

Analyzing Reliability of CGS Station by Continuous Time Markov Chains (CTMC)

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Abstract

Improving the system's reliability is one way to achieve a secure system. City Gas Station (CGS) has a key role in the timely and safe supply of Natural gas (NG) to residential, commercial, and industrial customers. With complexities inherent in systems, having a proper and all-embracing model of the entirety of a system is not readily possible. The continuous-time Markov chain (CTMC) model is regarded as a great help in communicating, comparing, and integrating partial system models. In this study, we have exploited CTMC for reliability analysis in CGS stations. The CTMC model can solve both time-dependent and stationary state probabilities. Therefore, it can potentially develop the state enumeration method into a sequential one. Implementing this procedure leads to identifying critical components and failure probability, eventually enhancing the station's reliability. Additionally, some suggestions are presented for optimizing the performance of the station.

Keywords: Reliability Assessment; Failure probability; Continuous time Markov chain (CTMC); Pipeline.

1. Nomenclature

<i>NG</i>	Natural gas
<i>NGTS</i>	Natural Gas Transportation Systems
<i>CGS</i>	City Gate Stations
<i>MBDA</i>	Model-Based Dependability Assessment
<i>CTMC</i>	Continuous Time Markov Chain

2. Introduction

Natural gas (NG) is one of the most pivotal sources among fossil energy all over the world, particularly for industry [1]. NG pipeline networks are important infrastructures connecting natural gas resources. Understanding the importance of natural gas pipeline networks for energy security and reliable and sustainable supply of natural gas has become a serious and global concern for economic, political, and technical reasons [2]. Due to the proximity and/or presence of natural gas

transportation systems (NGTS), especially city gate stations (CGSs), to/in urban areas, the consequences of any damage to the system can be disastrous. Therefore, public safety largely depends on the safe operation of NGTS [3].

Specific characteristics of natural gas have caused the accidents occurring in the NGTS to differ from other industrial accidents. For example, an explosion in an NG factory in Belgium in 2004 left 14 fatalities and more than 200 injuries; the conflagration of NG in Paraguay in 2004 caused more than 250 deaths, and in 2009, an explosion caused by NG leakage led to the largest conflagration in Moscow ever since the Second World War [4,5]. In 2010, a gas pipeline explosion in the Northeast of Iran left 19 casualties and more than 30 severe injuries, and an underground gas pipeline fire and explosion in Tehran, Iran, caused two fatalities and significant damage to surrounding residential buildings [4, 6]. Statistics show a significant increase in the failure

rate of NG pipelines in the last two decades [7]. Thus, it is very important to assess the reliability of NGTS facilities. This issue becomes extremely dangerous to stations adjacent to densely populated areas. Thus, CGS accidents threaten urban safety and have become an important issue for the general public [4, 7, and 8].

One of the important facilities in the NGTS is CGS station. CGS has a key role in the timely and safe supply of NG where to residential and industrial customers. Therefore, it's important to assess the reliability of the CGS operations to provide safe conditions and protect human lives and properties [4, 5]. Improving system reliability is one way to achieve a secure system [9]. To enhance reliability and adequate performance from repairable systems, it is necessary to provide regular maintenance [10]. Reliability engineering, as a sub-discipline of system engineering, includes the systematic application of engineering techniques throughout a product lifecycle, so reliability should be considered from the concept plan to wear out of the system [11].

Researchers working in the field of reliability have developed and investigated systems and assessed the performances based on reliability measures with a significant degree of satisfaction [12]. Several studies have been conducted to assess the reliability of NGTS [13-17]. However, most of the previous studies have merely focused on gas pipelines, whereas the work devoted to other important components of NGTS, such as CGS, has been very few. In addition to the limited scope of the previous work, most of the methods used for reliability in previous studies suffer from limitations such as statistics and the inability to consider dependencies.

However, modern reliability engineering emphasizes model-based dependability assessment (MBDA) [18, 19].

Reliability assessment methods such as logical modeling are employed to calculate system reliability [20-22]. The reliability of complex infrastructures, developed on a small scale and operating under different conditions, could be accurately assessed using these methods [23, 24].

