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Original Research Article

Step Direction Computation of the First-Order Reliability Analysis Based on Two-Step Root-Finding Methods

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Abstract

The iHLRF algorithm is a popular iterative algorithm for determining the failure probability in structural reliability problems. It belongs to the family of first-order reliability methods (FORM) and is known for its fast convergence and remarkable simplicity. However, in cases where the limit state function oscillates significantly near the design point, which often occurs in high nonlinear limit state functions, the iHLRF algorithm may suffer from convergence issues. To address these convergence issues, this paper proposes three two-step direction determination techniques for first-order analysis. These techniques are based on two-step root-finding methods with a higher convergence rate than existing methods. The proposed techniques aim to improve the accuracy and robustness of the iHLRF algorithm, especially in cases where the limit state function shows highly nonlinear behavior. A numerical example with high nonlinear limit state functions in standard normal space is presented to demonstrate the proposed techniques' efficiency and capability. The performance of each proposed technique is compared with other existing methods, highlighting the advantages and limitations of each approach. Overall, this paper aims to contribute to developing more accurate and reliable methods for determining the reliability index in structural reliability problems, with the potential to be applied in various engineering fields.

Keywords: Reliability analysis method; Two-step root-finding; Step direction; Convergence rate; Nonlinear limit state function.

1. Introduction

Accurately determining the probability of failure in reliability analysis problems requires multiple integrations, which is time-consuming. Simulation methods are among the alternative methods to direct integration. Monte Carlo simulation and importance sampling, are two well-known methods that require thousands of simulations to determine the probability of failure [1]–[9]. For this reason, the implementation of iterative approximate methods for determining the probability of failure has been investigated. Iterative approximation methods are among the other reliability analysis techniques to determine the failure probability based on the statistical moments of the random variables. These methods start by searching for an initial response vector and finding the design point in a few iterations with the intended approximation. One simple and fast iterative method for solving reliability problems is the first-order reliability method (FORM). This method involves expanding the limit state function around the design point, which is called the most probable point (MPP), using the Taylor series [10].

Choosing the appropriate method to achieve the right linearization around the design point is one of the key steps in the first-order reliability method (FORM), and the result is accessing an iterative process to reach the problem solution. Hasofer and Lind first introduced the method. They introduced the concept of the reliability index, which is equal to the shortest distance from the origin to the point located on the limit state function in standard normal space [11], [12]. Then, it was improved by Rackwitz and Fiessler to a comprehensive form for use in problems with non-normal random variables [13]–[15]. In their proposed method, due to considering the unit value for the step size, severe convergence problems arose, similar to the problems in the initial Newton root-finding methods. Zhang and Kiureghian addressed this issue by controlling the merit function and considering an appropriate step size value [16], using the Armijo rule to optimize the merit function.

However, FORM is limited in solving high-order nonlinear limit state functions due to its first-order approximation by Taylor series expansion, which is used in the linear fitting of the limit state function. Another method proposed to overcome this limitation is the

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second-order reliability method (SORM) [17], which employs a second-order Taylor series expansion and higher-order derivatives. However, this approach faces challenges in problem-solving when the number of random variables increases.

Another approach presented by researchers to improve the efficiency of the FORM method is to provide new linearization of the limit state function. For example, in the reference [18], the univariate function method was implemented to linearize the limit state function. Another solution is to use statistical moments of the random variables, such as the third and fourth-moment methods, where the limit state function is transformed from physical space to standard normal space using statistical information of the random variables before starting the problem-solving process. The failure probability is determined in the next step using the abovementioned methods [19]–[21]. The numerical nonlinear optimization algorithm is another approach to finding design points in the structural reliability problem. Well-known instances in this regard include Gradient Projection (GP), Augmented Lagrangian Method (ALM), and Sequential Quadratic Programming (SQP) [22]–[25]. SQP, a popular robust optimization method, is a gradient-based method to implement inequality and equality constraint problems. Indeed, the initial idea of employing the first-order reliability method was triggered by the SQP algorithm.

