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Original Research Article

Markov Modeling and Reliability analysis of solar photovoltaic system Using Gumbel Hougaard Family Copula

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Abstract

The present work illustrated the reliability analysis of solar photovoltaic systems and the efficiency of medium grid-connected photovoltaic (PV) power systems with 1-out of- 2 PV panels, one out of one charge controller, 1- out of 3 batteries, 1- out of 2 inverters and one out one Distributor. The units that comprise the solar were studied. Gumbel Hougaard Family Copula method was used to evaluate the performances of solar photovoltaics. Other reliability metrics were investigated, including availability, mean time to failure, and sensitivity analysis. The numerical result was generated using the Maple 13 software. The numerical results were presented in tables, with graphs to go along with them. Failure rates and their effects on various solar photovoltaic subsystems were investigated. Numerical examples are provided to demonstrate the obtained results and to assess the influence of various system characteristics. The current research could aid companies, and their repairers overcome some issues that specific manufacturing and industrial systems repairers face.

Keywords: Availability; Efficiency; Inverter; Photovoltaic; Reliability; Sensitivity.

1. Introduction

The increase in linked PV's percentage growth can be attributed to various factors. Examples include low installation costs, quick energy, investment payback, and consumer stimulation. In this case, continuous output energy production must be demonstrated to satisfy the cost-benefit analysis of PV systems. As a result of the rapid expansion of PV system capacity on global grid systems, PV system technology is maturing and becoming more competitive in the power market. As a result, PV system engineers will prioritize PV operations in terms of reliability, efficiency, maintenance, and fault management. The sun's energy is one of the most ancient and cost-effective primary energy sources and has long been used for preservation and fabric drying. Agricultural commodities are dried, which is still done in most impoverished countries today (Solar energy as thermal). System Reliability is a metric that assesses how well a system performs under adverse conditions. Most complex systems are composed of components and subsystems linked in series, parallel, standby, or a combination of these, according to the specifications. In social, political, commercial, and technological settings, dependability terms express faith/trust in a person, firm, or piece of equipment. An analysis of a solar system can assist users in making timely decisions to ensure the system's optimal performance. The subject of dependability theory evolved as a result of operational research in the context of military studies. The terms "reliable" and "reliability" have been used interchangeably since antiquity. In reality, they are frequently used in the social, political, economic, and practical sectors to demonstrate the efficacy of a person or a piece of mechanical equipment. The word "reliability" was given a mathematical structure later that year, in 1950, in conjunction with its scientific use for military goals. Dependability theory was developed in the Western world due to its importance. The history of India's dependability technology development will be informative and exciting for academics. Almost every problem we encounter daily is influenced by dependability theory, either directly or indirectly. Power, transportation, medical services, steel, and communication networks are just a few examples of systems whose resiliency directly impacts society. System failures can occur in any discipline, according to modern engineering history.

Researchers have made significant contributions to improving the efficiency and performance of various solar systems and investigating the variables that impede photovoltaic system performance, as mentioned above. The dependability metrics used to assess solar system strength, efficacy, and performance are poorly understood. More research on the dependability metric for assessing solar system strength, effectiveness, and performance enhancement are required. The current work developed a reliability modeling technique to investigate the overall performance of the PV system due to a lack of PV system data. This paper presents a novel solar system model with four subsystems: Control charger, panel, inverter, and battery bank. The transition diagram is used to build and solve a system of partial differential equations, yielding strong reliability characteristics such as reliability, availability, mean time to failure (MTTF), sensitivity analysis, and profit function. This project aims to develop dependability models to assess the PV system's strength. The findings of this study will be useful to managers of residential, commercial, and industrial plants, as well as industries and manufacturing systems that plan to use photovoltaic energy and power sources.

Our primary goal in implementing solar energy is to reduce carbon dioxide emissions from traditional power generation. Furthermore, machine failure is a problem that industries face, resulting in slow technological advancement worldwide due to power fluctuations. To determine maintenance costs, dependability, availability, and power outages, distribution networks connected to photovoltaic systems will be investigated, necessitating methodology and tools to evaluate the reliability of grid-connected photovoltaic systems. For the development and operation of PV power plants as well as PV-connected distribution networks, risk assessment and reliability evaluation are critical.