Due to these shortcomings, it becomes difficult to apply the traditional methods for analyzing complex engineering systems, which are featured by a number of dependencies and uncertainties [25]. Therefore, we require an investigation to use continuous-time Markov chain (CTMC) in order to analyze [26]. Both time-dependent and stationary state probabilities can be solved by the CTMC model. Therefore, it can potentially develop the state enumeration method into a sequential method [27-29]. CTMC is an important class of stochastic processes that have been widely used in practice to determine system performance and dependability characteristics [30-32]. Based on the CTMC theory, a sequential analytical approach for composite system reliability assessment is proposed in this paper.

3. Methodology

This study was conducted in 2021 to assess the reliability of the pressure reduction station. In general, the framework of the research implementation method is based on Fig 1.

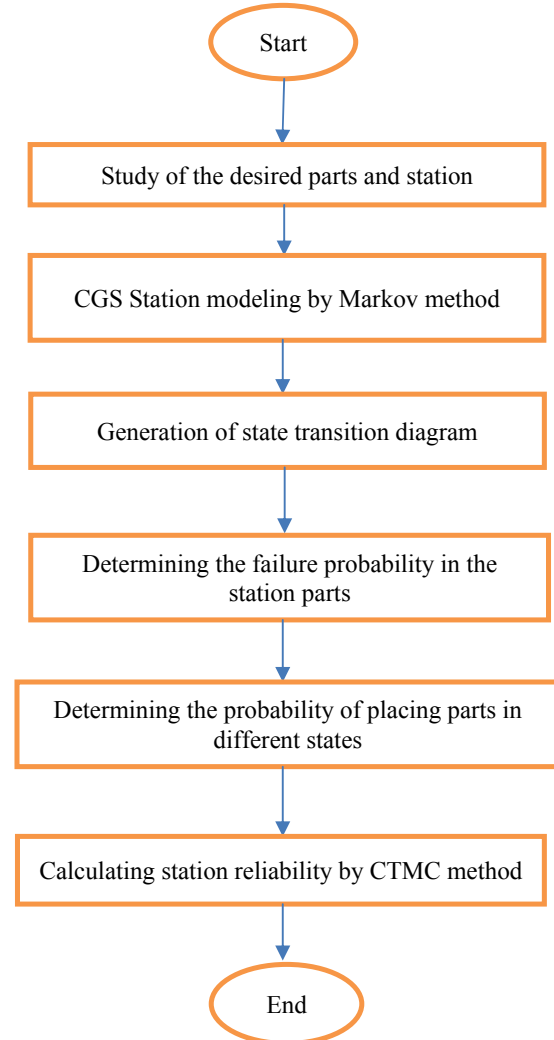


Figure 1. Framework of the research implementation method

Step 1: Study of the desired parts and station: It is necessary to get familiar with the system structure and performance. Safety specialists, occupational health engineers, operations engineers, maintenance engineers, and managers responsible for monitoring the system performance should be involved at this stage. Data tools such as flowcharts, block diagrams, fault trees, status charts, and timelines are very useful at this stage.

Step 2: CGS Station modeling by Markov method: To model the system using the Markov method, the different states of the system and their relationship with the states of the subsystems must be determined.

Step3: Generation of state transition diagram: To produce the matrix elements of the Kolmogorov, the

relationship between the states and how to transition between them should be determined. In a great number of models on estimating systems reliability, the assumption is that the system ingredients work "Independently" of one another.[34].

Step 4: Determining the failure probability in the station parts: station failure data were collected from 2011 to 2021 based on the reports of maintenance and instrumentation sectors. Next, based on the following equation1, the failure probability of each part was determined.

$$Failureprobability = \frac{\sum failuretime}{\sum accesstime} \quad (1)$$

Step 5: Determining the probability of placing parts in different states: In order to determine the probability, equation 1 was used. As well as, according to the opinions of station experts, all the time that the department is stopped due to breakdowns is not spent on repairs, and in these cases, a fraction of the time is lost. This time is spent in the absence of a repairman, the absence of a part needed for replacement, etc. So that, based on consultations with experts, only about 10% of the failure time of the parts is spent on repairs.