In all these methods, the first-order Taylor series expansion plays a key role in the initial convergence and efficiency. Therefore, it is necessary to reconsider the linearization of the limit state function and investigate higher-order and multi-step convergence methods. Modified Newton-Raphson methods with various convergence orders, which are modern root-finding methods in engineering for finding roots of nonlinear equations, can be used to linearize functions [26]–[28]. This function linearization is used in the FORM method to determine the distance from the design point to the origin in standard normal space and improve problem-solving convergence. To this end, two-step methods such as Double-Newton and Chun and Porta-Pták methods should be utilized to achieve better convergence, especially in high-order nonlinear functions, without significantly increasing computational cost [29]–[34]. These methods' main feature is increasing the convergence order while maintaining the first-order derivative.

In this article, three methods for determining the step direction are proposed to improve the convergence rate in the FORM method. These methods include Double-Newton, Chun, and Porta-Pták, which encompass classic and modern root-finding methods. Despite increasing the convergence order, these methods do not increase the derivative order. The main difference between the proposed methods lies in the approximation of the hyperplane used in solving the problem, which results in obtaining different values for the step direction, especially in the initial steps of problem-solving. In the second section, the modified FORM method or iHLRF is introduced, with the process of determining the step direction and step size. The third section presents the numerical procedure of the mentioned methods to determine the step direction. In the fourth section, a numerical example is given to examine the performance of the proposed methods. Finally, in the fifth section, the conclusions are presented.

2. Modified first-order method (iHLRF)

This method follows an iterative approach to find the design point using a recursive line search algorithm, expressed as Eq (1) [16].

$$u_{m+1} = u_m + s_m.d_m \tag{1}$$

u_m is the vector of random variables at the design point, m is the iteration number, s_m is the step size, and d_m is the step direction vector. The first step in this method is to determine the step direction. For this reason, the method starts with linearizing the limit state function using the first-order Taylor series expansion around the design point, as expressed in Eq (2).

$$G(u) \approx G(u_m) + \nabla G(u_m)^T .(u - u_m) \tag{2}$$

In this method, reaching the limit state surface is important as it determines the boundary between safety and failure. To determine the limit state surface, it is necessary to set Eq (2) equal to zero, and in this case, Eq (2) will be transformed into an equation of a hyperplane, as shown in Eq (3).

$$G(u) \approx G(u_m) + \nabla G(u_m)^T .(u - u_m) = 0 \tag{3}$$

According to geometric principles, the distance from the origin to the closest point on the limit state surface can be obtained by placing the geometric coordinates of the origin in the equation of the hyperplane and dividing by the magnitude of the gradient vector of the limit state function, which is presented in Eq (4).

$$\Delta = \frac{G(0) = 0}{\left\|\nabla G(u_m)\right\|} = \frac{G(u_m) - \nabla G(u_m)^T .u_m}{\left\|\nabla G(u_m)\right\|} \tag{4}$$

Given that the normalized and scaled value of the gradient vector is known as the vector α, Eq (4) can be rewritten as Eq (5).

$$\Delta = \frac{G(0) = 0}{\left\|\nabla G(u_m)\right\|} = \frac{G(u_m)}{\left\|\nabla G(u_m)\right\|} + \alpha.u_m \tag{5}$$

If the limit state function is linear, multiplying Eq (5) by the vector α will result in the final solution, which will be the same as the distance from the origin according to Eq (6).

$$u_{m+1} = \Delta\alpha \tag{6}$$

For problems with a nonlinear limit state function, Eq (6) is not the final solution but is used to determine the solution in the next step. Therefore, the value of the step direction can be defined as Eq (7).

$$d_m = u_{m+1} - u_m = \Delta\alpha - u_m = ...$$
$$\left(\frac{G(0)=0}{\|\nabla G(u_m)\|}\right)\alpha - u_m \quad (7)$$

The next step in this method is to determine the step size, which is estimated using Eq (8).

$$s_m = b^k, \quad b = 0.5 \quad (8)$$

Where k is an integer that increases if the convergence conditions are not obtained. The convergence criterion used for this method is the Armijo rule, in which the merit function is controlled by checking the step size. One of the simplest relations for determining the convergence condition is summarized as Eq (9).

$$m(u_{m+1}) \le m(u_m) \quad (9)$$

The function m represents the merit function obtained from Eq (10).

$$m(u) = 0.5\|u\|^2 + c.|G(u)| \quad (10)$$

In which the value of c is equal to Eq (11).

