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Original Research Article

Copula-Based Approach to Reliability Analysis of Phased-Mission Systems

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Abstract

A phased-mission system (PMS) involves several different tasks or phases that must be accomplished in sequence. The system configuration, task success criteria, and component failure characteristics may vary from phase to phase. Consequently, the reliability evaluation of PMSs is more challenging than that of single-phase in the field of system reliability analysis. The paper deals with the reliability evaluation of non-repairable Phased-Mission Systems with three phases and five phases involving dependent components in each phase. The cumulative exposure model has been used to model a PMS, and the dependency between components of a system in a phase is modeled using the Gumbel-Hougaard copula. Reliability importance analyses of the 3-PMS and 5-PMS have also been carried out. The method developed has been illustrated using numerical examples. The proposed methodology can also be generalized to PMSs with more than five phases.

Keywords: copulas; cumulative exposure model; phased-mission system; reliability; reliability importance measure.

1. Introduction

The increasing level of complexity and automation in engineering systems has resulted in dependencies among components within these systems. The operation of missions encountered in aerospace, nuclear power, chemical, electronic, navigation, military fields, and many other applications often involves several different tasks or phases that must be accomplished in sequence. The system configuration, task success criteria, and component failure characteristics may vary from phase to phase. During each mission phase, the system has to accomplish a specified task and may be subject to different stresses as well as different dependability requirements. This dynamic behavior requires a distinct model for each phase of the mission in the reliability analysis to be able to verify whether a system has met desired reliability.

Definition 1.1: A **phased-mission system (PMS)** is defined as a system where the mission consists of phased sub-missions whose relevant configuration changes during time periods (phases).

Definition 1.2: The **reliability of a PMS** is defined as the probability for all tasks in the PMS to complete successfully.

The evaluation of the reliability of a PMS must account for changes in configuration, component use, and stresses.

Some examples of PMSs are:

- An aircraft flight involves take-off, ascent, level-flight, descent, and landing phases. During each phase, the system has to accomplish a specified task and may be subject to different stresses, environmental conditions, and reliability requirements. For example, in a twin-engine airplane, one engine is required during the taxi phase, but both engines are necessary during the take-off phase. In addition, the engines are more likely to fail during the take-off period because they are generally under enormous stress in this phase as compared to other phases of the flight profile. See for example [1] and [2].
- The batch processing of jobs on a distributed computer system in which each job requires different system resources to be available, thus resulting in different success criteria for each task.
- In a boiling water reactor [3], a loss of coolant accident involves three phases for emergency core cooling - initial core cooling, suppression core cooling, and residual heat removal.

PMSs introduced by [4] have been studied extensively in the literature. There are broadly three classes of analytical approaches to analyze the reliability of PMSs, viz., Combinatorial approach, State-space oriented Method, and Phase Modular approach.

Combinatorial methods exploit Boolean algebra and various forms of decision diagrams and can handle any arbitrary types of distributions [2], [5]- [9]. State space-based methods (e.g., Markov chains, Petri nets) are powerful and flexible in modeling various dependencies but suffer from state explosion when modeling medium to large-scale systems [5], [10]. The phased Modular approach is the integrated approach combining the combinatorial approach and state space-based approach [2]. See also [6], [11]- [16]. Simulation methods can typically offer great generality in representing system behavior but can only provide approximate results [6]. The present paper uses the copula-based approach to capture dependencies amongst the components of the system in each phase. Copulas help model dependency between dependent components of a reliability system. The dependence structure relates the known marginal life distributions of components to their multivariate distribution [17]. The kind of dependence structure comes from the choice of an appropriate copula. There are many types of copula functions, such as **Gaussian copula**, **Student's t-copula**, **Frank copula**, **Clayton copula** and **Gumbel copula**. The copula-based approach in reliability theory has been studied by several authors, for example, [18]- [20]. However, this approach has not been used in PMSs so far. ***Gumbel-Hougaard Copula*** is used in this paper. The concept of equivalent age of a component to represent the cumulative damage it has accrued up to a given point of time is used [21].

The paper is organized as follows:

Section 2 describes the PMS models considered. Section 3 describes the copula function; Section 4 presents the method for evaluating the entire phased mission reliability; reliability importance analyses of the three PMSs have been carried out in Section 5, and Section 6 illustrates the proposed method.

2. PMSs Model Description

Two different three phases of mission systems (3-PMS) have been used, as depicted in Figure 1 see [10] and Figure 2 [9]. Also, the 5-PMS system representing the space application mission discussed by [22]- [23] (see also [9]) is shown in Figure 3.

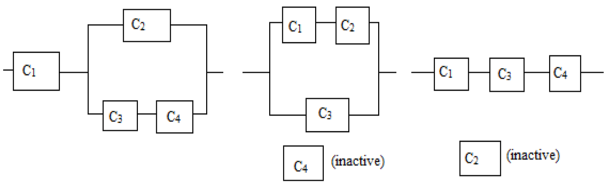

Figure 1. 3-PMS with inactive components.

Figure 1 comprises three phases with:

- The first phase comprises two subsystems in series, with the first subsystem composed of one component, C_1, and the second subsystem being a parallel-series system of two subsystems with one composed of one component, C_2, and the other two components, viz., C_3 and C_4,
- the second phase is composed of a parallel-series configuration of two subsystems in which one is composed of two components, viz., C_1 and C_2, and the other one component, C_3; component C_4 being inactive,
- the third phase consists of a series configuration of three components, C_1, C_3, and C_4; component C_2 is inactive.

Figure 2 comprises three phases with:

- the first phase comprises a series configuration of three components, viz., A, B, and C,
- the second phase comprises a parallel configuration of three components, viz., A, B, and C,
- The third phase is composed of a series-parallel configuration of two subsystems, with one comprising one component, A, and the other comprising two components, B and C.

Figure 3 comprises five phases with:

- the first phase is launch comprising 3-out-of-4 subsystems in series with a parallel subsystem of order 2,
- the second phase is Hibern.1 comprises a parallel system of order 2,
- the third phase is Asteroid comprising a 3-out-of-4 subsystem in series with a parallel subsystem of order 2,
- the fourth phase is Hibern.2 comprises a parallel system of order 2,
- the fifth phase is Comet comprising a 3-out-of-4 subsystem in series with a parallel subsystem of order 2.

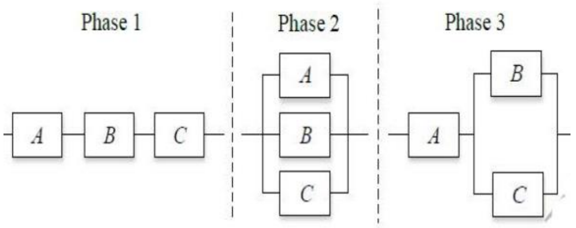

Figure 2. 3-PMS with active components.

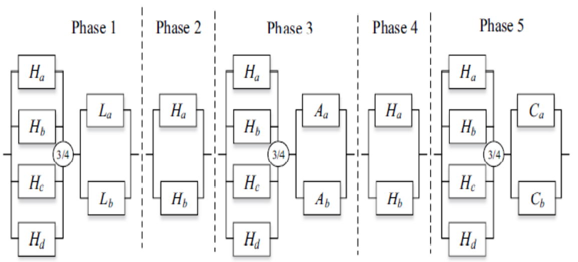

Figure 3. 5-PMS with active components (Spacecraft Application).

Assumptions

The reliability of these PMSs is derived using the following assumptions:

- The lifetimes of all the components in the subsystems are dependent.
- The components in a phase follow a Weibull or exponential life distribution.
- The structure of the system varies across the phases.

3. Copula Function

The dependency existing between the marginal random variables in bivariate and multivariate distributions is described by a copula [17]. The copula describes the way in which the marginal are linked together on the basis of their association.

The Weibull life distribution is widely used in the industrial situation, and exponential life distribution is its particular case. The reason for using Gumbel-Hougaard Copula in this work is the existence of the following relationship:

Weibull life distribution ⇔ Gumbel-Hougaard Copula

for the bivariate case, which can be extended to n dimensions, see [24].

Let X_1, X_2 and X_3 be the random variables with $\bar{G}_1(x_1), \bar{G}_2(x_2)$ and $\bar{G}_3(x_3)$ as their marginal reliability functions, respectively. Let $\bar{H}(x_1,x_2,x_3)$ be their corresponding joint reliability function. Then, according to Sklar's Theorem, there exists a copula reliability function $C(\cdot,\cdot,\cdot)$ such that for all (X_1, X_2, X_3) in the defined range,

$$\bar{H}(x_1,x_2,x_3) = C\left(\bar{G}_1(x_1), \bar{G}_2(x_2), \bar{G}_3(x_3)\right), \quad (1)$$

Three- dimensional Gumbel-Hougaard copula [25] is defined as:

$$C_\theta(u,v,w) = exp\left[-\left((-log_e[u])^\theta + (-log_e[v])^\theta + (-log_e[w])^\theta\right)^{1/\theta}\right], \quad (2)$$

where $\theta\epsilon[1,\infty)$ characterizes the association between the two variables.

Similarly, the four-dimensional Gumbel- Hougaard copula is defined as:

$$C_\theta(u,v,wz) = exp\left[-\left((-log_e[u])^\theta + (-log_e[v])^\theta + (-log_e[w])^\theta + (-log_e[z])^\theta\right)^{\frac{1}{\theta}}\right], \quad (3)$$

Weibull marginal with reliability function

$$R(t) = \exp\left[-\left(\frac{t}{\alpha}\right)^\beta\right], t > 0;\ \alpha > 0;\ \beta > 0 \quad (4)$$

is used in the paper.

4. Mission Reliability Evaluation

Let 3-PMS in Figure 1 and Figure 2 be denoted as PMS-1 and PMS-2, respectively, and 5-PMS in Figure 3 be denoted as PMS-3. In Section 4.1, reliability, $\bar{F}_{PMS-I}(t)$, of PMS-1 is computed, in section 4.2, reliability, $\bar{F}_{PMS-II}(t)$, of PMS-2 is obtained, and finally, reliability, $\bar{F}_{PMS-III}(t)$, of PMS-3 is evaluated in section 4.3.