Step 6: Calculating reliability by CTMC method: To analyze the reliability of the Markov model based on the principles of probabilistic processes, which is based on mathematical principles, has been used. In the following, the reliability calculations by the CTMC method are given.

Based on this method, if the state space has n independent events, the probability of occurrence of a series of n consecutive events is shown in the following equation:

$$P(E_{j1}, E_{j2} \dots E_{jn}) = P_{j1} P_{j2} \dots P_{jn} \quad (2)$$

Markov generalizes this assumption to the case where the output of each event is directly dependent on the output of the previous event. In this generalized case, the conditional probability P_{ij} associated with the conditional occurrence of a pair of outputs or states is in the form (E_i, E_j) and is defined in the following equation:

$$P_{ij} = P\{X_{n+1} = E_j | X_n = E_i\} n = 0, 1, 2, \dots \quad (3)$$

The state θ space is used to define $\{X(t), t \geq 0\}$ the process, which means that θ space is the space of all possible values of X that the random variable X (t) can take. In this space, E is one of the events in the subset of θ space $(E \subset \theta)$, if the state space consists entirely of events that do not have anything in common with each other and are complementary to each other, that is, the community of all of them is the state space, using the rules Chances are, the following equations will hold.

$$[0 \leq P(E_i) \leq 1] \quad (4)$$

$$P\{\sum_{i=0}^{Nmax} E_i\} = \sum_{i=0}^{Nmax} P(E_i) = 1 \quad (5)$$

If the state space; is a discrete space. Continuous sets E_i can be represented by $\{X_i(t)\}_{t \in R}$ points or random

processes. These processes are discrete in space but continuous in time. The random process $X_i(t)$ is called a Markov process if the conditional probability expressed in the following equation is true:

$$P_{ij}(t) = P(X_j(t) = j | X_i(\tau) = i) \quad (6)$$

In the following, two probabilistic relative transfer functions of the Markov process and density are defined. The density function shows the probability of a change in the state under the condition that the change occurs. The specifications of the two introduced functions are shown in the following relations.

$$[0 \leq Q_{ij}(t) \leq 1] \quad \{\forall i, j \in z, t \in R\} \quad (7)$$

$$\sum_{j=0}^N Q_{ij}(t) = 1 \quad \{\forall j \in z, t \in R\} \quad (8)$$

Equations describing conditional probability functions of states are called Kolmogorov equations. The Kolmogorov equation can be expressed in matrix form [33-36]. This study used Excel and MATLAB software to calculate the Markov model.

4. Results

The results of the study, step by step based on the method section, are given below:

Step 1: Determining different parts of the CGS station: A schematic of the CGS station is shown in Fig 2.

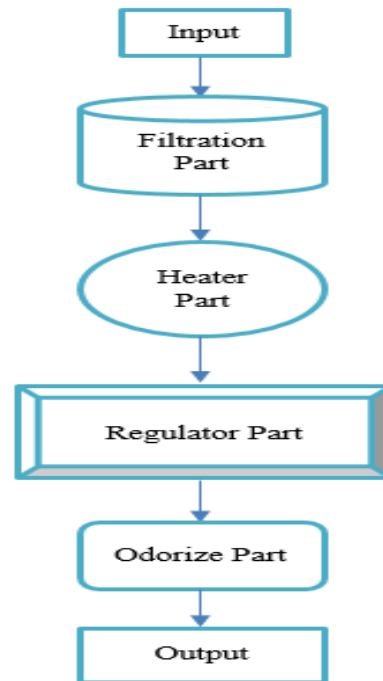


Figure 2. A schematic of CGS station

Step 2: CGS Station modeling by Markov method: The relationships between system survival failure and subsystem failure are given in Table1.