$$c = \gamma.\frac{\|u_m\|}{\|\nabla G(u_m)\|} + \eta \quad , \gamma = 2 \ \& \ \eta = 10 \quad (11)$$

If Eq (9) is not satisfied, the step size value, or s_m, will decrease in each iteration by increasing the value of k in Eq (8). It is also possible to consider a final value for the step size reduction direction; for example, the maximum value of k is 6. The steps described above will be used in all of the methods presented below. The difference between the methods presented is related to determin the value of Δ in Eq (5). The method used in the iHLRF algorithm is from the family of Newton methods in root-finding problems, which uses the first-order gradient. Despite its simplicity and high convergence rate, especially for highly nonlinear functions, it has some limitations. Therefore, in section three, a suitable solution will be adopted by introducing proposed methods to address these limitations.

3. Presenting methods to estimate step direction

This section presents some methods used in root-finding problems that can also be used for function linearization. These methods are in the class of two-step methods, whose goal is to determine the direction in finding the design point, including Double-Newton, Chun, and Porta-Pták methods. The reason for using these methods is that they do not cause a significant increase in computational cost up to convergence, especially for highly nonlinear functions [29]–[34]. The output of these methods is a new form of the hyperplane equation used in the iHLRF method, and they differ only in calculating the step direction.

3.1 The First Method: Double-Newton

The Double-Newton method is one of the advanced two-stage methods in the family of Newtonian root-finding methods. It has predictive and corrective items, and its one-dimensional form for finding the root of a function is expressed as Eq (12) [26], [29].

$$z_k = x_k - \frac{f(x_k)}{f'(x_k)}$$
$$x = x_k - \frac{f(x_k)}{f'(x_k)} - \frac{f(z_k)}{f'(z_k)} \quad (12)$$

In which z_k is the predictive point, calculated by the first-order Taylor expansion around the design point x_k, which is the current estimate of the root at iteration k, and x is the estimation of the root at the next step. The second part in Eq (12) can be rewritten and used to linearize the function, as shown in Eq (13). To this end, the value of the gradient z_k can be replaced by the gradient x_k, which does not make much difference in the results.

$$f(x) = \left[f(x_k) + f(z_k)\right] + ...$$
$$f'(x_k)(x - x_k) = 0 \quad (13)$$

The vector form of Eq (13) for G(u)=0, where u is a vector of random variables, is given by Eq (14).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$G(u) = \left[G(u_m) + bG(z_m)\right] + ... \quad (14)$$
$$\nabla G(u_m)^T.(u - u_m) = 0$$

Using Eq (14), a new form of the hyperplane equation and the calculation of Δ (Eq (4)) are obtained. The coefficient b in Eq (14) controls convergence in this method, and its value has been chosen empirically as -0.5. Thus, the step direction vector in this method is given by Eq (15).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$d_m = \left(\frac{G(0)}{\|\nabla G(u_m)\|}\right)\alpha - u_m = ... \quad (15)$$
$$\left[\frac{\left[G(u_m) + bG(z_m)\right] - \nabla G(u_m)^T.u_m}{\|\nabla G(u_m)\|}\right]\alpha - u_m$$

3.2 The Second Method: Chun

The second method is an iterative method with fourth-order convergence for solving nonlinear equations, and its root-finding form is expressed by Eq (16) [30].

$$z_k = x_k - \frac{f(x_k)}{f'(x_k)}$$
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} - ... \quad (16)$$
$$\left[1 + 2\frac{f(z_k)}{f(x_k)} + \frac{f(z_k)^2}{f(x_k)^2}\right]\frac{f(z_k)}{f'(x_k)}$$

The second part in Eq (16) can be rewritten and used to linearize the function, as shown in Eq (17).

$$f(x) = (x - x_k)f'(x_k) + f(x_k) + ...$$
$$\left[1 + 2\frac{f(z_k)}{f(x_k)} + \frac{f(z_k)^2}{f(x_k)^2}\right]f(z_k) = 0 \quad (17)$$