4.1 Reliability of PMS-1

Let T_1, T_2, T_3 and T_4 denote lifetimes of the components with reliabilities $\bar{G}_1(t), \bar{G}_2(t), \bar{G}_3(t)$, and $\bar{G}_4(t)$ respectively. Let $\bar{F}_{11}(t), \bar{F}_{21}(t), \bar{F}_{31}(t)$ be the reliability of subsystems in Phase 1, phase 2, and Phase 3, respectively. Then, the reliability of PMS-1 is:

$$\bar{F}_{PMS-I}(t) = \begin{cases} \bar{F}_{11}(t), 0 \le t \le \tau_1 \\ \bar{F}_{21}(t), \tau_1 \le t \le \tau_2, \\ \bar{F}_{31}(t), \tau_2 \le t \le \tau_3 \end{cases} \quad (5)$$

$$\bar{F}_{11}(t) = P[min\{T_1, T_1'\} > t] = P[T_1 > t, T_1' > t], \quad (6)$$

where,

$T_1' = max\{T_2, min\{T_3, T_4\}\}$

$$\bar{F}_{21}(t) = P[T_2' > t], \quad (7)$$

where

$T_2' = max\{min\{T_1, T_2\}, T_3\}$,

$$\bar{F}_{31}(t) = P[T_3' > t], \quad (8)$$

where,

$T_3' = min\{T_1, T_3, T_4\}$,

Consider

$\bar{F}_{11}(t) = P[min\{T_1, T_1'\} > t]$,

where, $T_1' = max\{T_2, min\{T_3, T_4\}\}$

$$\begin{aligned} &= P[T_1 > t, T_1' > t] \\ &= P[T_1 > t] - P[T_1 > t, T_1' \le t] \\ &= P[T_1 > t] - P[T_1 > t, T_2 \le t, min\{T_3, T_4\} \le t] \\ &= P[T_1 > t] \\ &\quad - P[T_1 > t, T_2 \le t] - P[T_1 > t, T_2 \le t, min\{T_3, T_4\} > t] \\ &= P[T_1 > t] - \{P[T_1 > t] - P[T_1 > t, T_2 > t]\} \\ &\quad + \{P[T_1 > t, min\{T_3, T_4\} > t] - P[T_1 > t, T_2 > t, min\{T_3, T_4\} > t]\} \\ &= P[T_1 > t, T_2 > t] + P[T_1 > t, min\{T_3, T_4\} > t] \\ &\quad - P[T_1 > t, T_2 > t, min\{T_3, T_4\} > t] \\ &= C(\bar{G}_{11}(t), \bar{G}_{21}(t), 1,1) + C\left(\bar{G}_{11}(t), 1, \bar{G}_{31}(t), \bar{G}_{41}(t)\right) - C\left(\bar{G}_{11}(t), \bar{G}_{21}(t), \bar{G}_{31}(t), \bar{G}_{41}(t)\right). \end{aligned} \quad (9)$$

Consider now

$\bar{F}_{21}(t) = P[T_2' > t]$,

where $T_2' = max\{min\{T_1, T_2\}, T_3\}$.

$$\begin{aligned} \Rightarrow \bar{F}_{21}(t) &= 1 - P[T_2' \le t] \\ &= 1 - P[min\{T_1, T_2\} \le t, T_3 \le t] \\ &= 1 - \{P[T_3 \le t] - P[min\{T_1, T_2\} > t, T_3 \le t]\} \end{aligned} \quad (10)$$

$$= 1 - \{\{1 - P[T_3 > t]\} - P[T_1 > t, T_2 > t, T_3 \le t]\}$$
$$= 1 - \{\{1 - P[T_3 > t]\} - \{P[T_1 > t, T_2 > t] - P[T_1 > t, T_2 > t, T_3 > t]\}\}$$
$$= 1 - \Big\{\{1 - C(1,1,\bar{G}_{32}(t),1)\} - \{C(\bar{G}_{12}(t),\bar{G}_{22}(t),1,1) - C(\bar{G}_{12}(t),\bar{G}_{22}(t),\bar{G}_{32}(t),1)\}\Big\}.$$

Finally, consider

$$\bar{F}_{31}(t) = P[T_3' > t], \text{ where, } T_3' = min\{T_1, T_3, T_4\}. \Rightarrow \bar{F}_{31}(t) = P[T_1 > t, T_3 > t, T_4 > t] = C(\bar{G}_{13}(t), 1, \bar{G}_{33}(t), \bar{G}_{43}(t)). \quad (11)$$

(9), (10), and (11) give the reliability of the three subsystems in PMS-1.

Thus, the reliability of 3-PMS-1 system with $\bar{G}_{ji}(t)$ denoting reliability of j^{th} component in i^{th} subsystem, $j = 1,2,3,4$; $i = 1,2,3$, is:

$$\bar{F}_1(\tau_3) = P[\bar{F}_{11} > \tau_1]P[\bar{F}_{21} > \tau_2 \mid \bar{F}_{11} > \tau_1]P[\bar{F}_{31} > \tau_3 \mid \bar{F}_{11} > \tau_1, \bar{F}_{21} > \tau_2]$$
$$= P[\bar{F}_{11} > \tau_1, \bar{F}_{21} > \tau_2, \bar{F}_{31} > \tau_3] = C(\bar{F}_{11}(\tau_1), \bar{F}_{21}(\tau_2), \bar{F}_{31}(\tau_3)), \quad (12)$$

where

$$\bar{F}_{11}(\tau_1) = C(\bar{G}_{11}(\tau_1), \bar{G}_{21}(\tau_1), 1,1) + C(\bar{G}_{11}(\tau_1), 1, \bar{G}_{31}(\tau_1), \bar{G}_{41}(\tau_1)) - C(\bar{G}_{11}(\tau_1), \bar{G}_{21}(\tau_1), \bar{G}_{31}(\tau_1), \bar{G}_{41}(\tau_1)),$$

$$\bar{F}_{21}(\tau_2) = 1 - \Big\{\{1 - C(1,1,\bar{G}_{32}(\tau_2 - \tau_1 + l_{32}),1)\} - \{C(\bar{G}_{12}(\tau_2 - \tau_1 + l_{12}), \bar{G}_{22}(\tau_2 - \tau_1 + l_{22}),1,1) - C(\bar{G}_{12}(\tau_2 - \tau_1 + l_{12}), \bar{G}_{22}(\tau_2 - \tau_1 + l_{22}), \bar{G}_{32}(\tau_2 - \tau_1 + l_{32}),1)\}\Big\},$$

$$\bar{F}_{31}(\tau_3) = C(\bar{G}_{13}(\tau_3 - \tau_2 + l_{13}), 1, \bar{G}_{33}(\tau_3 - \tau_2 + l_{33}), \bar{G}_{43}(\tau_3 - \tau_2 + l_{43})),$$

using cumulative exposure model [21].

l_{12} is determined in such a way that $\bar{G}_{12}(l_{12}) = \bar{G}_{11}(\tau_1)$,

l_{22} is determined in such a way that $\bar{G}_{22}(l_{22}) = \bar{G}_{21}(\tau_1)$,

l_{32} is determined in such a way that $\bar{G}_{32}(l_{32}) = \bar{G}_{31}(\tau_1)$,

l_{13} is determined in such a way that $\bar{G}_{13}(l_{13}) = \bar{G}_{12}(\tau_2 - \tau_1 + l_{12})$,

l_{33} is determined in such a way that $\bar{G}_{33}(l_{33}) = \bar{G}_{32}(\tau_2 - \tau_1 + l_{32})$,

l_{43} is determined in such a way that $\bar{G}_{43}(l_{43}) = \bar{G}_{41}(\tau_2 - \tau_1)$.

4.1.1 Computation of Reliability of PMS-1

The reliability of 3-PMS–1 is computed using a four-dimensional Gumbel-Hougaard copula with Weibull marginal:

$$C(\bar{G}_{1i}(t_1), \bar{G}_{2i}(t_2), \bar{G}_{3i}(t_3), \bar{G}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{G}_{1i}(t_1))\right)^\theta + \left(-log(\bar{G}_{2i}(t_2))\right)^\theta + \left(-log(\bar{G}_{3i}(t_3))\right)^\theta + \left(-log(\bar{G}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right],$$

$$C(1, \bar{G}_{2i}(t_2), \bar{G}_{3i}(t_3), \bar{G}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{G}_{2i}(t_2))\right)^\theta + \left(-log(\bar{G}_{3i}(t_3))\right)^\theta + \left(-log(\bar{G}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right],$$

$$C(1,1, \bar{G}_{3i}(t_3), \bar{G}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{G}_{3i}(t_3))\right)^\theta + \left(-log(\bar{G}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right],$$

$$C(1,1,1, \bar{G}_{4i}(t_4)) = \bar{G}_{4i}(t_4),$$

where,

$$\bar{G}_{ji}(t) = exp\left[-\left(\frac{t}{\alpha_{ji}}\right)^{\beta_{ji}}\right], t > 0; \alpha_{ji} > 0; \beta_{ji} > 0, j = 1,2,3,4, \ i = 1,2,3,$$

$\beta_{ji} = 1$ implies a constant failure rate, $\beta_{ji} > 1$ implies an increasing failure rate, and $\beta_{ji} < 1$ implies decreasing failure rate.

Further, a constant failure rate signifies an exponential life distribution.

Similarly, copulas with different placements of 1s in $C(\bar{G}_{1i}(t_1), \bar{G}_{2i}(t_2), \bar{G}_{3i}(t_3), \bar{G}_{4i}(t_4))$ can be obtained.

4.2 Reliability of PMS-2 system

Let T_1, T_2 and T_3 denote lifetimes of the components with reliabilities $\bar{H}_1(t), \bar{H}_2(t)$ and $\bar{H}_3(t)$, respectively. Let $\bar{F}_{12}(t), \bar{F}_{22}(t), \bar{F}_{32}(t)$ be the reliability of subsystems in phase 1, phase 2, and phase 3, respectively. Then, the reliability of PMS-2 is:

$$\bar{F}_{PMS-II}(t) = \begin{cases} \bar{F}_{12}(t), 0 \le t \le \tau_1 \\ \bar{F}_{22}(t), \tau_1 \le t \le \tau_2, \\ \bar{F}_{32}(t), \tau_2 \le t \le \tau_3 \end{cases} \quad (13)$$

$$\bar{F}_{12}(t) = P[T_1' > t], \quad (14)$$

$$\text{where, } T_1' = min\{T_1, T_2, T_3\}, \quad \bar{F}_{22}(t) = P[T_2' > t], \quad (15)$$

$$\text{where } T_2' = max\{T_1, T_2, T_3\}, \quad \bar{F}_{32}(t) = P[T_3' > t], \text{ where } \quad (16)$$

$$T_3' = min\{T_1, max\{T_2, T_3\}\},$$

Consider

$$\bar{F}_{12}(t) = P[T_1' > t], \text{ where } T_1' = min\{T_1, T_2, T_3\}. \Rightarrow \bar{F}_{12}(t) = P[T_1 > t, T_2 > t, T_3 > t] = C(\bar{H}_{11}(t), \bar{H}_{21}(t), \bar{H}_{31}(t)). \quad (17)$$

Consider now

$\bar{F}_{22}(t) = P[T_2' > t]$, where

$T_2' = max\{T_1, T_2, T_3\}$.

