi^3 + 4.5948\xi^4 - \\ 9.9736\xi^5 + 20.2948\xi^6 - 33.0351\xi^7 \\ + 47.1063\xi^8 - 40.7556\xi^9 + 19.6\xi^{10} \end{pmatrix} \tag{6}$$

The flexural crack compliance ($FBIL$) proposed by Bilello [19]as in Eq. (7):

$$FBIL = \frac{\xi(2-\xi)}{0.9(\xi-1)^2} \tag{7}$$

The flexural crack compliance ($FLOY$) reported by Loya et al [20] as in Eq. (8):

$$FLOY = \frac{2\xi^2}{(1-\xi)^2} \begin{pmatrix} 5.93 - 19.69\xi + 37.14\xi^2 \\ - 35.84\xi^3 + 13.2\xi^4 \end{pmatrix} \tag{8}$$

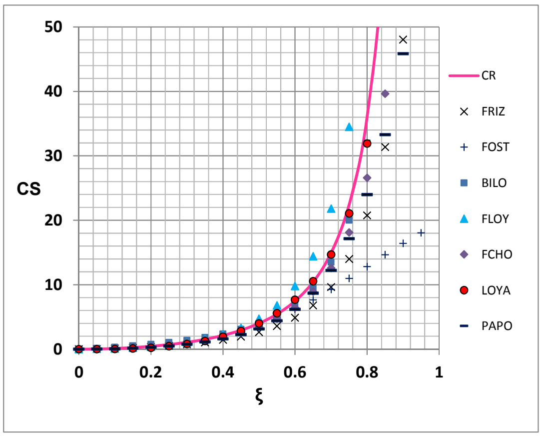

Fig.1. Comparison of crack compliances

Loya et al [20] reported the crack compliance ($LOYA$) for cracked bar as in Eq. (9):

$$LOYA = 4 \times \left(\frac{\xi}{1-\xi}\right)^2 \begin{pmatrix} -0.22 + 3.82\xi + 1.54\xi^2 \\ -14.64\xi^3 + 9.6\xi^4 \end{pmatrix} \quad (9)$$

The crack compliance ($PAPO$) defined by Papadopoulos [21] for the shaft in torsion as in Eq. (10):

$$PAPO = 4 \times \begin{pmatrix} 31.985\xi^6 - 18.846\xi^5 + 4.9219\xi^4 \\ +1.5285\xi^3 + 1.5785\xi^2 - 0.0301\xi \end{pmatrix} \quad (10)$$

Note that the original compliances, in Eq. (9) for cracked bar and in Eq. (10) for a cracked shaft are multiplied by 4 to be comparable with flexural compliances. Seven compliances in Eq. (4) to Eq. (10) are compared with each other in Fig. 1. Difference between the results is a clear flag for presence of epistemic uncertainty in classical fracture mechanics.

As will be shown later in this paper there should be one compliance (CR) as in Fig. 1. The investigations of the (ART) concluded in the birth of the state based philosophy which paved the way for proposition of a reliable fracture mechanics as follows.

Birthplace of the State Based Philosophy

Classical analysis of cracked structures is conducted via solution of a governing differential equation. The governing differential equation applies point wise along the intact part of the domain of the structure. When a crack introduced into the structure, a jump in a derivative of displacement at the point of crack occurs. This jump is used as continuity equation to combine the solutions of the intact parts on two sides of the cracked point. The conventional analysis is extensive, difficult, and time consuming. The aim of the (ART) is to combine the governing differential equation with the jump equation to obtain a single governing differential equation for the analysis of cracked structure. This is done as follows.

Strong Form of the Governing Equation

In common practice, investigation of cracked structures, is conducted via construction and solution of the governing equations subjected to the boundary and/or the initial conditions. The governing differential equation for a typical structure in its initial (intact) state, is defined as in Eq. (11), in which (ψ) is the displacement function of an independent-variable (x), and (n) is the core order of derivative with respect to (x) [40]. For example in structural mechanics, ($n=1$) for axial members, and ($n = 2$) for flexural members.

$$\left(\psi^{(n)}\right)^{(n)} + ... = 0 \quad (11)$$

The derivatives of the displacement function are divided into the essential (displacement), and the natural (force) as defined in Eq. (12). Moreover the force per unit of displacement is called

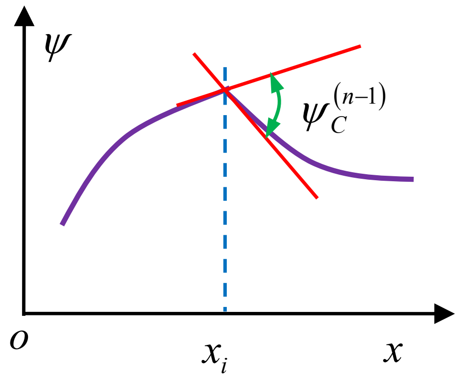

Fig. 2. Change in a derivative of the displacement function

The stiffness and the displacement per unit of force is called the flexibility.

$$\psi^{(m)} = \begin{cases} Essential = Displacement & for\ m = 0\ to\ n-1 \\ Natural = Force & for\ m = n\ to\ 2n-1 \end{cases} \quad (12)$$

A crack in a structure, introduces a change in a derivative of displacement as shown in Fig. 2, and as defined in Eq.(13), where ($\psi_C^{(n-1)}$)is the change in a derivative of displacement at the cracked point,(c_S)is the crack stiffness/flexibility, ($H\left(x - x_i\right)$) is the Heaviside unit step function, and (x_i)is the crack position [40].

$$\psi_C^{(n-1)} = c_S \psi^{(n)} H\left(x - x_i\right) \quad (13)$$

Apply the conventional rules of derivatives to obtain the derivative of Eq. (13) with respect to (x) as written in Eq. (14), in which ($\delta(x - x_i)$) is the Dirac delta (defined as derivative of the ($H\left(x - x_i\right)$)).

$$\psi_C^{(n)} = c_S \psi^{(n)} \delta\left(x - x_i\right) + c_S \psi^{(n+1)} H\left(x - x_i\right) \quad (14)$$

In view of Fig. 3(b), the derivative of ($\psi^{(n)}$) at the crack position (x_i) is written as in Eq. (15).

$$\psi^{(n+1)} = Limit \left.\frac{\psi_{+i}^{(n)} - \psi_{-i}^{(n)}}{\Delta x}\right|_{\Delta x = 0} \quad (15)$$

Noting that ($\psi_{+i}^{(n)} = \psi_{-i}^{(n)}$), therefore Eq. (15) is concluded in Eq. (16).