The vector form of Eq (17) for G(u)=0, where u is a vector of random variables, is given by Eq (18).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$G(u) = 0 = G(u_m) + \ldots$$
$$\left[1 + 2\frac{G(y_z)}{G(u_m)} + \frac{G(z_m)^2}{G(u_m)^2}\right] G(z_m) + \ldots \quad (18)$$
$$\nabla G(u_m)^T . (u - u_m)$$

The second part in Eq (18) is the hyperplane equation obtained in this method, which determines the step direction. Thus, using Eq (18), a new form of the hyperplane equation and the calculation of Δ (Eq (4)) is obtained, and using them, the step direction vector in this method is taken as Eq (19).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$G(u) = 0 = G(u_m) + \ldots$$
$$\left[1 + 2\frac{G(y_z)}{G(u_m)} + \frac{G(z_m)^2}{G(u_m)^2}\right] G(z_m) + \ldots \quad (19)$$
$$\nabla G(u_m)^T . (u - u_m)$$

3.3 Third Method: Porta-Pták

The third method is a third-order convergence method that combines Newton's and Steffensen's methods to determine the simple and real roots of nonlinear equations with one variable, and its relations are given by Eq (20) [21-24].

$$z_k = x_k - \frac{f(x_k)}{f'(x_k)}$$
$$x_{k+1} = x_k - \left(1 + \frac{f(z_k)}{f(x_k) - \alpha f(z_k)}\right)\frac{f(x_k)}{f'(x_k)} \quad (20)$$

The second part in Eq (21) can be rewritten and used to linearize the function, as shown in Eq (22).

$$f(x) = 0 = (x - x_k) f'(x_k) + \left(\frac{f(x_k)^2}{f(x_k) - f(z_k)}\right) \quad (21)$$

The vector form of Eq (21) for G(u)=0, where *u* is a vector of random variables, is given by Eq (22).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$G(u) = 0 = \nabla G(u_m)^T (u - u_m) + \ldots \quad (22)$$
$$\left(\frac{G(u_m)^2}{G(u_m) - G(z_m)}\right)$$

The second part in Eq (22) is the hyperplane equation obtained in this method, which determines the step direction. Thus, using Eq (22), a new form of the hyperplane equation and the calculation of Δ (Eq (4)) is obtained, and using them, the step direction vector in this method is taken as Eq (23).

$$z_m = u_m + \frac{G(u_m)}{\|\nabla G(u_m)\|}\alpha$$
$$d_m = u_{m+1} - u_m = \ldots$$
$$\frac{1}{\|\nabla G(u_m)\|}\left[-\nabla G(u_m)^T (u_m) + \left(\frac{G(u_m)^2}{G(u_m) - G(z_m)}\right)\right]\alpha - u_m \quad (23)$$

4. Numerical simulation

This section presents an example to evaluate and compare the efficiency and the convergence of the proposed methods. A nonlinear limit is employed. These cases will also exhibit greater severity after transforming the random variables to non-correlated standard normal variables. Due to neglecting the effects of transforming random variables between the physical space and the standard normal space and better observing the effects of the proposed methods, the third-moment method [19] transforms the limit state function into a function of non-correlated standard normal variables. Then the final results of the proposed methods are compared with iHLRF and importance sampling methods [1], [16], [35]. The starting point is the origin in the standard normal space.

This example has a linear limit state function in the physical space given by Eq (24), where coefficients are considered for random variables [19], [21], [36], [37].

$$G(X) = 18 - 3X_1 - 2X_2 \quad (24)$$

Table 1 presents the specifications of the random variables, statistical moments to use in the third-moment transformation method.

Table 1. Probability Distribution of Random Variable

Var	Distribution	Mean	Variance	Skewness
X_1	Exponential	1.0	1.0	2.0
X_2	Exponential	1.0	1.0	2.0

The correlation coefficient between the two variables is equal to -0.403653. To solve this example, firstly, using the statistical information in Table 1, the limit state function is transformed by the third-moment method into a function composed of non-correlated standard normal variables, resulting in Eq (25). This example is linear in the physical space. Still, it becomes nonlinear after transformation using the third-moment method, where the second-order terms and the product of random variables are observed.

$$G(u) = -1.26u_1 - 1.53u_1^2 + 0.72u_1u_2 - \ldots$$
$$1.10u_2 - 0.3u_2^2 + 14.83 \quad (25)$$

The solution to this example using the importance sampling method for Eq (25) with a coefficient of variation of 0.2 for 7800 simulations equals 2.70404. The final result is presented using the treated methods in Figure 1.