2. Literature Review

We studied the following material to better understand modeling, photovoltaics, and the Gumbel Haugaard Family copula. The Gumbel-Hougaard Family Copula was used to analyze the reliability and performance of a series-parallel system by Maihulla et al. [1]. Maihulla and Yusuf [2] studied Performance Analysis of Photovoltaic Systems Using (RAMD) Analysis. Goyal et al. [3] studied the Reliability, maintainability, and sensitivity analysis of the physical processing unit of the sewage treatment plant. The Reliability assessment in electrical power systems: the Weibull-Markov stochastic model was studied by Casteren et al. [4], Ogaji et al. [5] investigated the Reliability of the Afam electric power generating station, Nigeria. Ebelin [6] studied the introduction to reliability and maintainability engineering. Gupta and Tewari [7] established the Simulation modeling and analysis of the complex system of the thermal power plant. Tsarouhas et al. [8] Studied Reliability and maintainability analysis of strudel production line with experimental data. Carazas and Souza [9] studied the Availability analysis of gas turbines used in power plants. Lado and Singh [10] studied the Cost assessment of a complex repairable system consisting of two subsystems in the series configuration using the Gumbel Hougaard family copula. Singh et al. [11] studied the Performance analysis of a complex repairable system with two subsystems in a series configuration with an imperfect switch. Yusuf et al. [12] studied the Performance Analysis of Multi-computer System Consisting of Three Subsystems in Series Configuration Using Copula Repair Policy. Raghav[13] studied the Reliability Prediction of Distributed System with Homogeneity in Software and Server using Joint Probability Distribution via Copula Approach. The Gumbel–Hougaard family copula was investigated for reliability modeling and performance evaluation of solar photovoltaic systems by Maihulla et al. [14]. Reliability and Performance Analysis of Two Unit Active Parallel System Attended by Two Repairable Machines was studied by Yusuf et al. [15]. The Reliability, availability, maintainability, and dependability analysis of photovoltaic systems was studied by Maihulla and Yusuf [16]. Gumbel-Hougaard Family Copula Reliability Analysis of Multi-Workstation Computer Network Set Up as a Series-Parallel System [17].

From all the above literature, Markov modeling for the reliability analysis of the solar photovoltaic system was not addressed. Also, sensitivity analysis regarding the study of repairable solar Photovoltaic was very little or non in the existing literature.

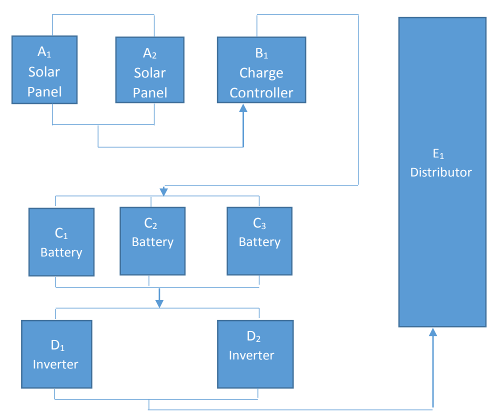

Figure 1. System Block Diagram

3. ASSUMPTIONS

Throughout the model's explanation, the following assumptions are made:

1. At first, all subsystems are in good functioning order.
2. For the system to be operational, two units from subsystems 3 and one from subsystems 1, 2, 4, and 5 must be used consecutively.

3. If one of the units in subsystems 1 and 4 fails, the system will be rendered reduced capacity.
4. The system will be rendered inoperable if all two units from subsystems 1, 3, and 5 fail.
5. A system's failing unit can be fixed when it is in a reduced capacity or failed state. Copula maintenance is required once a unit in a subsystem fails completely. A copula-repaired system is believed to operate like a new system, and no damage occurs during the repair.
6. Once the faulty unit has been fixed, it is ready to execute the task.

