



**Original Research Article** 

# Prediction of Wear Failure in Journalpin Bearings

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#### Abstract

Wear damage is a common failure in engine bearing shells. Wear failure induces changes in the bearing geometry and affects the oil film pressure and the durability of bearing shells. In this paper, wear failure in journalpin (main) bearings of a reciprocating engine is studied. Using a dynamic model, forces and torques in the journalpin bearings have been calculated and used in the analysis. To calculate the lubricating characteristics of the bearing, such as minimum oil film thickness and maximum oil film pressure, the Elasto-HydroDynamic (EHD) model that incorporate mass conservation algorithms is utilized. Wear failure in journalpin bearings of a reciprocating engine is assessed and the results are presented. Archard's model is used as a wear model of journalpin-bearing material. The asperity interaction of two rough surfaces is considered by the boundary lubrication model. The results show that the wear rate at the initial stage of engine running is high. The results indicate that the main reason for wear in the journalpin bearings is the applied torque on the bearing, leading to edge wear. In a V-12 or 6Inline engine, journalpin bearing number 4 has the highest amount of torque applied and therefore has the highest amount of wear.

Keywords: Wear Failure; Journalpin Bearing; Elasto-Hydrodynamic Model; Reciprocating Engine; Archard Model.

## Nomenclature

a, b and c	Constants of wear model
$C_R$	Bearing radial clearance
dW / dt	Wear rate
Ε	Elastic modulus
$E^*$	Composite elastic modulus
$e_x$	Eccentricity in the X direction
$e_v$	Eccentricity in the Y direction
Ň	Elastic factor
Ζ	Axial position of bearing nodes
α	Pressure viscosity coefficient
$\alpha_x$	Misalignment around the X axis
$\alpha_{v}$	Misalignment around the Y axis
β	Circumferential coordinate of bearing
δ	Radial deformation of the bearing surface
$\eta_0$	Viscosity at ambient pressure
η	Viscosity
$\dot{\theta}$	Fill ratio
$\sigma_{\rm s}$	Composite surface asperity height (r.m.s)
v	Poisson ratio

### 1. Introduction

Engine designers are increasingly using more advanced simulation techniques to reduce design time and costs. At the same time, they use them to improve the accuracy

of the design and limit the number of required validation tests. In recent years, the demand for higher specific power has enforced higher bearing loads in reciprocating engines. On the other hand, due to competition in the market, engine parts should be durable; therefore, the least amount of failure should occur in critical parts. Wear is a common failure in engine bearing shells; therefore, the assessment of this failure and introduction of the methods to prevent it is vital. Bearings of reciprocating engines are subject to high firing and dynamical loading. Design limitations for the weight of engine parts have been caused by to design of lighter and thus more flexible bearing housing, especially in journalpin bearings. This will highlight the importance of elastic deflection of bearing support and its consequent effects on clearance height between pin and shell. In addition, this will have significant effects on the lubricating performance of the bearing. The bearing housing design should be such that bearing and bearing housing deflection is not critical [1].

Wear has been recognized as the phenomenon of material removal from a surface due to interaction with a mating surface. Almost all machines lose their durability and reliability due to wear. Journalpin bearings, due to high loading conditions, in some cases, may have thinned the oil film between the bearing and journalpin. Under such conditions, the lubrication regime of engine bearings shifts from fully hydrodynamic to mixed lubrication, and the bearing is prone to wear failure. Figure 1 shows the wear failure in the journalpin bearing. Minimum oil film thickness and asperity contact stress are two factors that have been used to determine the possibility of wear failure in the bearing during the design process. Since many other factors are involved in wear failure of bearing, especially in running-in, thus, it is necessary to study the wear during the progression of wear failure.



Figure 1. Wear failure in journalpin bearing [2]

Iwasawa et al. [3] experimentally studied the effects of the surface conditions and the mechanical properties of the shafts and bearings on the wear amount of the bearing. Ushijima et al. [4] and Okamoto et al. [5-6] used a wear model for lubricated bearings similar to Archard's [7] model. They considered just the reaction force from asperity contacts in the wear calculation. Wang et al. [8] proposed a simple method to predict the dynamic wear in run-in contacts under partial-Elasto Hydrodynamic (EHD) conditions that included the considerations of lubrication effects. They included the real contact area of asperity contact pressure and the effect of surface roughness in their proposed model. However, the proposed model was based on line contact conditions and should be developed for general 3D contact conditions.