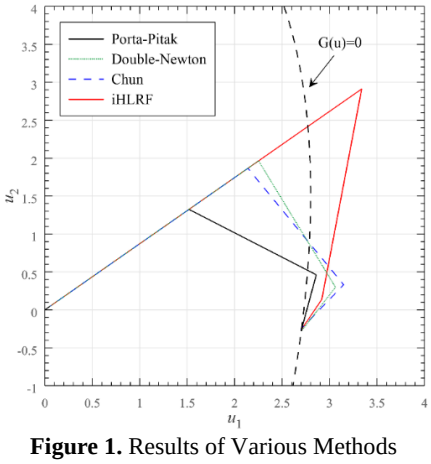
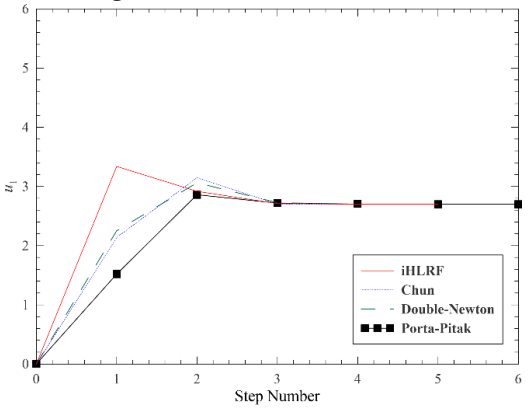
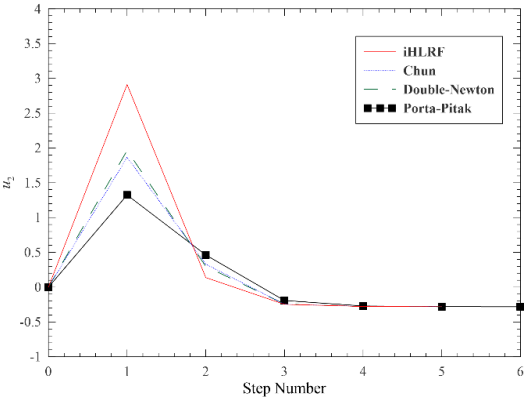


Figure 1. Results of Various Methods

Based on Fig 1, it can be seen that all methods have reached the final result, but the steps taken to reach this result, especially in the initial steps, are different from each other. In the first step, the iHLRF method had the largest movement, and the Double-Newton, Chun, and Porta-Pták methods had less movement, respectively. This is due to the difference in convergence order of each method compared to the others, which is reflected in the step direction and, ultimately, in the step size. After completing the second step and reaching the third step, the results are near the limit state function. In other words, different methods have converged to similar values. The contents discussed in this section and the response of each random variable until final convergence for each method are presented in Figure 2.



(a)



(b)

Figure 2. Results of Various Methods; **(a):** The First Variable; **(b):** The second variable Changes in each step

5. Conclusion

This article presented three types of mathematical relationships for determining the step direction using modern and classical two-step root-finding methods, which were used to solve complex problems. A single example was studied in this article to evaluate the methods and compare their differences with the iHLRF method. Although all methods achieved a similar final solution, the steps taken by these methods, especially in the initial steps, were different from each other. The presented example showed that the Chun and Porta-Piták methods had shorter steps and smoother convergence behavior compared with the iHLRF method. In contrast, the Double-Newton method resulted in larger movement in each computational step. As the results approached the limit state surface, the oscillations between the methods gradually decreased, and similar behavior were observed between the methods. However, the Double-Newton method was a simple method and continued to have larger displacements until it approached the limit state surface in problems with higher nonlinear orders. Although these methods achieved the final solution, the accuracy of this solution was still in the first-order methods due to the use of first-order derivatives to calculate the step direction. To improve the computational accuracy of the final result, combining methods such as the SORM method can be used after determining the final result using any of these methods. Ultimately, the results demonstrate the high flexibility of these methods in analyzing reliability problems. As a scope of future research, it is recommended to study other modern root-finding methods, such as 3 and 4-step methods with convergence orders of 6 and 8, is recommended.

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