$$\begin{aligned}\Rightarrow \bar{F}_{22}(t) &= 1 - P[T_2' \le t] \\ &= 1 - P[T_1 \le t, T_2 \le t, T_3 \le t] \\ &= 1 - \{P[T_1 \le t, T_2 \le t] - P[T_1 \le t, T_2 \le t, T_3 > t]\} \\ &= 1 - \{\{P[T_1 \le t] - P[T_1 \le t, T_2 > t]\} - \{P[T_1 \le t, T_3 > t] - P[T_1 \le t, T_2 > t, T_3 > t]\}\} \\ &= 1 - \{P[T_1 \le t] - \{P[T_2 > t] - P[T_1 > t, T_2 > t]\}\} \\ &\quad + \{\{P[T_3 > t] - P[T_1 > t, T_3 > t]\} - \{P[T_2 > t, T_3 > t] - P[T_1 > t, T_2 > t, T_3 > t]\}\} \\ &= 1 - \{[1 - C(\bar{H}_{12}(t), 1,1)] - \{C(1, \bar{H}_{22}(t), 1) - C(\bar{H}_{12}(t), \bar{H}_{22}(t), 1)\}\} + \\ &\quad \{\{C(1,1, \bar{H}_{32}(t)) - C(\bar{H}_{12}(t), 1, \bar{H}_{32}(t))\} - \{C(1, \bar{H}_{22}(t), \bar{H}_{32}(t)) - C(\bar{H}_{12}(t), \bar{H}_{22}(t), \bar{H}_{32}(t))\}\}.\end{aligned} \quad (18)$$

Finally, consider

$\bar{F}_{32}(t) = P[T_3' > t]$, where

$T_3' = min\{T_1, max\{T_2, T_3\}\}$.

$$\begin{aligned}\Rightarrow \bar{F}_{32}(t) &= P[T_1 > t, max\{T_2, T_3\} > t] \\ &= P[T_1 > t] - P[T_1 > t, max\{T_2, T_3\} \le t] \\ &= P[T_1 > t] - P[T_1 > t, T_2 \le t, T_3 \le t] \\ &= P[T_1 > t] - \{P[T_1 > t, T_3 \le t] - P[T_1 > t, T_2 > t, T_3 \le t]\} \\ &= P[T_1 > t] - \{P[T_1 > t] - P[T_1 > t, T_3 > t]\} + \{P[T_1 > t, T_2 > t] - P[T_1 > t, T_2 > t, T_3 > t]\} \\ &= C(\bar{H}_{13}(t), 1,1) - \{C(\bar{H}_{13}(t), 1,1) - C(\bar{H}_{13}(t), 1, \bar{H}_{33}(t))\} + \\ &\quad \{C(\bar{H}_{13}(t), \bar{H}_{23}(t), 1) - C(\bar{H}_{13}(t), \bar{H}_{23}(t), \bar{H}_{33}(t))\}.\end{aligned} \quad (19)$$

(17), (18), and (19) give the reliability of the three subsystems in PMS-2.

Thus, the reliability of the 3-PMS-2 system with $\bar{H}_{ji}(t)$ denoting reliability of j^{th} component in i^{th} subsystem $i = 1,2,3$; $j = 1,2,3$, is:

$$\begin{aligned}\bar{F}_2(\tau_3) &= P[\bar{F}_{12} > \tau_1] P[\bar{F}_{22} > \tau_2 \mid \bar{F}_{12} > \tau_1] P[\bar{F}_{32} > \tau_3 \mid \bar{F}_{12} > \tau_1, \bar{F}_{22} > \tau_2] \\ &= P[\bar{F}_{12} > \tau_1, \bar{F}_{22} > \tau_2, \bar{F}_{32} > \tau_3] \\ &= C(\bar{F}_{12}(\tau_1), \bar{F}_{22}(\tau_2), \bar{F}_{32}(\tau_3)),\end{aligned} \quad (20)$$

where

$\bar{F}_{12}(\tau_1) = C(\bar{H}_{11}(\tau_1), \bar{H}_{21}(\tau_1), \bar{H}_{31}(\tau_1))$,

$\bar{F}_{22}(\tau_2) = 1 - \{[1 - C(\bar{H}_{12}(\tau_2 - \tau_1 + l_{12}), 1,1)] - \{C(1, \bar{H}_{22}(\tau_2 - \tau_1 + l_{22}), 1) - C(\bar{H}_{12}(\tau_2 - \tau_1 + l_{12}), \bar{H}_{22}(\tau_2 - \tau_1 + l_{22}), 1)\}\} - \{\{C(1,1, \bar{H}_{32}(\tau_2 - \tau_1 + l_{32})) - C(\bar{H}_{12}(\tau_2 - \tau_1 + l_{12}), 1, \bar{H}_{32}(\tau_2 - \tau_1 + l_{32}))\} - \{C(1, \bar{H}_{22}(\tau_2 - \tau_1 + l_{22}), \bar{H}_{32}(\tau_2 - \tau_1 + l_{32})) - C(\bar{H}_{12}(\tau_2 - \tau_1 + l_{12}), \bar{H}_{22}(\tau_2 - \tau_1 + l_{22}), \bar{H}_{32}(\tau_2 - \tau_1 + l_{32}))\}\}$,

$\bar{F}_{32}(\tau_3) = C(\bar{H}_{13}(\tau_3 - \tau_2 + l_{13}), 1,1) - \{C(\bar{H}_{13}(\tau_3 - \tau_2 + l_{13}), 1,1) - C(\bar{H}_{13}(\tau_3 - \tau_2 + l_{13}), 1, \bar{H}_{33}(\tau_3 - \tau_2 + l_{33}))\} + \{C(\bar{H}_{13}(\tau_3 - \tau_2 + l_{13}), \bar{H}_{23}(\tau_3 - \tau_2 + l_{23}), 1) - C(\bar{H}_{13}(\tau_3 - \tau_2 + l_{13}), \bar{H}_{23}(\tau_3 - \tau_2 + l_{23}), \bar{H}_{33}(\tau_3 - \tau_2 + l_{33}))\}$,

using cumulative exposure model.

l_{12} is determined in such a way that
$\bar{H}_{12}(l_{12}) = \bar{H}_{11}(\tau_1)$,
l_{22} is determined in such a way that
$\bar{H}_{22}(l_{22}) = H_{21}(\tau_1)$,
l_{32} is determined in such a way that
$\bar{H}_{32}(l_{32}) = \bar{H}_{31}(\tau_1)$,
l_{13} is determined in such a way that
$\bar{H}_{13}(l_{13}) = H_{12}(\tau_2 - \tau_1 + l_{12})$,
l_{23} is determined in such a way that
$\bar{H}_{23}(l_{23}) = \bar{H}_{22}(\tau_2 - \tau_1 + l_{22})$,
l_{33} is determined in such a way that
$\bar{H}_{33}(l_{33}) = \bar{H}_{32}(\tau_2 - \tau_1 + l_{32})$.

4.2.1 Computation of Reliability of 3-PMS-2

The reliability of 3-PMS–2 is computed using a three-dimensional Gumbel-Hougaard copula with Weibull marginal:

$$C(\bar{H}_{1i}(t_1), \bar{H}_{2i}(t_2), \bar{H}_{3i}(t_3)) = exp\left[-\left(\left(-log(\bar{H}_{1i}(t_1))\right)^{\theta} + \left(-log(\bar{H}_{2i}(t_2))\right)^{\theta} + \left(-log(\bar{H}_{3i}(t_3))\right)^{\theta}\right)^{1/\theta}\right],$$

$$C(1, \bar{H}_{2i}(t_2), \bar{H}_{3i}(t_3)) = exp\left[-\left(\left(-log(\bar{H}_{2i}(t_2))\right)^{\theta} + \left(-log(\bar{H}_{3i}(t_3))\right)^{\theta}\right)^{1/\theta}\right],$$

$$C(\bar{H}_{1i}(t_1), 1, \bar{H}_{3i}(t_3),) = exp\left[-\left(\left(-log(\bar{H}_{1i}(t_1))\right)^{\theta} + \left(-log(\bar{H}_{3i}(t_3))\right)^{\theta}\right)^{1/\theta}\right],$$

$$C(\bar{H}_{1i}(t_1), \bar{H}_{2i}(t_2), 1) = exp\left[-\left(\left(-log(\bar{H}_{1i}(t_1))\right)^{\theta} + \left(-log(\bar{H}_{2i}(t_2))\right)^{\theta}\right)^{1/\theta}\right],$$

$C(\bar{H}_{1i}(t_1), 1,1) = \bar{H}_{1i}(t_1)$,
$C(1, \bar{H}_{2i}(t_2), 1) = \bar{H}_{2i}(t_2)$,
$C(1,1, \bar{H}_{3i}(t_3)) = \bar{H}_{3i}(t_3)$,
where,

$\bar{H}_{ji}(t) = exp\left[-\left(\frac{t}{\alpha_{ji}}\right)^{\beta_{ji}}\right], t > 0; \alpha_{ji} > 0; \beta_{ji} > 0, j = 1,2,3, i = 1,2,3.$

Thus,

$$C\left(\bar{H}_{ji}(t_1), \bar{H}_{ji}(t_2), \bar{H}_{ji}(t_3)\right) = exp\left[-\left(\left(\left(\left(\frac{t_1}{\alpha_{ji}}\right)^{\beta_{jk}}\right)\right)^{\theta} + \left(\left(\left(\frac{t_2}{\alpha_{ji}}\right)^{\beta_{ji}}\right)\right)^{\theta} + \left(\left(\left(\frac{t_3}{\alpha_{ji}}\right)^{\beta_{ji}}\right)\right)^{\theta}\right)^{1/\theta}\right].$$

4.3 Reliability of 5-PMS-3 system

Let T_1, T_2, T_3 and T_4 denote lifetimes of the components of subsystem $'1'$ with reliabilities $\bar{R}_1(t), \bar{R}_2(t), \bar{R}_3(t)$ and $\bar{R}_4(t)$, respectively, and T_1' and T_2' denote lifetimes of the components of subsystem $'2'$ with reliabilities $\bar{R}_1'(t)$and $\bar{R}_2'(t)$, respectively. Let $\bar{F}_{p1}(t), \bar{F}_{p2}(t), \bar{F}_{p3}(t), \bar{F}_{p4}(t)$, and $\bar{F}_{p5}(t)$ be the reliability of subsystems in Phase 1, phase 2, phase 3, phase 4, and Phase 5, respectively. Then, the reliability of PMS-3 is:

$$\bar{F}_{PMS-III}(t) = \begin{cases} \bar{F}_{p1}(t), 0 \le t \le \tau_1 \\ \bar{F}_{p2}(t), \tau_1 \le t \le \tau_2 \\ \bar{F}_{p3}(t), \tau_2 \le t \le \tau_3, \\ \bar{F}_{p4}(t), \tau_3 \le t \le \tau_4 \\ \bar{F}_{p5}(t), \tau_4 \le t \le \tau_5 \end{cases} \quad (21)$$

PHASE-1

Let $F_{11}(t)$, $F_{21}(t)$, $F_{31}(t)$, and $F_{41}(t)$ be life distribution of components 'H_a', 'H_b', 'H_c', and 'H_d', respectively in subsystem '1' further let $V_{11}(t)$ and $V_{21}(t)$ be life distribution of components 'L_a' and 'L_b', respectively, in subsystem '2'.