Karimaei et al. [9] studied wear and cavitation damage in big end bearings of two different designs of connecting rods structure. They showed that housing structure affects failure and should be considered in calculations. Moon et al. [10] evaluated the lubrication performance of the bearing in a marine two-stroke diesel engine to investigate the adhesion failure on the bearing. A numerical algorithm for the lubrication analysis is developed. In terms of the film parameter, the operating condition that can result in solid-solid contact is investigated and suggested the desirable operating conditions of clearance to prevent the solid-solid contact. Yu et al. [11] proposed a quasi-dynamic method to evaluate bearings' characteristics, including pressure distribution over the contact area, sliding velocity distribution, and lubrication parameters. The extent of permissible wear is confirmed based on the Archard wear equation. A mathematical model for wear-life analysis is then presented. The effects of axial loads, rotating speed, and structural parameters on the wear characteristics of bearings are analyzed. The wear life of a bearing decreases with increased axial load. The wear life of the bearing decreases as the rotational speed of the inner ring increases.

Karimaei and Chamani [12] simulated and studied the effects of bearing step (assembly errors) on the lubrication performance of main bearings and the probable wear damage has been investigated. Their results showed that for the under-study 12-cyl engine, considering step and bore relief in simulation has high importance so that the solution without step and bore relief does not predict the wear, but with step and bore, relief predicts medium wear damage in some of the main bearings. Ligier and Noel [13] reviewed several technical solutions to meet high-reliability bearings. Damage risks relating to customer use were also presented to check whether these risks were negligible or not. Moon et al. [14] evaluated the lubrication performance of the bearing in a marine two-stroke diesel engine to investigate the adhesion failure on the bearing. Their results showed that the lubricant temperature has a higher effect on film thickness than clearance. In terms of the film parameter, the operating condition that can result in solid-solid contact is investigated and suggested the desirable operating conditions of clearance and lubricant temperature are to prevent the solid-solid contact.

The current study studied wear failure assessment in journalpin bearing using the EHD model that incorporates mass conservation algorithms. To calculate the wear volume of journalpin bearing, a model based on the Archard wear model was utilized and the boundary lubrication model considered the asperity interaction of two rough surfaces. Variations of local hydrodynamic oil film pressure during the progression of bearing wear damage and the variation of local clearance height during the progression of wear damage were also extracted. Discussion on wear failure in the journalpin bearing of a reciprocating engine was performed in detail.

# 2. Elasto-Hydrodynamic (EHD) Model

AVL\Excite software is a powerful tool for bearing analysis and in current work has been employed for EHD analysis of the journalpin bearing of the engine. The EHD model is based on solving Reynold's Equation (1) [9] in the bearing surface. Equation (1) includes a mass conservation model reflected by the additional variable, clearance fill ratio  $\theta$ . For  $\theta = 1$ , the equation will become the ordinary Reynold's equation. Reynolds equation is solved for pressure p in the lubrication region and fill ratio  $\theta$ . The fill ratio is the fraction of oil filled with oil to the total volume [15]. A fill factor equal

to one indicates that the gap is filled with oil and zero indicates an empty gap.

$$\frac{\partial}{\partial x} \left( \frac{\theta h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\theta h^3}{12\eta} \frac{\partial p}{\partial z} \right) = \left( \frac{u_1 + u_2}{2} \right) \frac{\partial(\theta h)}{\partial x} + \frac{\partial(\theta h)}{\partial t}$$
(1)

For an EHD analysis, the effect of elastic displacements on the bearing surface must be included. The oil film thickness h is expressed by Equation (2) [15] considering the initial geometrical clearance, misalignment of the shaft, and elastic deformation.

$$h(\beta, z) = C_R - (e_x + \alpha_y Z) \cos\beta - (e_y + \alpha_x Z) \sin +$$
(2)

 $\delta(\beta, z)$  (2) Radial deformation of the bearing surface is obtained from the nodal displacements of the bearing surface along the radial and circumferential axes. The nodal displacements of the bearing surface are determined by solving the equations of motion for the condensed bearing structure. The current work uses Barus' equation [15] to define the oil viscosity.

$$\eta = \eta_0 e^{\alpha p} \tag{3}$$

Where,  $\eta_0$  is the viscosity at ambient pressure and  $\alpha$  is the pressure viscosity coefficient.