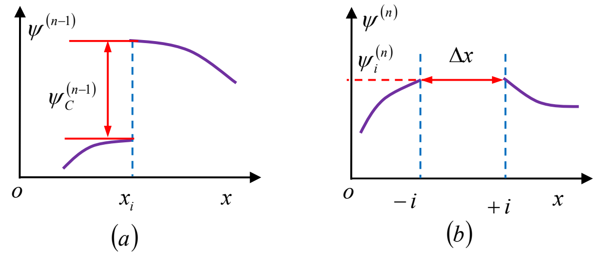

Fig. 3 Derivative of the displacement function at the discontinuous point

$$\psi^{(n+1)} = 0 \tag{16}$$

Substitution of Eq. (16) into Eq. (14) ended in the so called, the Golden Derivative, as defined in Eq. (17). The paramount importance of this derivative and its rule in the birth of the state based philosophy will be observed in later sections.

$$\psi_C^{(n)} = c_S \psi^{(n)} \delta(x - x_i) \tag{17}$$

The derivation of Eq. (11) to Eq. (17) were based on concise conventional mathematical rules, while the results introduced a new but nonconventional concept. For the first time the derivative at the point of discontinuity,(see Fig. 3(a), and compare Eq. (13) and Eq. (17)) is defined as in Eq. (17). The name Golden is selected because of this new finding and its effect in our future detections.

Eq. (11) and Eq. (17) have the same style. These are combined (the crack reduces the core derivative, therefore the crack-core-derivative in Eq. (17) is subtracted from the core derivative in Eq. (11)) as in Eq. (18).

$$\begin{gathered}\left(\psi^{(n)} - \psi_C^{(n)}\right)^{(n)} + \ldots = 0 \\ \left(\psi^{(n)} - c_S \psi^{(n)} \delta(x - x_i)\right)^{(n)} + \ldots = 0\end{gathered} \tag{18}$$

Eq. (18) is the governing differential equation for the cracked structure, which is derived, for the first time by the (ART) [40]. This equation has the same form as in Eq. (11), and can be solved by the same rules that are available for solution of Eq. (11). That is, Eq. (18) is the general equation and Eq. (11) is a special case, therefore from here on, Eq. (18) is placed on the table of human knowledge for the fracture analyses.

Analytical Solution by the Laplace Transform

As an example, the dynamic stability of beam-like cracked structures is considered as follows. The governing differential equation for the dynamic stability analysis of beam-like structures is defined in Eq. (19), in which (ψ) is the lateral displacement, (λ_P) is the load factor, and (λ_W) is the frequency factor [42, 43].]

$$\left(\psi^{(2)} - c_S \psi^{(2)} \delta(x - x_i)\right)^{(2)} + \lambda_P^2 \psi^{(2)} - \lambda_W^4 \psi = 0 \tag{19}$$

The method of Laplace Transform is used to solve Eq. (19). The solution is defined as in Eq. (20), in which the solution coefficient functions ($A_{mo}(x)$, $m, o = 0{,}4$) are defined as in Table 1.

$$\begin{aligned}
\psi^{(0)} &= A_{00}(x)\psi_0^{(0)} + A_{01}(x)\psi_0^{(1)} + A_{02}(x)\psi_0^{(2)} \\
&\quad + A_{03}(x)\psi_0^{(3)} + c_S A_{04}(x - x_i)\psi_i^{(2)} H(x - x_i) \\
\psi^{(1)} &= A_{10}(x)\psi_0^{(0)} + A_{11}(x)\psi_0^{(1)} + A_{12}(x)\psi_0^{(2)} \\
&\quad + A_{13}(x)\psi_0^{(3)} + c_S A_{14}(x - x_i)\psi_i^{(2)} H(x - x_i) \\
\psi^{(2)} &= A_{20}(x)\psi_0^{(0)} + A_{21}(x)\psi_0^{(1)} + A_{22}(x)\psi_0^{(2)} \\
&\quad + A_{23}(x)\psi_0^{(3)} + c_S A_{24}(x - x_i)\psi_i^{(2)} H(x - x_i) \\
\psi^{(3)} &= A_{30}(x)\psi_0^{(0)} + A_{31}(x)\psi_0^{(1)} + A_{32}(x)\psi_0^{(2)} \\
&\quad + A_{33}(x)\psi_0^{(3)} + c_S A_{34}(x - x_i)\psi_i^{(2)} H(x - x_i) \\
\psi^{(4)} &= A_{40}(x)\psi_0^{(0)} + A_{41}(x)\psi_0^{(1)} + A_{42}(x)\psi_0^{(2)} \\
&\quad + A_{43}(x)\psi_0^{(3)} + c_S A_{44}(x - x_i)\psi_i^{(2)} H(x - x_i) \\
\psi^{(4)} &= \lambda_P^2 \psi^{(1)} + \psi^{(3)}
\end{aligned} \tag{20}$$

Weak Form of the Governing Equation

In order to prepare for analysis of cracked structures by numerical methods, the governing differential equation and the natural boundary conditions should be combined to obtain the Weak Form equation, which is the starting point of all numerical methods, such as the finite element method, the boundary element method, and etc. The method of weighted residual is the tool for doing the job [71]. In a numerical method the solution is approximated, therefore the left side of Eq. (18) is nonzero, and is defined as the residual of the governing equation (R_{GE}), defined as in Eq. (21).

$$R_{GE} = \left(\psi^{(n)} - c_S \psi^{(n)} \delta(x - x_i)\right)^{(n)} + \ldots \neq 0 \tag{21}$$

The crack only affects the main differential (the core derivative such as the left term in Eq. (11)) in the governing differential equation, therefore the other terms i.e. those in the (…) is ignored. According to the rules of the method of weighted residual, the weighted error is set equal to zero as in Eq. (22), where (W)is the weighting function.

$$\int W \left(\psi^{(n)} - c_S \psi^{(n)} \delta(x - x_i)\right)^{(n)} dx + \ldots = 0 \tag{22}$$

Making use of the chain rules of derivatives, Eq. (22) is changed to the Weak Form equation as in Eq. (23).