Reliability of Subsystem-I,

$$\bar{F}_{sub11}(t) = p[T_1 > t, T_2 > t, T_3 > t, T_4 \le t] + p[T_1 > t, T_2 > t, T_4 > t, T_3 \le t] + p[T_1 > t, T_3 > t, T_4 > t, T_2 \le t] + p[T_2 > t, T_3 > t, T_4 > t, T_1 \le t] + p[T_1 > t, T_2 > t, T_3 > t, T_4 > t]$$

$$= p[T_1 > t, T_2 > t, T_3 > t] - p[T_1 > t, T_2 > t, T_3 > t, T_4 > t] + p[T_1 > t, T_2 > t, T_4 > t] - p[T_1 > t, T_2 > t, T_4 > t, T_3 > t] + p[T_1 > t, T_3 > t, T_4 > t] - p[T_1 > t, T_3 > t, T_4 > t, T_2 > t] + p[T_2 > t, T_3 > t, T_4 > t] - p[T_2 > t, T_3 > t, T_4 > t, T_1 > t] + p[T_1 > t, T_2 > t, T_3 > t, T_4 > t]$$

$$= C\left(\bar{F}_{11}(t), \bar{F}_{21}(t), \bar{F}_{31}(t)\right) + C\left(\bar{F}_{11}(t), \bar{F}_{21}(t), \bar{F}_{41}(t)\right) + C\left(\bar{F}_{11}(t), \bar{F}_{31}(t), \bar{F}_{41}(t)\right) + C\left(\bar{F}_{21}(t), \bar{F}_{31}(t), \bar{F}_{41}(t)\right) - 3C\left(\bar{F}_{11}(t), \bar{F}_{21}(t), \bar{F}_{31}(t), \bar{F}_{41}(t)\right).$$

Reliability of Subsystem-II,

$$\bar{F}_{sub21}(t) = 1 - p[\min(U_1, U_2) \le t]$$
$$= 1 - p[U_1 \le t, U_2 \le t]$$
$$= 1 - \{p[U_1 \le t] - p[U_1 \le t, U_2 > t]\}$$
$$= 1 - \{1 - p[U_1 > t] - \{p[U_2 > t] - p[U_1 > t, U_2 > t]\}\}$$
$$= p[U_1 > t] + p[U_2 > t] - p[U_1 > t, U_2 > t]$$
$$= C(\bar{V}_{11}(t), 1) + C\left(1, \bar{V}_{21}(t)\right) - C\left(\bar{V}_{11}(t), \bar{V}_{21}(t)\right).$$

Thus, the Reliability of phase-1,

$$\bar{F}_{p1}(t) = \bar{F}_{sub11}(t). \bar{F}_{sub21}(t). \quad (22)$$

PHASE-2

Let $F_{12}(t)$ and $F_{22}(t)$ be life distribution of components 'H_a' and 'H_b', respectively,

$$\bar{F}_{p2}(t) = C(\bar{F}_{12}(t), 1) + C(1, \bar{F}_{22}(t)) - C\left(\bar{F}_{12}(t), \bar{F}_{22}(t)\right). \quad (23)$$

PHASE-3

Let $F_{13}(t)$, $F_{23}(t)$, $F_{33}(t)$ and $F_{43}(t)$ be life distribution of components 'H_a', 'H_b', 'H_c' and 'H_d', respectively in subsystem '1'. Further, let $V_{13}(t)$ and $V_{23}(t)$ be life distribution of components 'A_a'and 'A_b',

$$\bar{F}_{sub13}(t) = C\left(\bar{F}_{13}(t), \bar{F}_{23}(t), \bar{F}_{33}(t)\right) + C\left(\bar{F}_{13}(t), \bar{F}_{23}(t), \bar{F}_{43}(t)\right) + C\left(\bar{F}_{13}(t), \bar{F}_{33}(t), \bar{F}_{43}(t)\right) + C\left(\bar{F}_{23}(t), \bar{F}_{33}(t), \bar{F}_{43}(t)\right) - 3C\left(\bar{F}_{13}(t), \bar{F}_{23}(t), \bar{F}_{33}(t), \bar{F}_{43}(t)\right).$$

Reliability of Subsystem-II,

$$\bar{F}_{sub23}(t) = C(\bar{V}_{13}(t), 1) + C\left(1, \bar{V}_{23}(t)\right) - C\left(\bar{V}_{13}(t), \bar{V}_{23}(t)\right).$$

Thus, the Reliability of phase-3,

$$\bar{F}_{p3}(t) = \bar{F}_{sub13}(t). \bar{F}_{sub23}(t). \quad (24)$$

PHASE-4

Let $F_{14}(t)$ and $F_{24}(t)$ be life distribution of components 'H_a' and 'H_b', respectively,

$$\bar{F}_{p4}(t) = C(\bar{F}_{14}(t), 1) + C(1, \bar{F}_{24}(t)) - C\left(\bar{F}_{14}(t), \bar{F}_{24}(t)\right). \quad (25)$$

PHASE-5

Let $F_{15}(t)$, $F_{25}(t)$, $F_{35}(t)$, and $F_{45}(t)$ be life distribution of components 'H_a', 'H_b', 'H_c', and 'H_d', respectively in subsystem '1' further let $V_{15}(t)$ and $V_{25}(t)$ be life distribution of components 'C_a' and 'C_b',

$$\bar{F}_{sub15}(t) = C\left(\bar{F}_{15}(t), \bar{F}_{25}(t), \bar{F}_{35}(t)\right) + C\left(\bar{F}_{15}(t), \bar{F}_{25}(t), \bar{F}_{45}(t)\right) + C\left(\bar{F}_{15}(t), \bar{F}_{35}(t), \bar{F}_{45}(t)\right) + C\left(\bar{F}_{25}(t), \bar{F}_{35}(t), \bar{F}_{45}(t)\right) - 3C\left(\bar{F}_{15}(t), \bar{F}_{25}(t), \bar{F}_{35}(t), \bar{F}_{45}(t)\right).$$

Reliability of Subsystem-II,

$$\bar{F}_{sub25}(t) = C(\bar{V}_{15}(t), 1) + C\left(1, \bar{V}_{25}(t)\right) - C\left(\bar{V}_{15}(t), \bar{V}_{25}(t)\right).$$

Thus, the Reliability of phase-5,

$$\bar{F}_{p5}(t) = \bar{F}_{sub15}(t). \bar{F}_{sub25}(t). \quad (26)$$

(22), (23), (24), (25), and (26) give reliability of the five phases in PMS-3.

Thus, the reliability of the 5-PMS-3 system with L_3 denoting its lifetime and $\bar{R}_{ji}(t)$ and $\bar{R}_{ji}'(t)$ denoting reliability of j^{th} component in i^{th} phases of $'1'$ and $'2'$ subsystems, respectively, $i = 1,2,3,4,5;\ j = 1,2,3,4$ is:

$$\bar{F}_3(\tau_5) = P[\bar{F}_{p1} > \tau_1]P[\bar{F}_{p2} > \tau_2 \mid \bar{F}_{p1} > \tau_1]P[\bar{F}_{p3} > \tau_3 \mid \bar{F}_{p1} > \tau_1, \bar{F}_{p2} > \tau_2]P[\bar{F}_{p4} > \tau_4 \mid \bar{F}_{p1} > \tau_1, \bar{F}_{p2} > \tau_2, \bar{F}_{p3} > \tau_3]P[\bar{F}_{p5} > \tau_5 \mid \bar{F}_{p1} > \tau_1, \bar{F}_{p2} > \tau_2, \bar{F}_{p3} > \tau_3, \bar{F}_{p4} > \tau_4] = P[\bar{F}_{p1} > \tau_1, \bar{F}_{p2} > \tau_2, \bar{F}_{p3} > \tau_3, \bar{F}_{p4} > \tau_4, \bar{F}_{p5} > \tau_5] = C(\bar{F}_{p1}(\tau_1), \bar{F}_{p2}(\tau_2), \bar{F}_{p3}(\tau_3), \bar{F}_{p4}(\tau_4), \bar{F}_{p5}(\tau_5)), \quad (27$$

where,

$\bar{F}_{sub11}(\tau_1) = C(\bar{F}_{11}(\tau_1), \bar{F}_{21}(\tau_1), \bar{F}_{31}(\tau_1)) + C(\bar{F}_{11}(\tau_1), \bar{F}_{21}(\tau_1), \bar{F}_{41}(\tau_1)) + C(\bar{F}_{11}(\tau_1), \bar{F}_{31}(\tau_1), \bar{F}_{41}(\tau_1)) + C(\bar{F}_{21}(\tau_1), \bar{F}_{31}(\tau_1), \bar{F}_{41}(\tau_1)) - 3C(\bar{F}_{11}(\tau_1), \bar{F}_{21}(\tau_1), \bar{F}_{31}(\tau_1), \bar{F}_{41}(\tau_1))$,

$\bar{F}_{sub21}(\tau_1) = C(\bar{V}_{11}(\tau_1), 1) + C(1, \bar{V}_{21}(\tau_1)) - C(\bar{G}_{11}(\tau_1), \bar{G}_{21}(\tau_1))$,

$\bar{F}_{p1}(\tau_1) = \bar{F}_{sub11}(\tau_1).\bar{F}_{sub21}(\tau_1)$,

$\bar{F}_{p2}(\tau_2) = C(\bar{F}_{12}(\tau_2 - \tau_1 + k_{12}), 1) + C(1, \bar{F}_{22}(\tau_2 - \tau_1 + k_{22})) - C(\bar{F}_{12}(\tau_2 - \tau_1 + k_{12}), \bar{F}_{22}(\tau_2 - \tau_1 + k_{22}))$,

$\bar{F}_{sub13}(\tau_3) = C(\bar{F}_{13}(\tau_3 - \tau_2 + k_{13}), \bar{F}_{23}(\tau_3 - \tau_2 + k_{23}), \bar{F}_{33}(\tau_3 - \tau_1 + k_{33})) + C(\bar{F}_{13}(\tau_3 - \tau_2 + k_{13}), \bar{F}_{23}(\tau_3 - \tau_2 + k_{23}), \bar{F}_{43}(\tau_3 - \tau_1 + k_{43})) + C(\bar{F}_{13}(\tau_3 - \tau_2 + k_{13}), \bar{F}_{33}(\tau_3 - \tau_1 + k_{33}), \bar{F}_{43}(\tau_3 - \tau_1 + k_{43})) + C(\bar{F}_{23}(\tau_3 - \tau_2 + k_{23}), \bar{F}_{33}(\tau_3 - \tau_1 + k_{33}), \bar{F}_{43}(\tau_3 - \tau_1 + k_{43})) - 3C(\bar{F}_{13}(\tau_3 - \tau_2 + k_{13}), \bar{F}_{23}(\tau_3 - \tau_2 + k_{23}), \bar{F}_{33}(\tau_3 - \tau_1 + k_{33}), \bar{F}_{43}(\tau_3 - \tau_1 + k_{43}))$,