#### **3. Boundary Lubrication Model**

When the lubrication regime of engine bearings is shifted from fully hydrodynamic to mixed lubrication, the bearing clearance values will drop to an extremely small level and the surface asperities on the shaft and bearing will start interacting with each other. Figure 2 depicts the contact of two sliding surface asperities in the lubricating boundary condition. In AVL/Excite software, the model, according to Greenwood and Tripp [16], is applied for the asperity interaction of two rough

surfaces in the mixed-lubrication regime [9]. This model considers the contact of two nominally flat surfaces with a Gaussian distribution of asperity height and a fixed radius of asperities curvature. The nominal pressure generated by the asperities at contact is expressed as Equation (4) [16].



Figure 2. boundary lubrication between two rough surfaces

$$P_a = KE^* F(\frac{h}{\sigma_s}) \tag{4}$$

Where K denoted elastic factor,  $E^*$  composite elastic modulus and F is a factor which is the function of  $\Delta = \frac{h}{\sigma_s}$ . h denotes the nominal clearance between the contacting surfaces and  $\sigma_s$  is the standard deviation of the asperity summits of two surfaces and is called composite surface asperity height. The elastic factor describes the surface topography which can be determined experimentally.

Table 1 shows the surface roughness of the journalpin and bearing. Elastic factor K is calculated for as-new and worn bearing conditions for each type of bearings. Surface roughness characteristics were obtained by measuring the roughness with a roughness tester, which uses the contact method.

Items	Unit	quantity
Ra- journalpin	μm	0.14
Rz- journalpin	μm	1.54
Rmaz- journalpin	μm	2.15
Rt- journalpin	μm	2.47
Ra-Bearing	μm	0.57
Rz-Bearing	μm	1.3
Rmax-Bearing	μm	1.33
Rt-Bearing	μm	1.42

Table 1. Surface roughness of bearing and journalpin

### 4. Wear Failure Mechanism

Wear is progressive damage to a surface caused by relative motion concerning another substance. The wear in a bearing system is the function of some parameters, including the bearing material, the surface finishing of the mating surfaces, oil film thickness, carried the load and sliding speed. However, the processes involved in lubricated wear aren't well understood and the influence of the above parameters is not easily quantified. In sliding of two contacting surfaces, in which oil film thickness is lower than the sum of the surface roughness of two surfaces, all asperities, in contact with each other, are broken during every cycle. Every asperity will slide across several others along the opposing surface. This damage was identified as abrasive wear.

The minimum oil film thickness value when the bearing becomes worn is highly related to the surface topology of two contacting surfaces. Providing only the Ra and Rmax values makes very different surface morphologies possible. Figure 3 compares three different surface morphologies with similar Ra and Rmax values. Only the surface, as shown at the top of this figure, represents a desirable running surface because this surface has very few peaks that could get in contact with the sliding partners. Also, the surface includes some valleys to enable an oil-retaining property. Thus, the wear model should consider the surface topology of two surfaces and the changing surface roughness during wear progress.



Figure 3. Different surface morphologies with similar Ra and Rmax values [17]

In the current work, the bearing area curve (BAC) of the journalpin bearing is measured and used as the main parameter for the wear investigation of the bearing material. Figure 4 depicts a sample of BAC for journalpin bearing. By the progression of wear damage, the new BAC of the journalpin bearing is estimated, and the updated BAC is used in the subsequent step of wear calculation.



Figure 4. Bearing area curve of journalpin bearing

## 5. Assessment of Wear Using EHD Model

The reciprocating engine under study is a 1000kw engine. The journalpin-bearing parameters are illustrated in Table 2. The nodal displacements of the bearing surface are determined by solving the equations of motion for the condensed bearing structure. Thus, the mass and stiffness matrixes of the condensed bearing and its housing should be extracted using the substructuring method. Linear tetrahedral elements are used for the meshing of the housing. To investigate the wear in the journalpin bearings of the engine, firstly, the elasto-hydrodynamic analysis of the dynamic model of the engine with bearings has been done and the forces and torques in each of the journalpin bearings of the crankshaft have been calculated. Then, each journalpin bearing has been modeled with its elasto-hydrodynamic connection. To reduce the analysis time, only that part of the structure around the bearing, which plays a role in the deformation of the bearing, has been modeled. Figure 5 shows the dynamic model of the 12-cylinder engine in AVL/Excite software. The crankshaft and engine block are considered flexible and condensed

using the CON6 and SMOT methods, respectively. EHD joint type with 33 axial and 210 circumferential FD mesh nodes is used in the journalpin bearing.