Table 1. Coefficient functions for solution of dynamic stability equation

A0 Coefficient of $\psi^{(0)}$	**A1 Coefficient of** $\psi^{(1)}$
$A_{00}(x)=C_{ab}\begin{pmatrix}+b^2\cos ax\\+a^2\cosh bx\end{pmatrix}$	$A_{10}(x)=C_{ab}(ab)\begin{pmatrix}-b\sin ax\\+a\sinh bx\end{pmatrix}$
$A_{01}(x)=C_{ab}(ab)^{-1}\begin{pmatrix}+b^3\sin ax\\+a^3\sinh bx\end{pmatrix}$	$A_{11}(x)=C_{ab}\begin{pmatrix}+b^2\cos ax\\+a^2\cosh bx\end{pmatrix}$
$A_{02}(x)=C_{ab}\begin{pmatrix}-\cos ax\\+\cosh bx\end{pmatrix}$	$A_{12}(x)=C_{ab}\begin{pmatrix}+a\sin ax\\+b\sinh bx\end{pmatrix}$
$A_{03}(x)=C_{ab}(ab)^{-1}\begin{pmatrix}-b\sin ax\\+a\sinh bx\end{pmatrix}$	$A_{13}(x)=C_{ab}\begin{pmatrix}-\cos ax\\+\cosh bx\end{pmatrix}$
$A_{04}(x)=C_{ab}\begin{pmatrix}+a\sin axx_i\\+b\sinh bxx_i\end{pmatrix}$	$A_{14}(x)=C_{ab}\begin{pmatrix}+a^2\cos axx_i\\+b^2\cosh bxx_i\end{pmatrix}$
A2 Coefficient of $\psi^{(2)}$	**A3 Coefficient of** $\psi^{(3)}$
$A_{20}(x)=C_{ab}a^2b^2\begin{pmatrix}-\cos ax\\+\cosh bx\end{pmatrix}$	$A_{30}(x)=C_{ab}a^2b^2\begin{pmatrix}+a\sin ax\\+b\sinh bx\end{pmatrix}$
$A_{21}(x)=C_{ab}ab\begin{pmatrix}-b\sin ax\\+a\sinh bx\end{pmatrix}$	$A_{31}(x)=C_{ab}a^2b^2\begin{pmatrix}-\cos ax\\+\cosh bx\end{pmatrix}$
$A_{22}(x)=C_{ab}\begin{pmatrix}+a^2\cos ax\\+b^2\cosh bx\end{pmatrix}$	$A_{32}(x)=C_{ab}\begin{pmatrix}-a^3\sin ax\\+b^3\sinh bx\end{pmatrix}$
$A_{23}(x)=C_{ab}\begin{pmatrix}+a\sin ax\\+b\sinh bx\end{pmatrix}$	$A_{33}(x)=C_{ab}\begin{pmatrix}+a^2\cos ax\\+b^2\cosh bx\end{pmatrix}$
$A_{24}(x)=C_{ab}\begin{pmatrix}-a^3\sin axx_i\\+b^3\sinh bxx_i\end{pmatrix}$	$A_{34}(x)=C_{ab}\begin{pmatrix}-a^4\cos axx_i\\+b^4\cosh bxx_i\end{pmatrix}$
A4 Coefficient of $\lambda_P^2\psi^{(1)}+\psi^{(3)}$	**Working parameters**
$A_{40}(x)=C_{ab}ab\begin{pmatrix}+b^3\sin ax\\+a^3\sinh bx\end{pmatrix}$	$EI\lambda_\omega^4=m\omega^2\quad EI\lambda_P^2=P$
$A_{41}(x)=C_{ab}\begin{pmatrix}-b^4\cos ax\\+a^4\cosh bx\end{pmatrix}$	$a^2=\sqrt{\lambda_P^4+4\lambda_\omega^4}+\lambda_P^2$
$A_{42}(x)=C_{ab}ab\begin{pmatrix}-b\sin ax\\+a\sinh bx\end{pmatrix}$	$b^2=\sqrt{\lambda_P^4+4\lambda_\omega^4}-\lambda_P^2$
$A_{43}(x)=C_{ab}\begin{pmatrix}+b^2\cos ax\\+a^2\cosh bx\end{pmatrix}$	$C_{ab}(a^2+b^2)=1$
$A_{44}(x)=C_{ab}a^2b^2\begin{pmatrix}-\cos axx_i\\+\cosh bxx_i\end{pmatrix}$	$xx_i=x-x_i$

$$\int W^{(n)}\left(\psi^{(n)}-c_S\psi^{(n)}\delta\left(x-x_i\right)\right)dx+\ldots=0 \qquad (23)$$

Derivation of equations up to this end were done according to the accepted conventional rules. Although the derivative at discontinuity was exceptional, but it is also derived according to the accepted conventions. Passing from the weak form equation to the finite element equation confronted with the serious lack of knowledge which is resolved in the next section.

Finite Element Equation

For a phenomenon with the given weak form equation, deriving the finite element equation, conventionally begins with the definition of the weight function and the displacement function in terms of the nodal values ($W_O\,\&\psi_D$), and the nodal shape functions ($N_O\,\&N_D$), as defined in Eq. (24), where the repeated indices denote summation over the number of element nodes (nen).

$$W=W_ON_O\left(x\right),O=1,nen$$
$$\psi=\psi_DN_D\left(x\right),D=1,nen \qquad (24)$$

The effective displacement function (whole terms in braces as in Eq. (18)), is different from that in Eq. (11), but since we were not able to find appropriate procedure for changing this function into the finite element terms in the literature, then for the time being we inserted Eq. (24) into Eq. (23) to obtain the finite element equation as defined in Eq. (25),

$$k_{OD}^{SS}\,\psi_D+\ldots=0$$
$$k_{OD}^{SS}=k_{OD}^{S}-k_{OD}^{IC} \qquad (25)$$

Where the structure stiffness matrix (k_{OD}^{S}) is defined as in Eq. (26),

$$k_{OD}^{S}=\int N_O^{(n)}N_D^{(n)}dx \qquad (26)$$

and the initial crack stiffness matrix (k_{OD}^{IC}) is defined as in Eq. (27).

$$k_{OD}^{IC}=c_S\int N_O^{(n)}\delta\left(x-x_i\right)N_D^{(n)}dx \qquad (27)$$

For the case of free vibration analysis of flexural members, and free vibration analysis of bar members, Eq. (25) is implemented in a personal software and applied for solution of free Vibration of members with the given analytical solutions. It is found that the initial crack-stiffness matrix as defined in Eq. (27) is not correct and should be modified [44-46]! For several bar members and several beam members, via hundreds of trial free vibration analyses, the numerical values for the survive function (S_R) (A coefficient to be multiplied by the initial crack integral to obtain the correct result) are defined as in Table 2.

Table 2. Numerical data for the survive function S_R

Flexural, C_S=1				Axial, C_S=2			
L	SFA	SFB	SFC	L	SAA	SAB	SAC
0.50	0.1111	2/18	0.50/(0.50+4)	0.50	0.1999	2/10	0.50/(0.50+2)
0.75	0.1575	3/19	0.75/(0.75+4)	0.75	0.2766	3/11	0.75/(0.75+2)
1.00	0.1999	4/20	1.00/(1.00+4)	1.00	0.3344	4/12	1.00/(1.00+2)
1.25	0.2393	5/21	1.25/(1.25+4)	1.25	0.3844	5/13	1.25/(1.25+2)
1.50	0.2761	6/22	1.50/(1.50+4)	1.50	0.4283	6/14	1.50/(1.50+2)
1.75	0.3111	7/23	1.75/(1.75+4)	1.75	0.4666	7/15	1.75/(1.75+2)
2.00	0.3444	8/24	2.00/(2.00+4)	2.00	0.4999	8/16	2.00/(2.00+2)
S_F=L/(L+4 C_S)				S_A=L/(L+ C_S)			

For flexural members numerical values are changed via several steps, from (SFA) to (SFC), and an equation for the survive function (S_F) in flexure is derived as in Eq. (28).

$$S_F = (L/4)/(L/4 + c_S) \tag{28}$$

Similarly for axial members, the numerical values are changed via several steps, from (SAA) to (SAC), and an equation for the survive function of axial member (S_A), is obtained as in Eq. (29).

$$S_A = (L)/(L + c_S) \tag{29}$$

The derived survive functions in Eq. (28) and Eq. (29), were changed into a unified equation for the survive function (S_R), and its twin function, the so called the failure function (F_R), as defined in Eq. (30).