Table1. State Description

State	Description
S_0	Units A1 is operational in its initial form. And the system is fully working. Unit B1 in subsystem 2 is operational. Units C1 and C2 are operational in subsystem 3. Unit D_1 of subsystem 4 is in operation, and E_1 from subsystem 5 is also in operation. While A2 from subsystem 1, C3 from subsystem 3, and D2 from subsystem 4 are on standby.
S_1	In this state, unit A1 has failed and is being repaired. And the total repair time is (x,t). A2, B1, C1, D1, and E1 are operational. While C2 and C3 from subsystem-3 and D2 from subsystem 4 are on standby.
S_2	The A1 and C1 have failed, and the total repair time is (x,t) and (z,t), respectively. While units A2, B1, C2, D1, and E1 are operational. C3 from subsystem-3 and D2 from subsystem 4 are on standby. While C3 from subsystem-3 and D2 from subsystem 4 are on standby.
S_3	The A1 C1 and C2 have failed, and the total repair time is (x,t), (z,t), and (z,t), respectively. While units A2, B1, C3, D1, and E1 are operational. D2 from subsystem 4 is on standby. The state is partially operational. S4 is a completely failed state caused by the collapse of subsystem 1.
S_4	The A1 and C1, C2, and D1 have failed, and the total repair time is (x,t), (z,t), (z,t), and (k,t), respectively. While units A2, B1, C3, D2, and E1 are operational. The state is partially operational. S4 is a completely failed condition caused by the failure of two units in subsystem 2.
S_5	S5 is a completely failed state caused by the failure of two units in subsystem 1.
S_6	S6 is a completely failed state caused by the breakdown of a unit in subsystem 2.
S_7	S7 is a completely failed state caused by the failure of three units in subsystem 3.
S_8	S8 is a completely failed state caused by the failure of two units in subsystem 4.
S_9	S9 is a completely failed state caused by the failure of a unit in subsystem 5.

The number of respective states in the state transition diagram in figure 2 below was illustrated in table 1 above.

P_0: Denote the initial state where the system is working perfectly.

P_1: Denote state with an incomplete failure in subsystem-1 due to failure of first unit and repair machine is busy repairing the failed unit.

P_2: Denote state with a complete failure in subsystem-1 due to failure of the second unit, and Copula repair is busy repairing the failed unit.

P_3: Denote state with a complete failure in subsystem-2 due to failure of the only unit in the subsystem.

P_4: Denote state with a degraded state in subsystem-3 due to failure of the first unit.

P_5: Denote state with an incomplete failure in subsystem 3. Previously first has failed.

P_6: Denote state with a complete failure in subsystem 3. This is due to the failure of the first and second units from the subsystem. The Copula repair is employed for automatic repair of the completely failed unit.

P_7: Denote the incomplete state of the system due to the failure of the first unit from subsystem 4. The repair machine is busy repairing the failed component.

P_8: Denote the complete state of the system due to the failure of the second unit from subsystem 4. The Copula repair is employed for automatic repair of the completely failed unit.

P_9: Denote an incomplete failure state of the system. This is due to the failure of the first units from subsystems 1 and 3. The repair machine is automatically busy repairing the failed component.

P_{10}: Denote an incomplete failure state of the system. This is due to the failure of the first and second units from subsystems-1 and the first unit from subsystem 3. The repair machine is automatically busy repairing the failed component.

P_{11}: Denote an incomplete failure state of the system. This is due to the failure of the first units from subsystems 3 and 4. The repair machine is automatically busy repairing the failed component.

P_{12}: Denote an incomplete failure state of the system. This is due to the failure of the first and second units from subsystems-3 and the first unit from subsystem 4. The repair machine is automatically busy repairing the failed component.

P_{13}: Denote an incomplete failure state of the system. This is due to the failure of the first units from subsystems 4 and 1. The repair machine is automatically busy repairing the failed component.