$\bar{F}_{sub23}(\tau_3) = C(\bar{V}_{13}(\tau_3), 1) + C(1, \bar{V}_{23}(\tau_3)) - C(\bar{V}_{13}(\tau_3), \bar{V}_{23}(\tau_3))$,

$\bar{F}_{p3}(\tau_3) = (\bar{F}_{sub13}(\tau_3).\bar{F}_{sub23}(\tau_3))$,

$\bar{F}_{p4}(\tau_4) = C(\bar{F}_{14}(\tau_4 - \tau_3 + k_{14}), 1) + C(1, \bar{F}_{24}(\tau_4 - \tau_3 + k_{24})) - C(\bar{F}_{14}(\tau_4 - \tau_3 + k_{14}), \bar{F}_{24}(\tau_4 - \tau_3 + k_{24}))$,

$\bar{F}_{sub15}(\tau_5) = C(\bar{F}_{15}(\tau_5 - \tau_4 + k_{15}), \bar{F}_{25}(\tau_5 - \tau_4 + k_{25}), \bar{F}_{35}(\tau_5 - \tau_3 + k_{35})) + C(\bar{F}_{15}(\tau_5 - \tau_4 + k_{15}), \bar{F}_{25}(\tau_5 - \tau_4 + k_{25}), \bar{F}_{45}(\tau_5 - \tau_3 + k_{45})) + C(\bar{F}_{15}(\tau_5 - \tau_4 + k_{15}), \bar{F}_{35}(\tau_5 - \tau_3 + k_{35}), \bar{F}_{45}(\tau_5 - \tau_3 + k_{45})) + C(\bar{F}_{25}(\tau_5 - \tau_4 + k_{25}), \bar{F}_{35}(\tau_5 - \tau_3 + k_{35}), \bar{F}_{45}(\tau_5 - \tau_3 + k_{45})) - 3C(\bar{F}_{15}(\tau_5 - \tau_4 + k_{15}), \bar{F}_{25}(\tau_5 - \tau_4 + k_{25}), \bar{F}_{35}(\tau_5 - \tau_3 + k_{35}), \bar{F}_{45}(\tau_5 - \tau_3 + k_{45}))$,

$\bar{F}_{sub25}(\tau_5) = C(\bar{V}_{15}(\tau_5), 1) + C(1, \bar{V}_{25}(\tau_5)) - C(\bar{V}_{15}(\tau_5), \bar{V}_{25}(\tau_5))$

$\bar{F}_{p5}(\tau_5) = (\bar{F}_{sub15}(\tau_5).\bar{F}_{sub25}(\tau_5))$,

using cumulative exposure model.

k_{ji} is determined in such a way,

$R_{11}(\tau_1) = R_{12}(k_{12})$
$R_{21}(\tau_1) = R_{22}(k_{22})$
$R_{13}(k_{13}) = R_{12}(\tau_2 - \tau_1 + k_{12})$
$R_{23}(k_{23}) = R_{22}(\tau_2 - \tau_1 + k_{22})$
$R_{31}(\tau_1) = R_{33}(k_{33})$
$R_{41}(\tau_1) = R_{43}(k_{43})$
$R_{14}(k_{14}) = R_{13}(\tau_3 - \tau_2 + k_{13})$
$R_{24}(k_{24}) = R_{23}(\tau_3 - \tau_2 + k_{23})$
$R_{15}(k_{15}) = R_{14}(\tau_4 - \tau_3 + k_{14})$
$R_{25}(k_{24}) = R_{24}(\tau_4 - \tau_3 + k_{24})$
$R_{35}(k_{35}) = R_{33}(\tau_3 - \tau_1 + k_{33})$
$R_{45}(k_{45}) = R_{43}(\tau_3 - \tau_1 + k_{43})$.

4.3.1 Computation of Reliability of 5-PMS-3

The reliability of 5-PMS–3 is computed using a four-dimensional Gumbel-Hougaard copula with Exponential marginal:

For Subsystem-I,

$C(\bar{R}_{1i}(t_1), \bar{R}_{2i}(t_2), \bar{R}_{3i}(t_3), \bar{R}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{R}_{1i}(t_1))\right)^\theta + \left(-log(\bar{R}_{2i}(t_2))\right)^\theta + \left(-log(\bar{R}_{3i}(t_3))\right)^\theta + \left(-log(\bar{R}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right]$,

$C(1, \bar{R}_{2i}(t_2), \bar{R}_{3i}(t_3), \bar{R}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{R}_{2i}(t_2))\right)^\theta + \left(-log(\bar{R}_{3i}(t_3))\right)^\theta + \left(-log(\bar{R}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right]$,

$C(1,1, \bar{R}_{3i}(t_3), \bar{R}_{4i}(t_4)) = exp\left[-\left(\left(-log(\bar{R}_{3i}(t_3))\right)^\theta + \left(-log(\bar{R}_{4i}(t_4))\right)^\theta\right)^{1/\theta}\right]$,

$C(1,1,1, \bar{R}_{4i}(t_4)) = \bar{R}_{4i}(t_4)$,

where,

$\bar{R}_{ji}(t) = exp[-(t\alpha_{ji})], t > 0; \alpha_{ji} < 0; j = 1,2,3,4,\ i = 1,2,3,4,5.$

For Subsystem-II

$C(\bar{R}'_{1i}(t_1), \bar{R}'_{2i}(t_2)) = exp\left[-\left(\left(-log(\bar{R}'_{1i}(t_1))\right)^\theta + \left(-log(\bar{R}'_{2i}(t_2))\right)^\theta\right)^{1/\theta}\right]$,

$C(\bar{R}'_{1i}(t_1), 1) = \bar{R}'_{1i}(t_1)$,

$C(1, \bar{R}'_{2i}(t_2)) = \bar{R}'_{2i}(t_2)$,

where, $\bar{R}'_{ji}(t) = exp[(t\lambda_{ji})], t > 0; \lambda_{ji} < 0; j = 1,2,3, i = 1,2,3,4,5.$

5. Reliability Importance Analysis

Reliability importance analysis is used to identify a system's weakness and quantify the impact of component failures. These importance measures provide a numerical rank to determine which components are more important to system reliability improvement or more critical to system failure. This helps to allocate resources for inspection, maintenance, and repairs in an optimal manner over the lifetime of a system [9], [26],[27].

In this paper, the theory of Birnbaum importance measure is used to perform a reliability importance analysis of PMSs with respect to each component in each phase.

Birnbaum's measure is the partial derivative of the system's reliability with respect to the reliability of an individual component. Let the reliability importance index of PMS-1 and PMS-2 with respect to component j, $j \in \{1,2,\dots,m\}$ in phase i, $i \in \{1,2,\dots,M\}$, be denoted by $I^B_{CjPhasei\_PMS1}$ *and* $I^B_{CjPhasei\_PMS2}$, respectively, and that of PMS-3 with respect to component j, $j \in \{1,2,\dots,m\}$ in subsystem k, $k = 1{,}2$ of phase i, $i \in \{1,2,\dots,M\}$, be denoted by $I^B_{CkjPhasei\_PMS3}$.

For PMS-1, as defined in section 4.1, $\bar{F}_{11}(t), \bar{F}_{21}(t), \bar{F}_{31}(t)$ are the reliability of subsystems in phase 1, phase 2, and phase 3, respectively. Then, the reliability of PMS-1 is:

$$\bar{F}_{PMS-I}(t) = \begin{cases} \bar{F}_{11}(t), 0 \le t \le \tau_1 \\ \bar{F}_{21}(t), \tau_1 \le t \le \tau_2. \\ \bar{F}_{31}(t), \tau_2 \le t \le \tau_3 \end{cases}$$

Also, $\bar{G}_{ji}(t)$ denotes reliability of j^{th} component in i^{th} Phase, $j = 1{,}2{,}3{,}4$; $i = 1{,}2{,}3$.

Since we are using the cumulative exposure model, the reliabilities of j^{th} components for phase i is $\bar{G}_{ji}\left(t - \tau_{i-1} + l_{ji}\right), \tau_{i-1} \le t \le \tau_i$, $i = 2{,}3$.

The reliability importance index of PMS-1 is defined as follows:

$$I^B_{CjPhasei_{PMS1}} = \begin{cases} \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{G}_{ji}(t)}, \ \tau_{i-1} \le t \le \tau_i, i = 1 \\ \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{G}_{ji}(t-\tau_{i-1}+l_{ji})}, \ \tau_{i-1} \le t \le \tau_i, i = 2{,}3 \end{cases}. \quad (28)$$

For PMS-2, as defined in section 4.2, $\bar{F}_{12}(t), \bar{F}_{22}(t), \bar{F}_{32}(t)$ are the reliability of subsystems in phase 1, phase 2, and phase 3, respectively. Then, the reliability of PMS-2 is:

$$\bar{F}_{PMS-II}(t) = \begin{cases} \bar{F}_{12}(t), 0 \le t \le \tau_1 \\ \bar{F}_{22}(t), \tau_1 \le t \le \tau_2. \\ \bar{F}_{32}(t), \tau_2 \le t \le \tau_3 \end{cases}$$

Also, $\bar{H}_{ji}(t)$ denotes reliability of j^{th} component in i^{th} Phase, $j = 1{,}2{,}3$; $i = 1{,}2{,}3$.

Since we are using the cumulative exposure model, the reliabilities of j^{th} components for phase i is $\bar{H}_{ji}\left(t - \tau_{i-1} + l_{ji}\right), \tau_{i-1} \le t \le \tau_i, i = 2{,}3$.

The reliability importance index of PMS-2 is defined as follows:

$$I^B_{CjPhasei\_PMS2} = \begin{cases} \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{H}_{ji}(t)}, \ \tau_{i-1} \le t \le \tau_i, i = 1 \\ \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{H}_{ji}(t-\tau_{i-1}+l_{ji})}, \ \tau_{i-1} \le t \le \tau_i, i = 2{,}3 \end{cases}. \quad (29)$$

For PMS-3, as defined in section 4.3, Let $\bar{F}_{p1}(t), \bar{F}_{p2}(t), \bar{F}_{p3}(t), \bar{F}_{p4}(t)$, and $\bar{F}_{p5}(t)$ be the reliability of subsystems in Phase 1, Phase 2, Phase 3, Phase 4, and Phase 5, respectively. Then, the reliability of PMS-3 is:

$$\bar{F}_{PMS-III}(t) = \begin{cases} \bar{F}_{p1}(t), 0 \le t \le \tau_1 \\ \bar{F}_{p2}(t), \tau_1 \le t \le \tau_2 \\ \bar{F}_{p3}(t), \tau_2 \le t \le \tau_3. \\ \bar{F}_{p4}(t), \tau_3 \le t \le \tau_4 \\ \bar{F}_{p5}(t), \tau_4 \le t \le \tau_5 \end{cases}$$

Also, $\bar{R}_{ji}(t)$ denoting reliability of j^{th} component of subsystem $'1$ in i^{th} Phase, $j = 1{,}2{,}3{,}4$; $i = 1{,}2{,}3{,}4{,}5$ and $\bar{R}'_{ji}(t)$ denoting reliability of j^{th} component of subsystem $'2$ in i^{th} Phase, $j = 1{,}2$; $i = 1{,}3{,}5$.