$$\begin{aligned} F_R &= (c_S)/(f_S + c_S) \\ S_R &= (f_S)/(f_S + c_S) \end{aligned} \tag{30}$$

Finally the modified form of the cracked-stiffness-matrix (k_{OD}^C) are defined as in Eq. (31).

$$k_{OD}^C = \begin{cases} F_R f_S \int N_O^{(n)} \delta(x - x_i) N_D^{(n)} dx \\ S_R c_S \int N_O^{(n)} \delta(x - x_i) N_D^{(n)} dx \end{cases} \tag{31}$$

The present formulation shown that, the state of the art for analysis of behavior of cracked structures, is far from complete. Therefore the analysis of cracked structures needs more fundamental investigation! The achievements up to this point, concluded to the birth of the so called phenomenon functions (F_R & S_R), which opened the door to the world of the state based philosophy. Replacing the (k_{OD}^C) in place of the (k_{OD}^{IC}) into Eq. (25) concluded into the innovative finite element equation for analysis of the cracked structures. In view of the definition of the phenomenon functions, Eq. (25) may be written as in Eq. (32), in terms of the survived stiffness (k_{OD}^{SS}).

$$\begin{aligned} & k_{OD}^{SS} \psi_D + \ldots = 0 \\ & k_{OD}^{SS} = S_R k_{OD}^S \end{aligned} \tag{32}$$

Eq. (32) shows that the survived stiffness of a cracked-structure is proportional to that of the intact one. As will be shown in the subsequent sections, the survive function is developed explicitly and therefore the survived stiffness is determined in a simple way. Moreover, Eq. (32) is an alternative finite element equation in place of the extended finite element method (XFEM). The formulation for the state based philosophy in this section is exact, but since it is managed by an extensive numerical experimentation, then it is confronted with some speculation by those who were accustomed with the conventional methods of analyses which are based on conventional mathematics. To pave the way for universal acceptance of the state based philosophy, the derivations is based on a sound mathematical basis as described in the following sections.

Logical Basis of the State Based Philosophy

In this section we derive the basic formulation for the state based philosophy via a sound logical basis. The traditional formulations in the academic universe are divided into, the so called stiffness method, in which the system characteristics (called the stiffness) reduces to zero during the change of state, and the flexibility (the inverse of the stiffness) method, where the flexibility goes toward the infinity during the change of state. Near the end of the phenomenon, the former includes error from the small numbers and the latter includes error from the large numbers. The basis of the proposed universal method is a trivial relationship, i.e. the survived-stiffness ($k_{SS} = k_S - k_C$) times the survived-flexibility ($f_{SS} = f_S + c_S$) is set equal to one (product of a quantity and its inverse is equal to one), defined as in Eq. (33), in which (k_S)is the structure-stiffness, (k_C) is the crack-stiffness, (f_S) is the structure -flexibility, and (c_S)is the crack-flexibility. Eq. (33), and the proposed formulation, which is shown in Fig. 4, is free of epistemic uncertainty.

$$(k_S - k_C)(f_S + c_S) = 1 \tag{33}$$

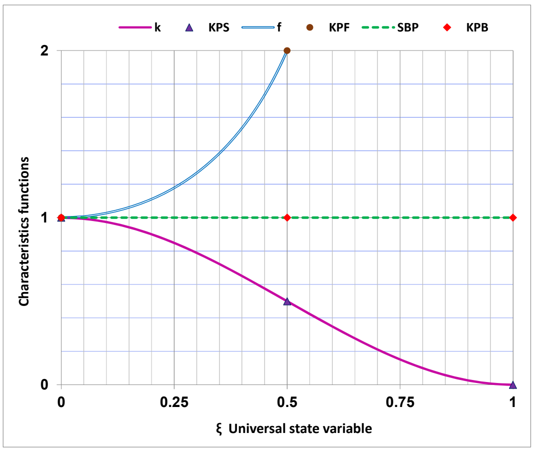

Fig.4. State based philosophy basic equation

Solve Eq. (33) for (k_C), to obtain the crack-stiffness (k_C), and the survived-stiffness (k_{SS}), as defined in Eq. (34),

$$k_C = F_R k_S$$
$$k_{SS} = S_R k_S \tag{34}$$

In which the phenomenon functions (the failure function (F_R) and the survive function (S_R)) are defined as in Eq. (35). Note that these phenomenon functions are exactly the same, as those defined in Eq. (30).

$$F_R = (c_S)/(f_S + c_S) \in [0 \quad 1]$$
$$S_R = (f_S)/(f_S + c_S) \in [1 \quad 0] \tag{35}$$

The structure-flexibility, is expressed as product of the dimensioned-flexibility (F_S), and the dimensionless-structure –flexibility (f_N), moreover the crack-flexibility is defined in terms of the dimensioned-flexibility (F_S), and the dimensionless-crack-flexibility (c_N), as in Eq. (36).

$$f_S = f_N F_S$$
$$c_S = c_N F_S \tag{36}$$

Substitution of Eq. (36) into Eq. (35) concluded into the dimensionless form of the phenomenon functions as defined in Eq. (37).

$$F_R = (c_N)/(f_N + c_N)$$
$$S_R = (f_N)/(f_N + c_N) \tag{37}$$

The proposed form of phenomenon functions as in Eq. (37) are defined in terms of (f_N) and (c_N), which are unknown to this end. The process for explicit definition of these functions is continued in the next section via definition and construction of the state functions.

State functions

The phenomenon functions are customized for ($f_N = 1$) to define the state functions ((D)the destination, (O) the origin, and (R)the state ratio) as in Eq. (38).

$$F_R = D$$
$$S_R = O \tag{38}$$
$$c_N = R$$

Consequently the state functions are defined in terms of the state ratio as in Eq. (39).

$$D = (R)/(1+R)$$
$$O = (1)/(1+R) \tag{39}$$
$$R = D/O$$

The state functions may be considered as the solution of the boundary value problems as expressed in Eq. (40), and shown in Fig. 5, where min denotes minimum and max denotes maximum.

$$D = \begin{cases} \min = 0 & @R = 0 \\ \max = 1 & @R = \infty \end{cases}$$
$$O = \begin{cases} \max = 1 & @R = 0 \\ \min = 0 & @R = \infty \end{cases} \tag{40}$$

The state ratio (R), with the far end in the infinity see Fig. 5, is not a good working parameter.