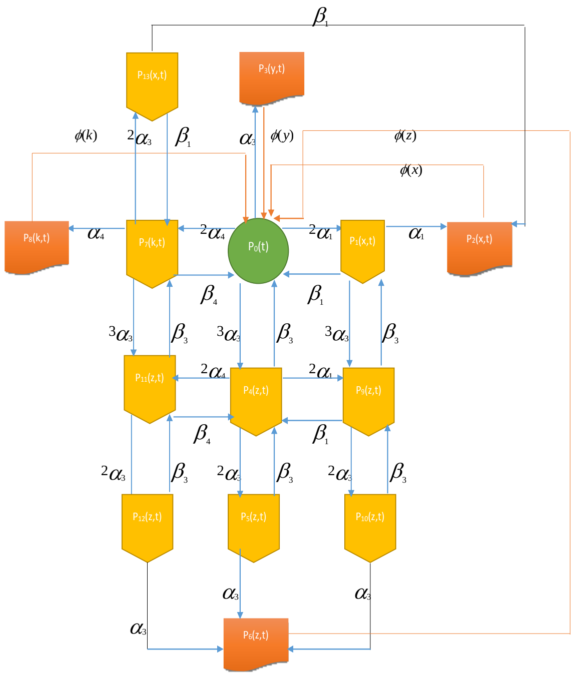

Figure 2. Systems Transition Diagram

$$\left[\frac{\partial}{\partial t}+2\alpha_1+\alpha_2+3\alpha_3+2\alpha_4\right]P_0(t) = \int_0^\infty \beta_1 P_1(x,t)dx + \int_0^\infty \varphi(y)P_3(y,t)dy + \int_0^\infty \beta_3 P_4(z,t)dz + \int_0^\infty \beta_4 P_7(k,t)dk + \int_0^\infty \beta_4 P_7(k,t)dk + \int_0^\infty \beta_1 P_1(x,t)dx + \int_0^\infty \varphi(x)P_2(x,t)dx + \int_0^\infty \varphi(k)P_8(k,t)dk + \int_0^\infty \varphi(z)P_6(z,t)dz \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\beta_1+3\alpha_3\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right]P_2(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\varphi(y)\right]P_3(y,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_3+2\alpha_1+2\alpha_4+2\alpha_3\right]P_4(z,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_3+\alpha_3\right]P_5(z,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\varphi(z)\right]P_6(z,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial k}+\alpha_4+3\alpha_3+\beta_4+2\alpha_1\right]P_7(k,t)=0 \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial k}+\varphi(k)\right]P_8(k,t)=0 \quad (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_3+2\alpha_3+\varphi(x)\right]P_9(z,t)=0 \quad (10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_3+\alpha_3\right]P_{10}(z,t)=0 \quad (11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\alpha_3+2\alpha_3+\beta_4\right]P_{11}(z,t)=0 \quad (12)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\alpha_3+\beta_3\right]P_{12}(z,t)=0 \quad (13)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\beta_1\right]P_{13}(x,t)=0 \quad (14)$$

Boundary condition

$$P_1(0,t)=2\alpha_1 P_0(t) \quad (15)$$

$$P_2(0,t)=2\alpha_1^2 P_0(t) \quad (16)$$

$$P_3(0,t)=\alpha_2 P_0(t) \quad (17)$$

$$P_4(0,t)=3\alpha_3 P_0(t) \quad (18)$$

$$P_5(0,t)=6\alpha_3^2 P_0(t) \quad (19)$$

$$P_6(0,t)=6\alpha_3^2 P_0(t) \quad (20)$$

$$P_7(0,t)=2\alpha_4 P_0(t) \quad (21)$$

$$P_8(0,t)=2\alpha_4^2 P_0(t) \quad (22)$$

$$P_9(0,t)=12\alpha_1\alpha_3 P_0(t) \quad (23)$$

$$P_{10}(0,t)=24\alpha_1\alpha_3^2 P_0(t) \quad (24)$$

$$P_{11}(0,t)=12\alpha_3\alpha_4 P_0(t) \quad (25)$$

$$P_{12}(0,t)=24\alpha_3^2\alpha_4 P_0(t) \quad (26)$$

$$P_{13}(0,t)=4\alpha_1\alpha_4 P_0(t) \quad (27)$$

By taking the Laplace transform of (1) to (27), we've

$$\left[S+2\alpha_1+\alpha_2+3\alpha_3+2\alpha_4\right]\overline{P}_0(S)=1+D \quad (28)$$