Since we are using the cumulative exposure model, the reliabilities of j^{th} components of subsystem $'1'$ for phase i is:

$$\begin{cases} \bar{R}_{ji}\left(t - \tau_{i-1} + l_{ji}\right), \tau_{i-1} \le t \le \tau_i, i = 2{,}3{,}4{,}5, j = 1{,}2 \\ \bar{R}'_{ji}\left(t - \tau_{i-2} + l_{ji}\right), \tau_{i-1} \le t \le \tau_i, i = 3{,}5, j = 3{,}4 \end{cases}.$$

The reliability importance index of PMS-3 is defined as follows:

$$I^B_{CjPhasei\_PMS3} = \begin{cases} \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{R}_{ji}}, \tau_{i-1} \le t \le \tau_i, i = 1, j = 1{,}2{,}3{,}4 \\ \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{R}'_{ji}}, \tau_{i-1} \le t \le \tau_i, i = 1, j = 1{,}2, \\ \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{R}_{ji}(t-\tau_{i-1}+l_{ji})}, \tau_{i-1} \le t \le \tau_i, i = 2{,}3{,}4{,}5, j = 1{,}2 \\ \frac{\partial \bar{F}_{ji}(t)}{\partial \bar{R}'_{ji}(t-\tau_{i-2}+l_{ji})}, \tau_{i-1} \le t \le \tau_i, i = 3{,}5, j = 3{,}4 \end{cases}. \quad (30)$$

Reliability importance for each component of PMS-1:

For Phase 1:

$I^B_{CjPhase1\_PMS1} = \frac{\partial \bar{F}_{11}(t)}{\partial \bar{G}_{j1}(t)}, \ \tau_0 \le t \le \tau_1$,

where

$\bar{G}_{j1}(t) = exp\left[-\left(\frac{t}{\alpha_1}\right)^{\beta_j}\right], t > 0; \alpha_1 > 0; \beta_j > 0, j = 1{,}2{,}3{,}4$

for Phase 2:

$I^B_{CjPhase2\_PMS1} = \frac{\partial \bar{F}_{21}(t)}{\partial \bar{G}_{j2}(t-\tau_1+l_{j2})}, j = 1{,}2{,}3, \tau_1 \le t \le \tau_2$

for Phase 3:

$I^B_{CjPhase3\_PMS1} = \frac{\partial \bar{F}_{31}(t)}{\partial \bar{G}_{j3}(t-\tau_2+l_{j3})}, j = 1,3,4, \tau_2 \le t \le \tau_3$.

Reliability importance for each component of PMS-2:

For Phase 1:

$I^B_{CjPhase1\_PMS2} = \frac{\partial \bar{F}_{12}(t)}{\partial \bar{H}_{j1}(t)}, \ \tau_0 \le t \le \tau_1,$

where

$\bar{H}_{j1}(t) = exp\left[-\left(\frac{t}{\alpha_1}\right)^{\beta_j}\right], t > 0; \alpha_1 > 0; \beta_j > 0, j =$ 1,2,3,

for Phase 2:

$I^B_{CjPhase2\_PMS2} = \frac{\partial \bar{F}_{22}(t)}{\partial \bar{H}_{j2}(t-\tau_1+l_{j2})}, j = 1,2,3, \tau_1 \le t \le \tau_2,$

for Phase 3:

$I^B_{CjPhase3\_PMS2} = \frac{\partial \bar{F}_{32}(t)}{\partial \bar{H}_{j3}(t-\tau_2+l_{j3})}, j = 1,2,3, \tau_2 \le t \le \tau_3$.

Reliability importance for each component of PMS-3:

For components H_a, H_b, H_c, and H_d of Phase 1:

$I^B_{C1jPhase1\_PMS3} = \frac{\partial \bar{F}_{p1}(t)}{\partial \bar{R}_{j1}(t)}, j = 1,2,3,4, \ \tau_0 \le t \le \tau_1,$

for components 'L_a' and 'L_b' of phase 1:

$I^B_{C2jPhase1\_PMS3} = \frac{\partial \bar{F}_{p1}(t)}{\partial \bar{R}'_{j1}(t)}, j = 1,2, \ \tau_0 \le t \le \tau_1,$

for components H_a and H_b of phase 2:

$I^B_{C1jPhase2\_PMS3} = \frac{\partial \bar{F}_{p2}(t)}{\partial \bar{R}_{j2}(t-\tau_1+k_{j2})}, j = 1,2, \tau_1 \le t \le \tau_2,$

for components H_a and H_b of phase 3:

$I^B_{C1jPhase3\_PMS3} = \frac{\partial \bar{F}_{p3}(t)}{\partial \bar{R}_{j3}(t-\tau_2+k_{j3})}, j = 1,2, \tau_2 \le t \le \tau_3,$

for components H_c and H_d of phase 3:

$I^B_{C1jPhase3\_PMS3} = \frac{\partial \bar{F}_{p3}(t)}{\partial \bar{R}_{j3}(t-\tau_1+k_{j3})}, j = 3,4, \tau_2 \le t \le \tau_3,$

for components 'A_a' and 'A_b' of phase 3:

$I^B_{C2jPhase3\_PMS3} = \frac{\partial \bar{F}_{p3}(t)}{\partial \bar{R}'_{j3}(t)}, j = 1,2, \ \tau_2 \le t \le \tau_3,$

for components H_a and H_b of phase 4:

$I^B_{C1jPhase4\_PMS3} = \frac{\partial \bar{F}_{p4}(t)}{\partial \bar{R}_{j4}(t-\tau_3+k_{j4})}, j = 1,2, \tau_3 \le t \le \tau_4,$

for components H_a and H_b of phase 5:

$I^B_{C1jPhase5\_PMS3} = \frac{\partial \bar{F}_{p5}(t)}{\partial \bar{R}_{j5}(t-\tau_4+k_{j5})}, j = 1,2, \tau_4 \le t \le \tau_5,$

for components H_c and H_d of phase 5:

$I^B_{C1jPhase5\_PMS3} = \frac{\partial \bar{F}_{p5}(t)}{\partial \bar{R}_{j5}(t-\tau_3+k_{j5})}, j = 3,4, \tau_4 \le t \le \tau_5,$

for Components 'C_a' and 'C_b' of phase 5:

$I^B_{C2jPhase5\_PMS3} = \frac{\partial \bar{F}_{p5}(t)}{\partial \bar{R}'_{j5}(t)}, j = 1,2, \ \ \tau_4 \le t \le \tau_5.$

6. Numerical Illustrations

The method developed has been illustrated using different parametric sets. The reliability values of PMS-1, PMS-2, and PMS-3 are depicted in Tables 1 & 2, 3 & 4, and 5 & 6, respectively. See for reference [9]. We are taking the same scale and shape parameters for each component across the phases.

Table 1. Data Set for PMS-1

S. No.	T	Parametric Set α_j = Scale β_i = Shape parameter, i=1,2,3,4, j=1,2,3		θ	Reliability of PMS 1
1.	τ_0=0	$\alpha_1 = 10^6$,	$\beta_1 = 1.4$	1.17	0.911102
2.	τ_1=1000	$\alpha_2 = 10^5$,	$\beta_2 = 1.7$	2	0.937666
3.	τ_2=2000	$\alpha_3 = 10^4$	$\beta_3 = 1.5$	2.5	0.943312
4.	τ_3=3000		$\beta_4 = 1.6$	3	0.946628
5.				4	0.950215

Table 2. Reliability of PMS-1

θ	$\bar{F}11(\tau_0)$	$\bar{F}11(\tau_1)$	$\bar{F}21(\tau_1)$	$\bar{F}21(\tau_2)$	$\bar{F}31(\tau_2)$	$\bar{F}31(\tau_3)$	Reliability of PMS 1
1.17	1	0.999937	0.999991	0.999707	0.997306	0.911202	0.911102
2	1	0.999937	0.999976	0.999173	0.997855	0.937671	0.937666
2.5	1	0.999937	0.999973	0.999058	0.997976	0.943312	0.943312
3	1	0.999937	0.999971	0.998989	0.998047	0.946628	0.946628
4	1	0.999937	0.999969	0.998917	0.99812	0.950215	0.950215

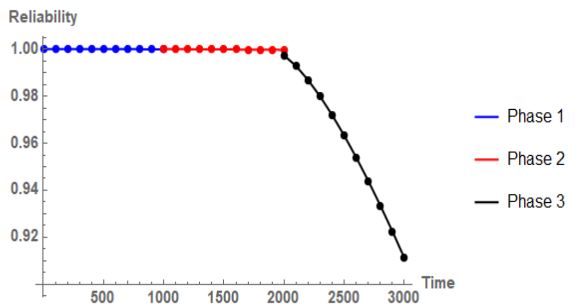

Figure 4. Reliability Plot of the PMS-1 for data set of Table 2 for $\theta = 1.17$.

The result of analyzing the reliability of PMS-1 is shown in Tables 1 and 2 and Figure 4.

Table 3. Data Set for PMS-2

S. No.	τ	Parametric Set α_j = Scale, β_i = Shape parameter, i=1,2,3, j=1,2,3		θ	Reliability of PMS 2
1.	τ_1=1000,	$\alpha_1 = 10^6$,	$\beta_1 = 1.4$	1.17	0.953321
2.				2	0.953799
3.	τ_2=2000,	$\alpha_2 = 10^5$,	$\beta_2 = 1.7$	2.5	0.95423
4.	τ_3=3000	$\alpha_3 = 10^4$	$\beta_3 = 1.5$	3	0.954513
5.				4	0.954798

Table 4. Reliability of PMS-2

θ	$\bar{F}12(\tau_0)$	$\bar{F}12(\tau_1)$	$\bar{F}22(\tau_1)$	$\bar{F}22(\tau_2)$	$\bar{F}32(\tau_2)$	$\bar{F}32(\tau_3)$	Reliability of PMS 2
1.17	1	0.999909	0.999998	0.999886	0.998171	0.95338	0.953321
2	1	0.999929	0.999993	0.999632	0.998181	0.953801	0.953799
2.5	1	0.999933	0.999992	0.999583	0.998187	0.95423	0.95423
3	1	0.999934	0.999992	0.999559	0.998189	0.954513	0.954513
4	1	0.999936	0.999992	0.99954	0.99819	0.954798	0.954798

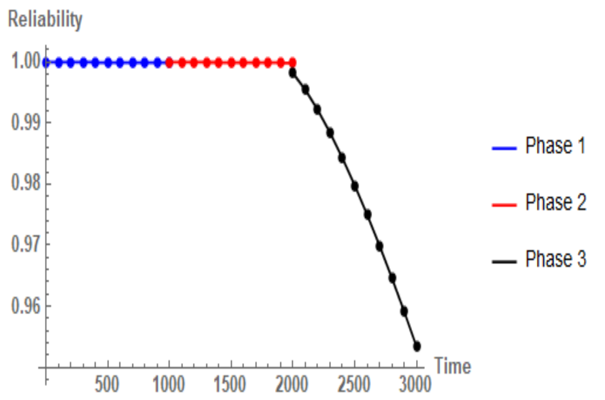

Figure 5. Reliability Plot of the PMS-2 for data Set of table 4 for $\theta = 1.17$.