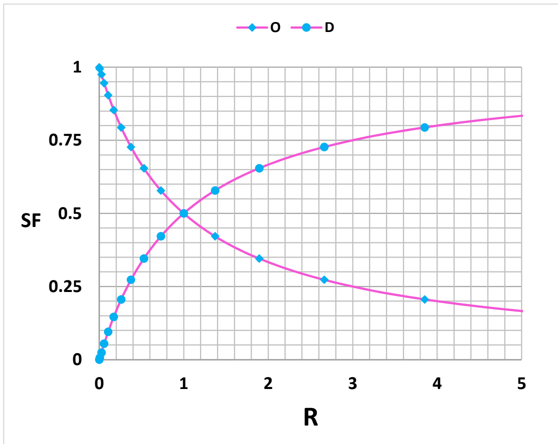

Fig.5 State Functions D and O versus the state ratio R

Moreover, this ratio is itself a function, so it is not wise to be used as an independent variable. Therefore, the state variable ($\xi \in [0 \quad 1]$) with a zero value ($\xi = 0$) at the origin and a unit value ($\xi = 1$) at the destination is innovatively defined. In term of the state variable, the boundary value problems in Eq. (40) is rewritten as in Eq. (41).

$$D = \begin{cases} \min = 0 & @\xi = 0 \\ \max = 1 & @\xi = 1 \end{cases}$$
$$O = \begin{cases} \max = 1 & @\xi = 0 \\ \min = 0 & @\xi = 1 \end{cases} \tag{41}$$

Polynomial functions with the conditions as in Eq. (41), first defined by Hermite [28] and used as the shape functions in numerical methods, are defined as in Eq. (42).

$$D = (\xi)^2 (1 + 2(1-\xi)) \quad O = (1-\xi)^2 (1 + 2(\xi)) \tag{42}$$

The reviewers of papers containing Eq. (42), and our colleagues rejected this equation, and said more than thousands other equations, with the conditions as in (42), may be derived. Therefore it is not acceptable! Our extensive investigation, detected Eq. (43), which is selected from the well accepted field of the strength of material, as a solution for Eq. (41).

$$D = 0.50(1 - \cos \pi\xi) \quad O = 0.50(1 + \cos \pi\xi) \tag{43}$$

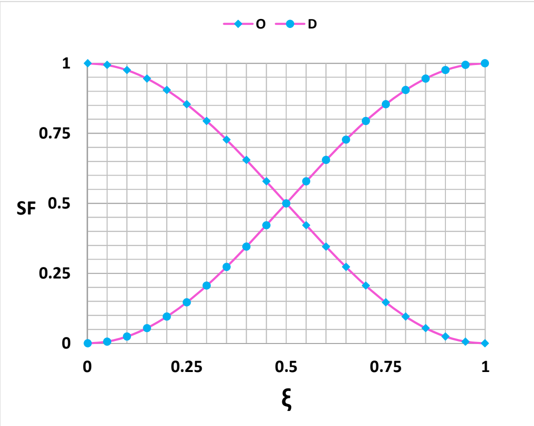

Fig.6. State Functions D and O versus the State Variable ξ

Finally, via taking average of Eq. (42) and Eq. (43), an explicit, symmetric, and beautiful form of the state functions are defined as in Eq. (44), and shown in Fig. 6.

$$D = 0.25\left(2 - 1 + 6\xi^2 - 4\xi^3 - \cos \pi\xi\right) \quad O = 0.25\left(2 + 1 - 6\xi^2 + 4\xi^3 + \cos \pi\xi\right) \tag{44}$$

It is clear that, any other formulation which is proposed for the state functions should fit those as in Eq. (44).

In view of the definition of (k_{SS}) and(f_{SS}), the (ART) detected (great detection)a fact that, the flexibility- and the stiffness-of the crack, as shown in Eq. (45),are proportional to each other (while for structures the stiffness and the flexibility are inverse of each other).

$$k_C = c_N K_S \quad k_S = k_N K_S \tag{45}$$

Based on the definition of the state functions, the (stiffness) ratio of (k_C) over (k_S) is set equal to the state ratio (R), which after some manipulation led into an explicit definition for the (c_N)as defined in Eq. (46).

$$k_C / k_S = c_N / k_N \quad c_N = k_N D/O \tag{46}$$

Substitution of Eq. (46) into Eq. (37) concluded in the general definition for the phenomenon functions as in Eq. (47).

$$F_R = \left(k_N^2 D\right) / \left(O + k_N^2 D\right) \quad S_R = (O) / \left(O + k_N^2 D\right) \tag{47}$$

Similar to the state ratio, the (k_N) is not also explicitly known and so it is not a feasible working parameter. Therefore Eq. (47) is rewritten in a unified form as in Eq. (48), in terms of the control parameters (a_M), and (b) [39-70].

$$F_R = \left(a_M D^b\right) / \left(O^b + a_M D^b\right) \quad S_R = \left(O^b\right) / \left(O^b + a_M D^b\right) \tag{48}$$

To this end the proposed formulation is mathematically in an abstract form, so it is a universal one in the sense that it is independent of geometrical and material properties, and the changing agent. Therefore, it applies to all natural phenomena.

Persian curves

For a given phenomenon, the lifetime is truncated at a workable interval, and the truncated lifetime, ($\lambda \in [\lambda_O \quad \lambda_T]$), is mapped onto the state variable as in Eq. (49), where (λ_O) is the origin of lifetime and (λ_T)is the end of lifetime.

$$\lambda = (1-\xi)\lambda_O + (\xi)\lambda_T \quad \xi = (\lambda - \lambda_O)/(\lambda_T - \lambda_O) \tag{49}$$

In terms of the lifetime, the phenomenon functions are renamed as Persian (the (F_R) is renamed as Fasa (P_F), and the (S_R) is renamed as Shiraz (P_S)) curves. In this way the Persian-Shiraz curve is the unified equation for the survived ratio of the system characteristics (or the capacity ratio). The feasible strength (capacity) data of the failure phenomenon is managed in the decreasing order. For the decreasing data the survive function is casted into the Persian-Shiraz curve, (P_S), as defined in Eq. (50), in which (P_T)is the ordinate of the end point (T).

$$P_S = \left(O^b + P_T a_M D^b\right) / \left(O^b + a_M D^b\right) \tag{50}$$

The twin function of the Shiraz curve is the called the Persian-Fasa curve as defined in Eq. (51).

$$P_F = \left(P_T a_M D^b\right) / \left(O^b + a_M D^b\right) \tag{51}$$

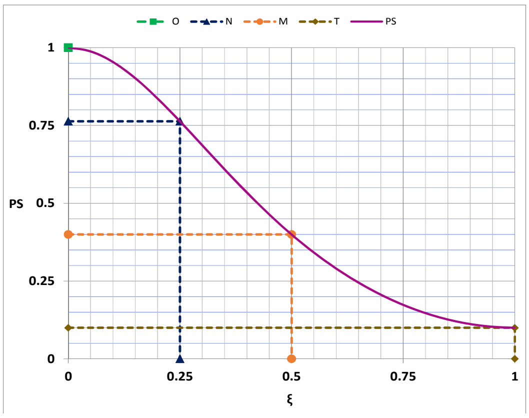

Fig.7. Key points on Persian (Shiraz) curve

Moreover, if the reliable data is managed in decreasing order as shown in Fig. 7, then the control parameters may be obtained as in Eq. (52) in terms of the coordinates of the key-points (KPS).

$$a_N = \frac{1 - P_N}{P_N - P_T} \quad a_M = \frac{1 - P_M}{P_M - P_T} \quad b = \frac{Log\left(a_N / a_M\right)}{Log\left(D(\xi_N)/O(\xi_N)\right)} \tag{52}$$