The first boundary condition requests the pressure at the bearing's edges to be 0. The second boundary condition for the rotation requests the pressure in the widest gap to be 0. At the end of the pressure distribution of the rotation (after the narrowest gap), the pressure must become 0 there, where the pressure gradient in the circumferential direction becomes 0. The oil flow inlet is from the oil hole by adjusting its supply pressure. The walls are not rigid and are flexible. Compressive and tensile loadings on the bearing can also be seen in Table 2. Of course, their distributions are also obtained from the same dynamic analysis. However, in this analysis, oil supply temperature, clearance height, and oil bore position are also given as the inputs.

There are some wear models based on fatigue wear of bearing material. Sastry et al. [18] proposed a fatigue wear model for the wear of lubricated surfaces and considered the plastic strain in asperities as the main reason for low cycle fatigue of asperities. Zhu et al. [19] classified the sliding wear models and concluded that Equation (5) could be used as the general form of sliding wear:

$$\frac{dW}{dt} = k \frac{p^a u^b}{H^c} \tag{5}$$

Table 2. Journalpin bearing parameters

dt

Items	Unit	Value
Diameter	mm	160
Width	mm	40
Nominal diametric clearance	μm	120
Lubricant	-	SAE 15W40



Figure 5. Dynamic model of 12-cylinder engine with bearings in AVL-Excite

Where dW/dt is wear rate, p contact pressure, u relative sliding velocity, H material hardness, k wear coefficient, and a.b and c are constants. Archard's model a = b = c = 1 is frequently referenced because of its simplicity and wide application in different practical cases [19]. Therefore it is used as a wear model of journalpin-bearing material.

Wear damage causes a change in the surface roughness of the bearing and increases the local radial clearance between the bearing and shaft (journalpin). This, in turn, leads to an adjustment in oil film performance, oil film pressure distribution, and oil film thickness. Thus, along with wear progression, a new EHD analysis should be performed to investigate the updated oil film performance.

Initially, an EHD analysis of the as-new bearing (unworn bearing) is performed, including the asperity contact properties of the as-new bearing condition. Results in each node are imported in a wear code, which is written in Matlab software, and calculate the wear of bearing and update the radial clearance in each node. The bearing area curve represents the distribution of bearing surface roughness; therefore, it is used as the key parameter for surface roughness evolution during wear progression. Figure 6 depicts the surface roughness of the As-new bearing and wear-down bearing conditions. The calculated worn shape of the bearing is mapped on the nodes of the bearing surface. The wear amounts are added to the diametric clearance between the bearing and shaft. Then, a new EHD analysis is carried out, and this process will be repeated until the wear rate is decreased to a small value.



Figure 6. Bearing surface roughness, a) As-new bearing, b) Wear down bearing.

The wear rate in the initial stage of engine operation is high and then decreases due to adaptation in the geometry of the bearing surface and oil film performance characteristics too. Thus, the time interval for calculating wear amount at the initial stage is small and then increases.

#### 6. Results and Discussions

In most of the analyzes that are performed on journalpin bearings of engines, the rigid dynamic model of the engine is used to obtain the forces in the journalpin bearings and the effects of the flexibility of the crankshaft and the engine block in the distribution of the forces on the journalpin bearings of the engine is ignored. Also, the torque applied to the journalpin bearings of the engine, which is a function of the flexibility of the crankshaft and the engine block, as well as the clearance between the bearing and its pin, is ignored. In this paper, the comprehensive, flexible model of the engine is used to obtain the forces and torques in the crankshaft journalpin bearings. The torque applied to the journalpin bearings of the engine causes asymmetry in the behavior of the oil film in the bearing, which is important in the investigation of the wear in the bearing.

The analysis results indicate that bearing number 5 has the highest bearing force and bearing number 4 has the highest torque applied to the bearings. Therefore, in this paper, the wear is studied just for these two journalpin bearings. Figure 7 shows the pressure distribution of the oil film in the journalpin bearing number 4 at the crank angles of 376 and 697 degrees. At the crank angle of 376 degrees, due to the torque applied to the bearing, the oil film pressure distribution is asymmetrical at the crank angle of 697 degrees; due to the high torque applied, boundary lubrication occurs at the edge of the bearing. It should be noted that in the elasto-hydrodynamic analysis compared to the hydrodynamic one, due to the change of the shape of the bearing and the structure around it, the bearing surface to withstand the forces becomes wider; therefore, the pressure profile also becomes wider and consequently, the maximum pressure value decreases.