The results of analyzing the reliability of PMS-2 are shown in Tables 3 & 4 and Figure 5.

Table 5. Data Set for PMS-3

S. No.	Time τ (in days)	Parametric Set α_i = Failure rate of components of subsystem 1, λ_i= Failure rate of components of subsystem 2, i=1,2,3,4,5	θ	Reliability of PMS 2
1.	τ_0=0,	$\alpha_1 = 10^{-6}, \alpha_2 = 10^{-4}, \alpha_3 = 10^{-6}$, $\alpha_4 = 10^{-4}, \alpha_5 = 10^{-6}, \lambda_1 = 10^{-6}$, $\lambda_3 = 10^{-6}$, $\lambda_5 = 10^{-6}$	1.17	0.884973
2.	τ_1=2,		2	0.840661
3.	τ_2=732,		2.5	0.8370071
4.	τ_3=760, τ_4=1883,		3	0.835322
5.	τ_5=1911		4	0.833509

Table 6. Reliability of PMS-3

θ	$\bar{F}_{p1}(\tau_0)$	$\bar{F}_{p1}(\tau_1)$	$\bar{F}_{p2}(\tau_1)$	$\bar{F}_{p2}(\tau_2)$	$\bar{F}_{p3}(\tau_2)$	$\bar{F}_{p3}(\tau_3)$	$\bar{F}_{p4}(\tau_3)$	$\bar{F}_{p4}(\tau_4)$	$\bar{F}_{p5}(\tau_4)$	$\bar{F}_{p5}(\tau_5)$	Reliability of PMS 3
1.17	1	0.999999	0.99996	0.982871	0.982576	0.982556	0.982863	0.946428	0.945542	0.945517	0.884973
2	1	0.999997	0.999997	0.957288	0.956873	0.956841	0.957271	0.892223	0.891228	0.891197	0.840661
2.5	1	0.999997	0.999997	0.951031	0.950557	0.95052	0.951012	0.878599	0.877473	0.877438	0.837007
3	1	0.999997	0.999997	0.947072	0.946559	0.946519	0.947052	0.869903	0.868692	0.868655	0.835322
4	1	0.999996	0.999996	0.942351	0.941792	0.941749	0.942329	0.859459	0.858148	0.858108	0.833509

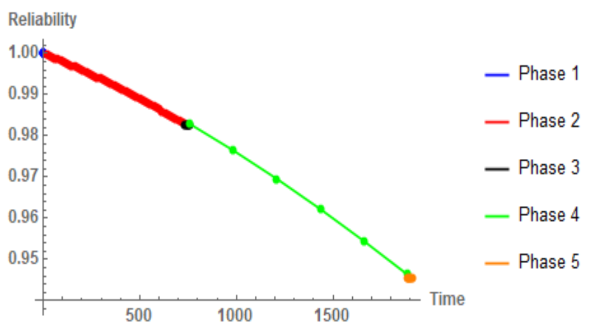

Figure 6. Reliability Plot of the PMS-3 for data set of Table 6 for $\theta = 1.17$.

The result of analyzing the reliability of PMS-3 is shown in Tables 5 & 6 and Figure 6.

Independent Case:

Figures 7(a)-7(c) depict the component-wise reliability importance plot of each phase in PMS-1.

Table 7. Reliability of PMS-1 for independent case

θ	$\bar{F}11(\tau_0)$	$\bar{F}11(\tau_1)$	$\bar{F}21(\tau_1)$	$\bar{F}21(\tau_2)$	$\bar{F}31(\tau_2)$	$\bar{F}31(\tau_3)$	Reliability of PMS 1
1	1	0.999937	1	0.999997	0.997024	0.89711	0.897051

Table 8. Reliability of PMS-2 for independent case

θ	$\bar{F}12(\tau_0)$	$\bar{F}12(\tau_1)$	$\bar{F}22(\tau_1)$	$\bar{F}22(\tau_2)$	$\bar{F}32(\tau_2)$	$\bar{F}32(\tau_3)$	Reliability of PMS 2
1	1	0.999897	0.999937	0.998191	0.998191	0.954972	0.953146

Table 9. Reliability of PMS-3 for independent case

θ	$\bar{F}_{p1}(\tau_0)$	$\bar{F}_{p1}(\tau_1)$	$\bar{F}_{p2}(\tau_1)$	$\bar{F}_{p2}(\tau_2)$	$\bar{F}_{p3}(\tau_2)$	$\bar{F}_{p3}(\tau_3)$	$\bar{F}_{p4}(\tau_3)$	$\bar{F}_{p4}(\tau_4)$	$\bar{F}_{p5}(\tau_4)$	$\bar{F}_{p5}(\tau_5)$	Reliability of PMS 3
1	1	0.999999	1	0.982871	0.982732	0.982556	0.982863	0.946428	0.945896	0.945517	0.884973

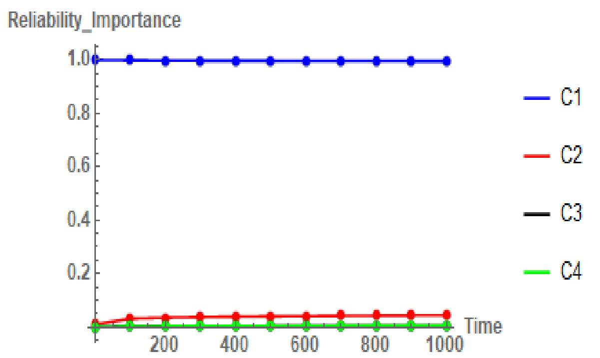

Figure 7(a). Reliability importance plot of the PMS-1 for each component of phase 1.

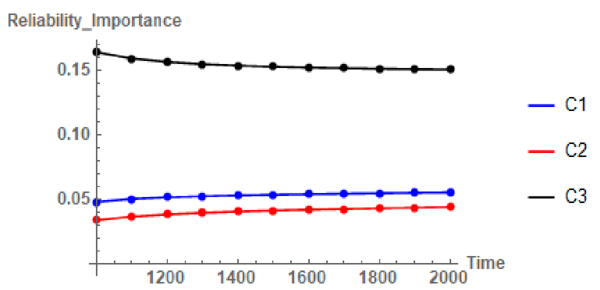

Figure 7(b). Reliability importance plot of the PMS-1 for each component of phase 2.

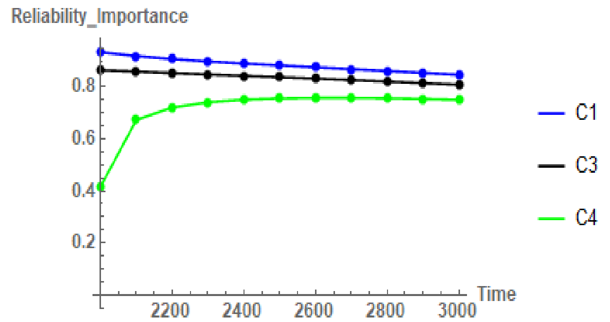

Figure 7(c). Reliability importance plot of the PMS-1 for each component of phase 3.

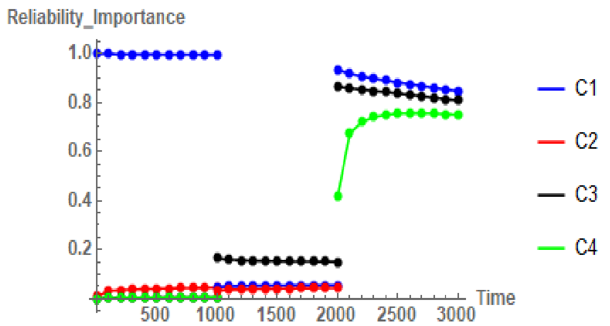

Figure 7(d). Reliability importance of each component phase-wise in PMS-1.

Figure 7(d) shows the reliability importance of each component phase-wise in PMS-1, and it can be seen that 'C_1' has the most significant influence on the reliability of the PMS-1 in phase 1 and phase 3, and 'C_3' has the most significant impact on the reliability in phase 2.

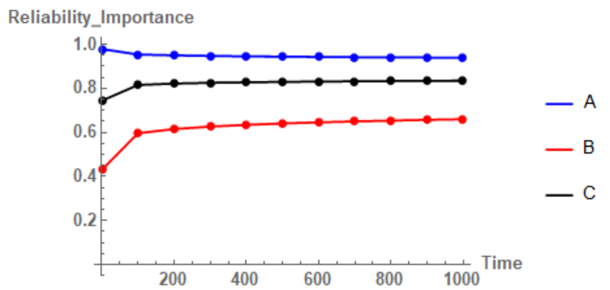

Figure 8(a). Reliability importance plot of the PMS-2 for each component of phase 1.

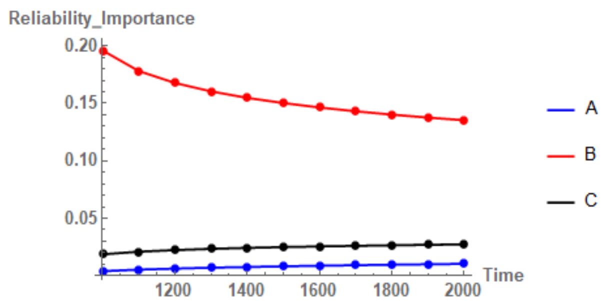

Figure 8(b). Reliability importance plot of the PMS-2 for each component of phase 2.

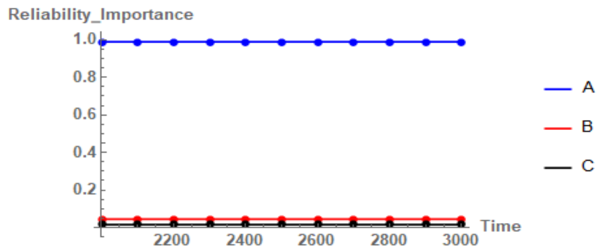

Figure 8(c). Reliability importance plot of the PMS-2 for each component of phase 3.