The key points are defined as the origin point (O), the middle point (M), the end point (T), and the other point (N) (a point between the other three), defined as in Eq. (53) and shown in Fig. 7.

$$\begin{aligned} &O\left(\xi_O = 0.00,\ 1.00\right) \quad N\left(\xi_N = 0.25,\ P_N\right) \\ &M\left(\xi_M = 0.50,\ P_M\right) \quad T\left(\xi_T = 1.00,\ P_T\right) \end{aligned} \tag{53}$$

Similarly for the increasing reliable data as shown in Fig. 8, the control parameters may be obtained as in Eq. (54) in terms of the coordinates of the key-points (KPS).

$$\begin{aligned} &a_N = \frac{P_N}{P_T - P_N} \quad a_M = \frac{P_M}{P_T - P_M} \\ &b = \frac{Log\left(a_N / a_M\right)}{Log\left(D(\xi_N)/O(\xi_N)\right)} \end{aligned} \tag{54}$$

Where the key points are defined as the origin point (O), the middle point (M), the end point (T), and the other point (N) (a point between the other three), defined as in Eq. (55) and shown in Fig. 8.

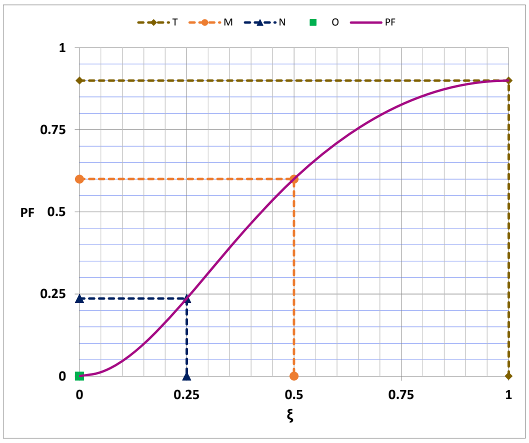

Fig.8. Key points on Persian (Fasa) curve

$$\begin{aligned} &O\left(\xi_O = 0.00,\ 0.00\right) \quad N\left(\xi_N = 0.25,\ P_N\right) \\ &M\left(\xi_M = 0.50,\ P_M\right) \quad T\left(\xi_T = 1.00,\ P_T\right) \end{aligned} \tag{55}$$

Moreover, for both cases, the Persian (Zahedan) curve, (P_Z), is defined as the derivative of the phenomenon functions with respect to the state variable, and expressed as in Eq. (56), in which ($D^{(1)}(\xi)$)is the derivative of ($D(\xi)$)with respect to the state variable (ξ).

$$\begin{aligned} &\frac{dF_R}{d\xi} = +P_Z \quad \frac{dS_R}{d\xi} = -P_Z \\ &P_Z = \frac{b a_M D^{b-1} O^{b-1} D^{(1)}(\xi)}{\left(O^b + a_M D^b\right)^2} \end{aligned} \tag{56}$$

Here is the end of the proposed logical formulation. It is applied to fracture mechanics problems in the subsequent sections.

Reliable Crack Compliance

The crack compliance is directly determined from Eq. (46) which is inserted in Eq. (57), for completeness as follows.

$$c_S = k_N \frac{D(\xi)}{O(\xi)} \tag{57}$$

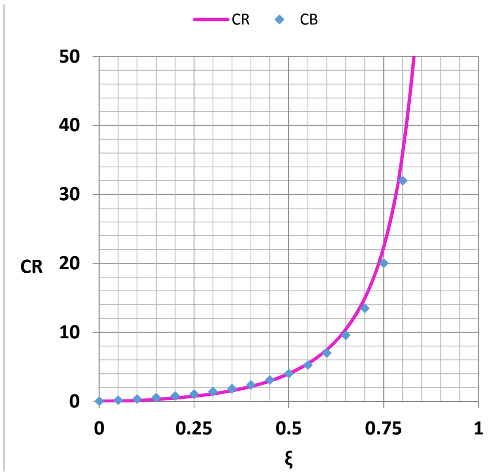

Fig.9. Comparison of crack compliances CB and CR

Eq. (57) is verified via comparison with the results of the others in the literature, in the following examples.

Example 1: Compare Eq. (57) with the flexural crack compliances computed by Bilello [19] as follows.

Solution 1: Bilello [19] proposed compliance for a cracked beam as in Eq. (7) which is modified as in Eq. (58) (state variable plays the role of crack depth).

$$CB = \frac{\xi(2-\xi)}{0.75(\xi-1)^2} \tag{58}$$

The reliable state based compliance from Eq. (57), i.e. ($CR = 4D(\xi)/O(\xi)$) is compared with the (CB)

in Fig. 9. The results are in close agreements with each other. The simplicity and accuracy and hence the reliability of the proposed formulation is apparent. Moreover the reliable crack compliance is derived as the sole characteristics of the structure. After computation of the crack compliance from Eq. (57) there is no need for other parameters, but in comply with the classical fracture mechanics they are derived as follows.

Extensive research of the (ART) concluded into the fact that main parameters of fracture mechanics, including the stress intensity factor, the energy release rate, the crack compliance, and others are all abstract mathematical entities. Consequently these parameters may be derived accurately without any sort of singularity or epistemic uncertainty.

Energy release rate

Energy is defined for a volume of the considered region. The volume for the given stress at the crack tip is not known! Therefore in analyses regarding the crack tip, the energy density is used. Based on the definition of energy, the tip energy density (E_T) is defined in terms of the crack tip displacement (Δ_T), as in Eq. (59).

$$E_T = k_N K_S \Delta_T^2 \quad (59)$$

Similarly the released energy density (E_R), is defined as in Eq. (60).

$$E_R = c_N K_S \Delta_T^2 \quad (60)$$

Divide Eq. (60) by Eq. (59) and substitute for (c_N) from Eq. (57) into the ratio to obtain the released energy density (E_R)in terms of the tip energy density (E_T)as in Eq. (61).

$$E_R = E_T D(\xi)/O(\xi) \quad (61)$$

For the case of elastic behavior the tip energy density is defined, in terms of the effective stress (σ)and the elastic modulus (E), as in Eq. (62).

$$E_T = \sigma^2/2E \quad (62)$$

While the energy release density is defined in terms of the energy release rate (G_R) as in Eq. (63).

$$E_R = \int_0^{\xi} G_R dx \quad (63)$$

Substitute Eq. (62) and Eq. (63) into Eq. (61) as in Eq. (64).

$$\int_0^{\xi} G_R dx = \sigma^2 D(\xi)/2E\,O(\xi) \quad (64)$$

Now take derivative of Eq. (64) with respect to (ξ), to obtain the state based energy release rate G_R as in Eq. (65).

$$G_R = \frac{\sigma^2}{2E} \times \frac{D^{(1)}(\xi)}{O^2(\xi)} \quad (65)$$

Stress intensity factor

The stress intensity factor (K), is defined in terms of the effective stress (σ), via the application of a mathematical operator (L) (there is no need for any information regarding this operator) as in Eq. (66).