Figure 7. Oil film pressure distribution in journalpin bearing No. 4 at crank angles of 376 and 697 degrees

Figure 8 and Figure 9 show the boundary lubrication pressure distribution graphs for the journalpin bearings 4 and 5 at full load. Also in Figure 10 and Figure 11 the graphs of the minimum thickness of the oil film versus the crank angle of the crankshaft are shown for the journalpin bearings number 4 and 5. The minimum thickness of the oil film in both bearings is less than the total height of the roughness of the contact surfaces: therefore, boundary lubrication will occur. According to the obtained results, it is predicted that the amount of wear in bearing number 4 will be higher than that of bearing number 5 because the amount and extent of boundary lubrication pressure distribution are greater in this bearing. The minimum oil film thickness graph may not indicate the location and extent of potential bearing wear; however, it is good design guidance to investigate the bearing failure. Minimum oil film thickness usually occurs at the vicinity of the maximum oil film pressure crank angle. The minimum oil film thickness value is about 0.5 micrometer, which is lower than surface roughness; consequently, the boundary lubrication regime and asperity contact pressure would be expected at this location. The deformation of the journalpin bearing structure locally forms a concave shape in the axial direction, and the oil film is thick enough at the center line and gradually becomes thinner toward the edges of the bearing. Asperity contact pressure happens at the edges of the bearing.



**Figure 8.** Boundary lubrication pressure graph versus crank angle in journalpin bearing number 4



Figure 9. Boundary lubrication pressure graph versus crank angle in journalpin bearing number 5



Figure 10. Minimum oil film thickness graph versus crank angle in journalpin bearing number 4



Figure 11. Minimum oil film thickness graph versus crank angle in journalpin bearing number 5





Figure 12 shows the wear profile of journalpin bearing number 4 at the beginning of engine operation and after engine operation in running-in. The wear is in the form of an edge and is more on one side of the bearing than the other side. The wear profile in journalpin bearing No. 5 is also in the form of an edge, but the amount of wear is small. Therefore, it can be said that the main reason for wear in journalpin bearings of the crankshaft is the applied torque on the bearing, which leads to edge wear in the bearings. Wear failure locally happens at the edges of the bearing. By comparing the wear distribution, oil film thickness distribution, and asperity contact location, it is clear that the overall trend of wear damage location is similar to minimum oil film thickness and asperity contact locations. On the other hand, wear damage locations coincide with minimum oil film thickness positions. Wear at the edges of the bearing changes the bearing surface geometry and surface roughness so that the distribution of oil film pressure and thickness will be modified.

Along with the progress in wear damage, oil film pressure distribution in the axial direction is adjusted. Due to material removal from the bearing surface, the local clearance height increases, and the surface roughness of the bearing decreases. Results exhibit that by the progression of wear, the amount of oil film thickness increases and the asperity contact pressure decreases.

### 7. Conclusion

Elasto-Hydrodynamic (EHD) model is utilized to consider the effect of stiffness of the bearing housing and journalpin in the model to obtain the required characteristics of the bearing lubrication. Archard's model is used as a wear model of journalpin (main) bearing material. The asperity interaction of two rough surfaces is considered by the boundary lubrication model. The numerical simulation results confirmed that the wear rate at the initial stage of engine running is high. Wear at the edge of the bearing changes the surface geometry and surface roughness so that the distribution of oil film pressure and oil film thickness is modified. At the first operating engine cycle, the value of minimum oil film thickness is about 0.5 micrometer which is lower than surface roughness; consequently, the boundary lubrication regime and asperity contact pressure would be expected at the vicinity of the location of the corresponding crank angle to the maximum oil film pressure. The progress of wear has caused the clearance in the edge and near the edge of the bearing to reach more than 1 micrometer; as a result, the pressure peak has decreased, and the pressure distribution has become more uniform. Using a dynamic model, forces and torques in the journalpin bearings have been calculated and used in the analysis. The results indicate that the main reason for wear in the journalpin bearings is the applied torque on the bearing, leading to edge wear. In a V-12 or 6Inline engine, journalpin bearing number 4 has the highest amount of torque applied and therefore has the highest amount of wear.

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