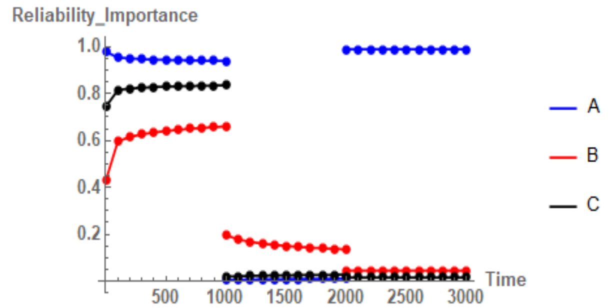

Figure 8(d). Reliability importance of each component phase-wise in PMS-2.

Figures 8(a)-8(c) depict the component-wise reliability importance plot of each phase in PMS-2. Figure 8(d) shows the reliability importance of each component phase-wise in PMS-2, and it can be seen that 'A' has the most significant influence on the reliability of the PMS-2 in phase 1 and phase 3, and 'B' has the most significant impact on the reliability in phase 2.

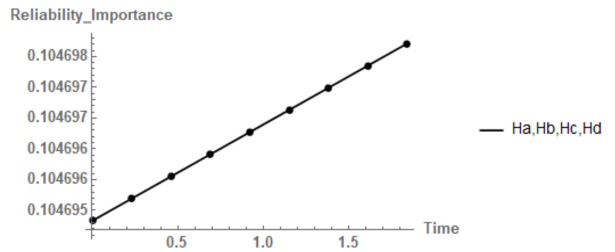

Figure 9 (a). Reliability importance plot of the PMS-3 for component H_a, H_b, H_c, H_d of subsystem 1 of phase 1.

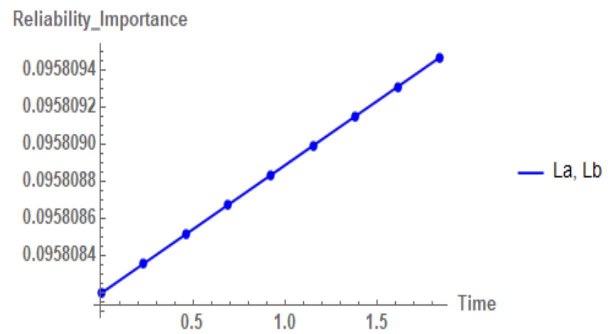

Figure 9 (b). Reliability importance plot of the PMS-3 for component 'L_a' & 'L_b' of subsystem 2 of phase 1.

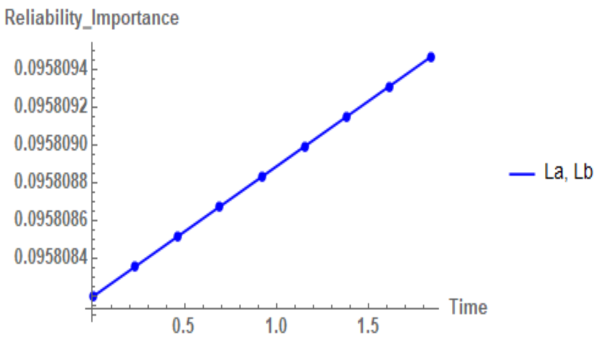

Figure 10. Reliability importance plot of the PMS-3 for component ‘H_a’ & ‘H_b’ of subsystem 1 of phase 2.

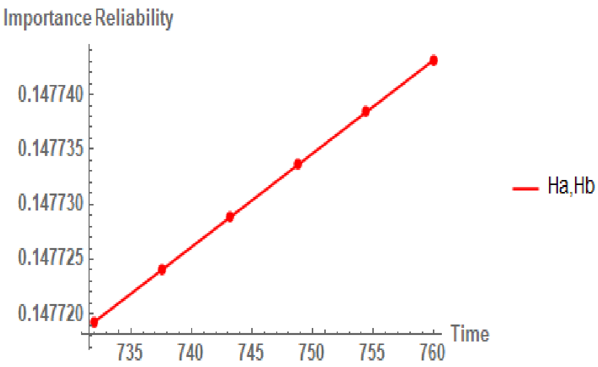

Figure 11(a). Reliability importance plot of the PMS-3 for component ‘H_a’ & ‘H_b’ of subsystem 1 of phase 3.

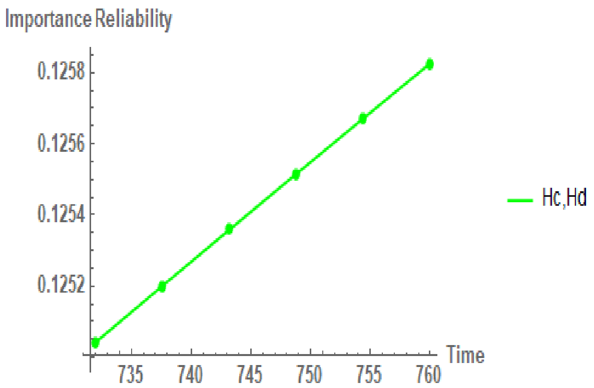

Figure 11(b): Reliability importance plot of the PMS-3 for component ‘H_c’ & ‘H_d’ of subsystem 1 of phase 3.

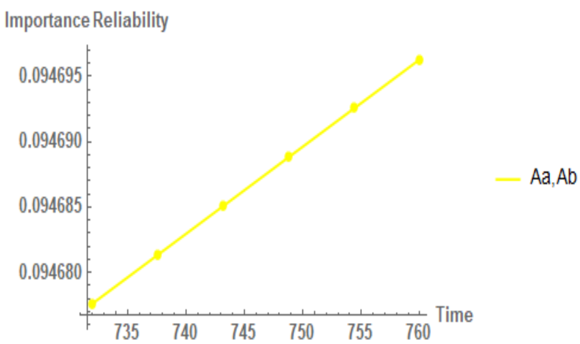

Figure 11(c). Reliability importance plot of the PMS-3 for component ‘A_a’ & ‘A_b’ of subsystem 2 of phase 3.

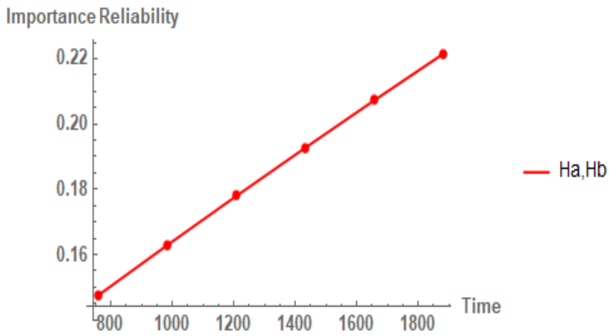

Figure 12: Reliability importance plot of the PMS-3 for component ‘H_a’ & ‘H_b’ of subsystem 1 of phase 4.

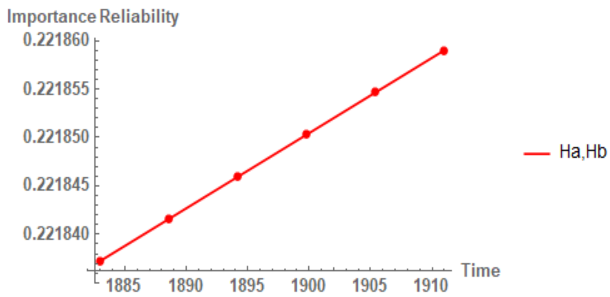

Figure 13 (a). Reliability importance plot of the PMS-3 for component ‘H_a’ & ‘H_b’ of subsystem 1 of phase 5.

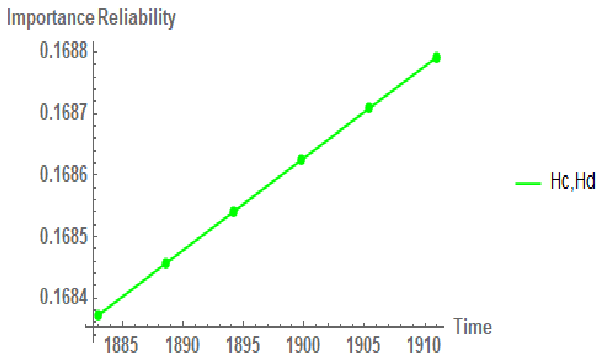

Figure 13 (b). Reliability importance plot of the PMS-3 for component ‘H_c’ & ‘H_d’ of subsystem 1 of phase 5.

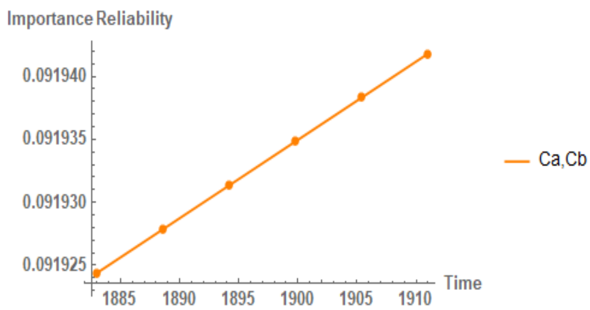

Figure 13 (c). Reliability importance plot of the PMS-3 for component ‘C_a’ and ‘C_b’ of subsystem 2 of phase 5.

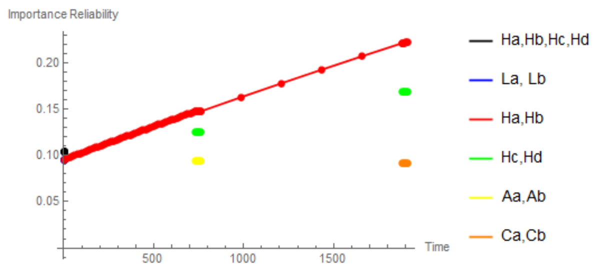

Figure 14. Reliability importance plot of each component phase-wise in PMS-3.

Figures 9(a)-13(c) depict each phase's component-wise reliability importance plot in PMS-3. Figure 14 shows the reliability importance of each component phase-wise in PMS-3, and it can be seen that 'H_a' and 'H_b' have the most significant influence on the reliability of the PMS-3.

7. Conclusion

In this paper copula-based approach has been used to obtain the reliability of phased-mission systems. Two 3-PMSs with and without inactive components and 5-PMS representing space application have been used with dependency between components modeled using the Gumbel-Hougaard copula and cumulative exposure model. Reliability importance analyses of the three PMSs based on the Birnbaum importance measure have been conducted to quantify the influence of the reliability of each component on the reliability of the PMSs. The method developed has been described using numerical examples. The expected results regarding the reliability and importance of components have been obtained for the hypothetical data set used. For instance, in space application PMS, H_a & H_b are found to be most important, implying that failure of both of them will result in failure of the PMS. In engineering practice, it would be advisable to prioritize these components in different phases to ensure the successful completion of the PMS's mission. The information about the reliability and importance of the components of the PMS can assist in formulating different maintenance strategies in different phases, thereby reducing the risk of failure. The proposed methodology can also be generalized to PMSs with more than five phases.

8. Acknowledgments

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9. Declaration of Conflict Interest

The authors have declared that no conflict interests exist.

10. References

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