$$LK_R = \sigma \quad (66)$$

In order to write the (K) in integral form, in terms of the effective stress (σ), the weight function (W_R) is defined as in Eq. (67),

$$LW_R = \delta(x - x_i) \quad (67)$$

in which ($\delta(x - x_i)$) is the Dirac delta with the integral free property defined as in Eq. (68).

$$\int R(x)\delta(x - x_i)dx = R(x_i) \quad (68)$$

Define a parameter (Q) in terms of the stress (σ) and the weight function (W_R) as in Eq. (69).

$$Q = \int \sigma W_R\, dx \quad (69)$$

Apply the operator (L) on Eq. (69) as in Eq. (70).

$$LQ = \int \sigma L W_R\, dx \quad (70)$$

Substitute for (LW_R) from Eq. (67) and make use of the integral free property of the Dirac delta as in Eq. (68) to obtain the mathematical Eq. (71).

$$LQ = \sigma \quad (71)$$

Compare Eq. (71) with Eq. (66) to obtain an integral definition for the stress intensity factor as in Eq. (72).

$$K_R = \int \sigma W_R dx \quad (72)$$

Eq. (72) is derived based on sound mathematical basis. The necessary and sufficient condition for its computation is the presence of the weight function (W_R) [6, 7, 29, and 44]. According to the classical fracture mechanics definition of (K_R) the stress intensity factor has a unit of stress times the square root of unit of length, which introduces ambiguity! Since the derivation of Eq. (72) is based on mathematical logics, then it is under our control, and we may select its units ourselves.

Substitution of Eq. (65) into Eq. (2) concluded in the state based stress intensity factor as in Eq. (73).

$$K_R = \frac{\sigma}{O(\xi)} \times \sqrt{\frac{D^{(1)}(\xi)}{2}} \quad (73)$$

Note that the proposed stress intensity factor as defined in Eq. (73) has the unit of stress as promised before. Two of the main parameters of state based fracture mechanics, i.e. (G_R) and (K_R), are independent of the crack compliance ($c_N(\xi)$). That was a great detection, because the main outcome of the classical fracture mechanics is the compliance ($c_N(\xi)$), which is directly obtained from Eq. (57). From now on, the state based compliance may be used in place of all compliance formulations in papers, handbooks, and etc. in the literature!

Since (K_R) and (G_R) are related to each other, therefore only one is verified as follows.

Example 2: The stress intensity factor ($ALIO$) proposed by Aliabadi [26] for the plate with central crack as in Eq. (74).

$$ALIO = \sqrt{\pi\xi}\begin{pmatrix} 1+0.043\xi+0.491\xi^2 \\ +7.125\xi^3-28.403\xi^4 \\ +59.583\xi^5-65.278\xi^6 \\ +29.762\xi^7 \end{pmatrix} \quad (74)$$

The same problem is analyzed by Patricio [27] via numerical analysis, and exactly the same result is obtained. Compare the ($ALIO$) with the (KR)?

Solution2: The ($ALIO$) and the (KR) are compared in Fig. 10. For ($\xi > 0.5$) there is great difference between the two. Making use of the proposed formulation, Eq. (74) is modified as defined in Eq. (75), to be in compliance with Eq. (73).

$$ALIM = \frac{\sqrt{\pi\xi}}{(1-\xi)^{1.06}(1+2\xi)}\begin{pmatrix} 1+0.043\xi+0.491\xi^2 \\ +7.125\xi^3-28.403\xi^4 \\ +59.583\xi^5-65.278\xi^6 \\ +29.762\xi^7 \end{pmatrix} \quad (75)$$

The ($ALIM$) is shown in Fig. 10.

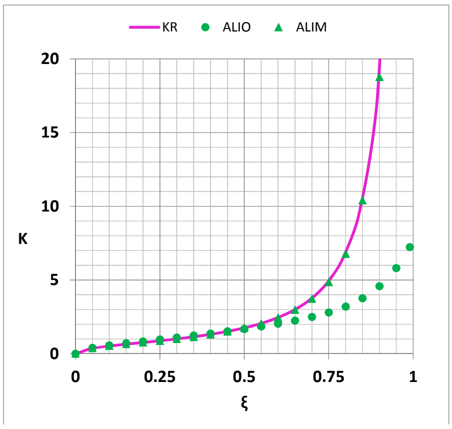

Fig.10. Comparison of stress intensity factors $ALIO$, $ALIM$ and KR

Example 3: The stress intensity factor defined by the ($ASTM$) special committee E-399 [22] for bending specimen ($BENO$) is given as in Eq. (76).

$$BENO = \frac{3\xi^{1/2}}{2(1+2\xi)(1-\xi)^{3/2}}\left(1.99-\xi(1-\xi)\begin{pmatrix} 2.15-3.93\xi \\ +2.7\xi^2 \end{pmatrix}\right) \quad (76)$$

Compare the ($BENO$) with the (KR)?

Solution3: The ($BENO$) and the (KR) are compared in Fig. 11.

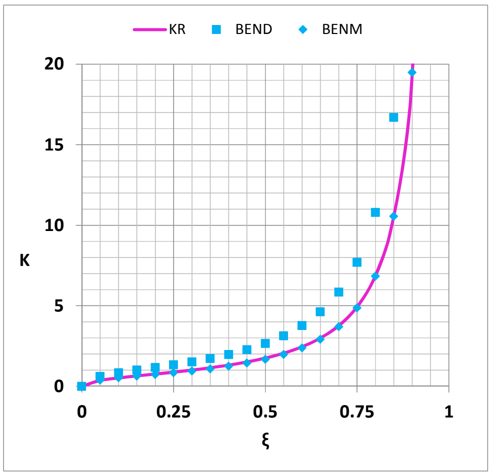

Fig.11. Comparison of stress intensity factors $BENO$, $BENM$ and KR

Again here, there is difference between the two. Since the proposed formulation is based on sound foundation and is reliable, then it is used to modify Eq. (76) for better compliance as in Eq. (77).

$$BENM = \frac{3\sqrt{0.4}\,\xi^{1/2}}{2(1+2\xi)(1-\xi)^{3/2}}\left(1.99-\xi(1-\xi)\begin{pmatrix} 2.15-3.93\xi \\ +2.7\xi^2 \end{pmatrix}\right) \quad (77)$$

The modified stress intensity factor is shown in Fig. 11. Close agreement of the results verified the proposed formulation.

The weight function for the stress intensity factor

The state based philosophy is quite universal, in the sense that it is applicable to all natural phenomena including the fracture mechanics. But in order to pave the way for gradual transfer between the classical fracture mechanics and the state based fracture mechanics the weight function as the key function in classical fracture mechanics is derived in this section. The pioneers of computations of the weight function for computation of parameters of fracture mechanics were Bueckner [6] and Rice [7]. Their formulation as the other formulations in fracture mechanics are based on assumption that are partly incorrect. In the present section the aim is to propose a weight function, which is free of epistemic uncertainty as in Ranjbaran [52].