Table 1. CGS Station parts failure states

Failure State				
state	Filtration	Heater	Regulator	Odorize
1	S	S	S	S
2	S	S	S	F
3	S	S	F	S
4	S	F	S	S
5	F	S	S	S
6	S	S	F	F
7	S	F	S	F
8	S	F	F	S
9	F	S	S	F
10	F	S	F	S
11	F	F	S	S
12	S	F	F	F
13	F	S	F	F
14	F	F	S	F
15	F	F	F	S
16	F	F	F	F

Step3: Generation of state transition diagram:Fig3 presents CGS Station state transition diagram.

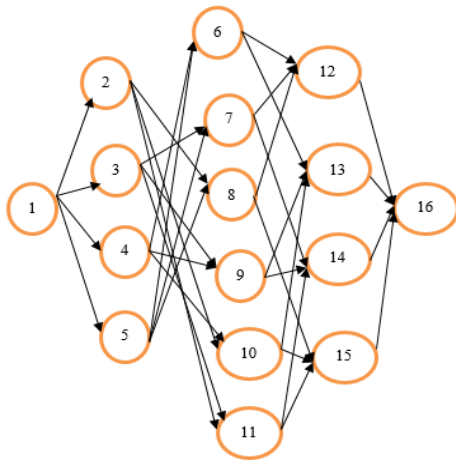


Figure 3. CGS Station state transition diagram

Step 4: Determining the failure probability in the station parts: Table2 shows the failure probability of each part of the station based on equation 1.

Table 2. Failure probability of each part of the station

Part	Failure Probability
Filtration	0.95
Heater	0.994
Regulator	0.974
Odorize	0.9963

Step 5: Determining the probability of placing parts in different states: In order to determine the probability of placing parts in different states, equation 1 was used.

As well as, about 10% of the failure time of the parts is spent on repairs. The results of these calculations are given in Table 3.

Table 3. Results from the probability of placing parts in different situations

part	Active	Inactive	Repaired	Under repaired
Filtration	0.95	0.05	0.995	0.005
Heater	0.994	0.006	0.9994	0.0006
Regulator	0.974	0.026	0.9974	0.0026
Odorize	0.9963	0.0037	0.99963	0.00037

Active mode → active mode ⇒active state
 Active mode → inactive mode ⇒inactive state
 Inactive mode → active mode ⇒repaired state
 Inactive mode → inactive mode ⇒repair under state

Step 6: Calculating station reliability by CTMC method: After calculating the different probabilities, the transfer matrix can be formed. The transfer matrix is a square matrix whose number of rows and columns equals the number of possible state spaces. Each component of this matrix indicates the probability of converting one state to another. The P transfer matrix formed for this case is in accordance with Table 4.

Table 4. Transmission matrix of different states of CGS station

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16
S1	0.92	0.003	0.02	0.006	0.05	0	0	0	0	0	0	0	0	0	0	0
S2	0.92	0	0.02	0.01	0.05	0	0	0	0	0	0	0	0	0	0	0
S3	0.94	0.002	0.002	0.006	0.05	0	0	0	0	0	0	0	0	0	0	0
S4	0.921	0.003	0.026	0	0.05	0	0	0	0	0	0	0	0	0	0	0
S5	0.96	0.004	0.025	0.006	0.005	0	0	0	0	0	0	0	0	0	0	0
S6	0.94	0	0.004	0.006	0.05	0	0	0	0	0	0	0	0	0	0	0
S7	0.92	0	0.03	0	0.05	0	0	0	0	0	0	0	0	0	0	0
S8	0.94	0.003	0.002	0.005	0.05	0	0	0	0	0	0	0	0	0	0	0
S9	0.96	0	0.03	0.005	0.005	0	0	0	0	0	0	0	0	0	0	0
S10	0.98	0.004	0.004	0.007	0.005	0	0	0	0	0	0	0	0	0	0	0
S11	0.96	0.004	0.03	0	0.006	0	0	0	0	0	0	0	0	0	0	0
S12	0.95	0	0	0	0.05	0	0	0	0	0	0	0	0	0	0	0
S13	0.99	0	0.001	0.004	0.005	0	0	0	0	0	0	0	0	0	0	0
S14	0.97	0	0.025	0	0.005	0	0	0	0	0	0	0	0	0	0	0
S15	0.99	0.004	0.002	0.004	0.004	0	0	0	0	0	0	0	0	0	0	0
S16	0.99	0	0.003	0.002	0.005	0	0	0	0	0	0	0	0	0	0	0

Using MATLAB software, the results showed that the reliability of the CGS station in the state that all systems are active is equal to 95%.