Set Eq. (72) equal Eq. (73) as in Eq. (78).

$$\int \sigma W_R dx = \sigma \times \sqrt{\frac{D^{(1)}(\xi)}{2O^2(\xi)}} \quad (78)$$

Omit (σ) from both sides and take derivative with respect to (ξ), to obtain the explicit definition for the weight function as in Eq. (79).

$$W_R = \frac{D^{(2)}(\xi)O(\xi)+2D^{(1)}(\xi)D^{(1)}(\xi)}{2\sqrt{2}\,O^2(\xi)\sqrt{D^{(1)}(\xi)}} \tag{79}$$

The proposed weight function is verified via comparison with the results of the others in the literature as follows.

Example 4: The weight function, (WFT), for determination of the stress intensity factor is defined by Fett [25] as in Eq. (80). Compare the (WFT) with the proposed weight function (WR)as defined in Eq. (79)?

$$WFT \times \sqrt{2\pi\xi} = \left(1 + \left(\begin{array}{l} 0.84683 - 0.07567\xi \\ +11.7732\xi^2 - 11.6391\xi^3 \\ +12.2052\xi^4 \end{array}\right)(1-\xi)^{-3/2}\right) \tag{80}$$

Solution4: The proposed weight function (WR) defined as in Eq. (79) is compared with the (WFT) in Fig. 12. Close agreement of the results verified the work. Eq. (80) is completely problem dependent. This is clear from the decimal numbers of coefficients and the style and power of the other terms. In brief, if the data is available, it is not easy, if not impossible, to reconstruct the equation! Since possibility of reconstruction of the results is a desirable property of equation construction, then the proposed Eq. (79) which can be derived directly and can be used for any problem, is undoubtedly the final choice, for the whole era of fracture mechanics.

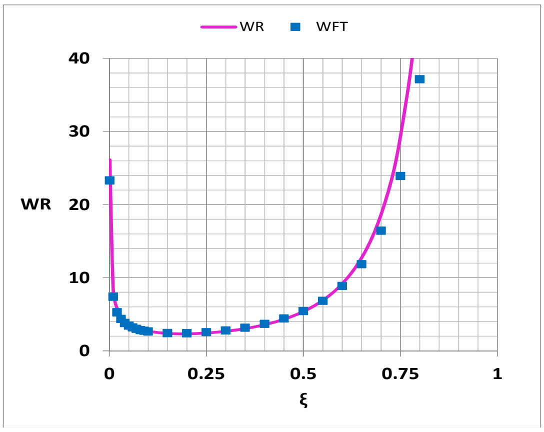

Fig.12. Comparison of weight function WR and WFT

Example 5: Yuan R. [24], in the University of Berkeley, in his PhD thesis derived the weight function for determination of the stress intensity factor for a single edge notched plate (WYU) as in Eq. (81).

$$\begin{aligned} A &= \left(2\tan(0.5\pi\xi)\right)^{1/2}\left(\begin{array}{l} 0.752 + 1.2\times 2.02\xi \\ +0.37\left(1-\sin(0.5\pi\xi)\right)^3 \end{array}\right)\Big/\cos(0.5\pi\xi) \\ B &= \xi\left(1.46 + 1.2\times 3.42\left(1-\cos(0.5\pi\xi)\right)\right)\Big/(0.5\pi\xi)^2 \\ WYU &= A\times B \end{aligned} \tag{81}$$

Compare the (WYU) with the proposed weight function (WR) as defined in Eq. (79)?

Solution5: The proposed weight function (WR) defined as in Eq. (79) is compared with the (WYU) in Fig. 13. The results are in good agreement with each other, then the work is verified.

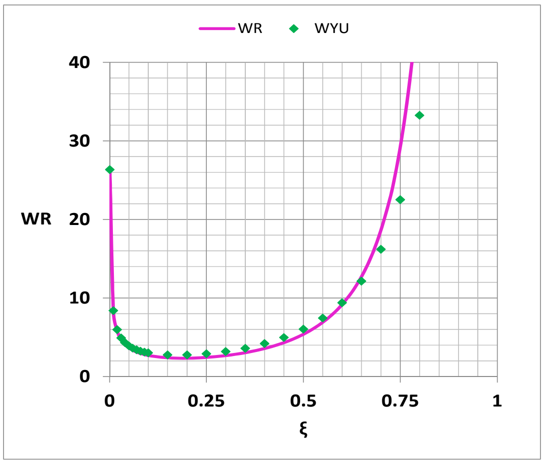

Fig.13 Comparison of weight function WR and WYU

Example 6: Analysis of stress intensity factor for a plate with an inclined crack.

Solution6: A plate with an inclined crack is selected for study. This plate is analyzed by Azevedo [23]. The properties as in Eq. (82) are used.

$$\begin{aligned} &a = 0.5\,mm \quad b = 10\,mm \quad h = 10\,mm \\ &\sigma = 200\,MPa \quad E = 7\times 10^4\,MPa \quad \nu = 0.33 \\ &\alpha(\deg) = 0 \;\; 10 \;\; 20 \;\; 25.56 \;\; 37 \;\; 45 \;\; 53 \;\; 63.44 \;\; 70 \;\; 80 \;\; 90 \end{aligned} \tag{82}$$

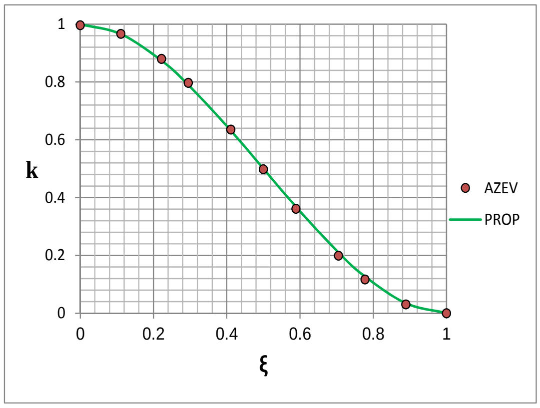

Fig.14. Comparison of stress intensity factors $PROP$ and $AZEV$

Corresponding to each angle the plate is modeled by two meshes each with about 20000 elements. The stress intensity factors were computed and shown as ($AZEV$) in Fig. 14. In this figure the abscissa is ($\xi = \alpha/90$) and the ordinate is the stress intensity factor ratio ($k = K_\alpha/K_0$) where (K_0) is the factor for (

$\alpha = 0$) and (K_α) is the factor for ($\alpha \neq 0$). The proposed result ($PROP = O(\xi)$) is compared with the ($AZEV$) in Fig. 14. Close agreement of the results verified the work.

Conclusions

The following conclusions were obtained from the present work.

- The highly advanced classical fracture mechanics is looked via a different perspective in which there is no problem of singularity and no need for calculation of difficult integrals.
- The state functions and the generic compliances are introduced and defined as explicit functions of the crack depth ratio.
- The crack energy is equal to the strain energy of the structure affected region times the generic compliance.
- The crack compliance is equal to the product of the stiffness coefficient and the generic compliance.
- The stress intensity factor is defined as an explicit function of the crack depth ratio.
- The proposed formulation is derived in generic form and is applicable to all structures.
- Through analysis of typical examples the validity of the work is verified.

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