5. Discussion

Improving the system's reliability is one way to achieve a secure system. City Gas Station (CGS) has a key role in the timely and safe supply of Natural gas (NG) where to residential, commercial, and industrial customers. With complexities inherent in systems, having a proper and all-embracing model of the entirety of a system is not readily possible. Continuous-time Markov chain (CTMC) models are regarded as a great help in communicating, comparing, and integrating partial system models. In this study, we have exploited CTMC for reliability analysis in the CGS station. Both time-dependent and stationary state probabilities can be solved by the CTMC model. Therefore, it can

potentially develop the state enumeration method into a sequential one. Implementing this procedure leads to identifying critical components and failure probability, eventually enhancing the system's reliability.

In order to analyze the reliability of the CGS station, the station was divided into four parts: filtration, heater, regulator, and odorize, so that the reliability of the filtration, heating, regulator, and odorize parts were 0.95, 0.994, 0.974 and 0.9963, respectively. Eventually, the reliability of the CGS station when all systems are active is 95%.

The filtration and regulator parts were among the most critical parts of the station. Zarei et al. found that the failure of the filtration and regulator parts was the worst risk scenario at the CGS station (4), which was consistent with the results of this research so that the failure of the filtration and regulators could increase gas pressure in station pipelines and cause accidents such as explosions if the safety and shut-off valves do not operate properly.

The results revealed the filtration part had the lowest level of reliability at the station, so that reliability was 0.95. Moreover, this system had the lowest mean time between failures. Thus, this was considered the first critical part of the station. An extra component was proposed to improve the reliability so that the extra component could perform the task assigned to the main component when it fails to operate.

The regulator part was the second critical part of the station, and its reliability was 0.974. The reliability of this equipment could be improved by optimizing its timely maintenance and repairs. Performing the lock-up test on regulators periodically eliminates the need to open and close the regulators and, consequently, leads to less failure in other devices and provides more safety.

Due to the nature of such studies and the presence of uncertainties in their results, there was a limited number of studies to compare the results of our research. Implementing this method in one of the important and sensitive centers of gas transmission lines was among the strengths of this research. Using modeling at the CGS station based on the solutions proposed for optimizing the system has brought useful experiences that could be employed in CGS station design, construction and operation. Although the reliability was assessed by modeling, there were some limitations. In future studies, random simulation methods such as Monte Carlo Simulation (MCS) or Monte Carlo Chain Markov (MCMC) methods are recommended. Given that some components have a non-constant and time-varying failure rate at different stages of their life, the effects of temporal changes on the failure rate, which is a continuous-time feature of complex systems with long life, could be accurately calculated using these methods.

6. Conclusion

In this study, the reliability was estimated by the CTMC model. Implementing this procedure leads to identifying

critical components and failure probability, eventually enhancing the system's reliability. The results showed the filtration and regulator parts required more attention to improve the system. Therefore, the reliability of the CGS station could be improved using the redundancy method and regular maintenance program, among the most important methods for improving reliability.

7. Conflict of interest

The present study did not have any conflict of interest for the authors.

8. Acknowledgment

This study, taken from the Ph.D. dissertation on occupational health, was approved by the Tehran University of Medical Sciences with ethical code no. IR.TUMS.SPH.REC.1400.151. We would like to appreciate the National Iranian Gas Company (North Khorasan Province) for providing financial support, as well as the Department of Occupational Health Engineering of Tehran University of Medical Sciences for their cooperation in this study.

9. Contribution of the authors

The responsible author was the main participant in all stages of project implementation and article writing, and the first and second authors were responsible for the necessary guidance and supervision as supervisors.

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