Where

$$D = \int_0^\infty \beta_1 \overline{P}_1(x,s)dx + \int_0^\infty \varphi(y)\overline{P}_3(y,s)dy + \int_0^\infty \beta_3 \overline{P}_4(z,s)dz + \int_0^\infty \beta_4 \overline{P}_7(k,s)dk + \int_0^\infty \beta_1 \overline{P}_2(x,s)dx + \int_0^\infty \varphi(x)\overline{P}_2(x,s)dx + \int_0^\infty \varphi(k)\overline{P}_8(k,s)dk + \int_0^\infty \varphi(z)\overline{P}_6(z,s)dz \tag{29}$$

$$\left[S + \frac{\partial}{\partial x} + \alpha 1 + \beta 1 + 3\alpha 3\right]\overline{P}1(x,s) = 0 \tag{30}$$

$$\left[S + \frac{\partial}{\partial x} + \phi(x)\right]\overline{P}_2(x,s) = 0 \tag{31}$$

$$\left[S + \frac{\partial}{\partial y} + \phi(y)\right]\overline{P}_3(y,s) = 0 \tag{32}$$

$$\left[S + \frac{\partial}{\partial y} + \beta_3 + 2\alpha_1 + 2\alpha_4 + 2\alpha_3\right]\overline{P}_4(z,s) = 0 \tag{33}$$

$$\left[S + \frac{\partial}{\partial y} + \beta_3 + \alpha_3\right]\overline{P}_5(z,s) = 0 \tag{34}$$

$$\left[S + \frac{\partial}{\partial z} + \phi(z)\right]\overline{P}_6(z,s) = 0 \tag{35}$$

$$\left[S + \frac{\partial}{\partial k} + \alpha_4 + 3\alpha_3 + \beta_4 + 2\alpha_1\right]\overline{P}_7(k,s) = 0 \tag{36}$$

$$\left[S + \frac{\partial}{\partial k} + \phi(k)\right]\overline{P}_8(k,s) = 0 \tag{37}$$

$$\left[S + \frac{\partial}{\partial z} + \beta_3 + 2\alpha_3 + \phi(x)\right]\overline{P}_9(z,s) = 0 \tag{38}$$

$$\left[S + \frac{\partial}{\partial z} + \beta_3 + \alpha_3\right]\overline{P}_{10}(z,s) = 0 \tag{39}$$

$$\left[S + \frac{\partial}{\partial z} + \alpha_3 + 2\alpha_3 + \beta_4\right]\overline{P}_{11}(z,s) = 0 \tag{40}$$

$$\left[S + \frac{\partial}{\partial z} + \alpha_3 + \beta_3\right]\overline{P}_{12}(z,s) = 0 \tag{41}$$

$$\left[S + \frac{\partial}{\partial x} + 2\beta_1\right]\overline{P}_{13}(x,s) = 0 \tag{42}$$

Boundary condition

$$\overline{P}_1(0,s) = 2\alpha_1 \overline{P}_0(s) \tag{43}$$

$$\overline{P}_2(0,s) = 2\alpha_1^2 \overline{P}_0(s) \tag{44}$$

$$\overline{P}_3(0,s) = \alpha_2 \overline{P}_0(s) \tag{45}$$

$$\overline{P}_4(0,s) = 3\alpha_3 \overline{P}_0(s) \tag{46}$$

$$\overline{P}_5(0,s) = 6\alpha_3^2 \overline{P}_0(s) \tag{47}$$

$$\overline{P}_6(0,s) = 6\alpha_3^2 \overline{P}_0(s) \tag{48}$$

$$\overline{P}_7(0,s) = 2\alpha_4 \overline{P}_0(s) \tag{49}$$

$$\overline{P}_8(0,s) = 2\alpha_4^2 \overline{P}_0(s) \tag{50}$$

$$\overline{P}_9(0,s) = 12\alpha_1\alpha_3 \overline{P}_0(s) \tag{51}$$

$$\overline{P}_{10}(0,s) = 24\alpha_1\alpha_3^2 \overline{P}_0(s) \tag{52}$$

$$\overline{P}_{11}(0,s) = 12\alpha_3\alpha_4 \overline{P}_0(s) \tag{53}$$

$$\overline{P}_{12}(0,s) = 24\alpha_3^2\alpha_4 \overline{P}_0(s) \tag{54}$$

$$\overline{P}_{13}(0,s) = 4\alpha_1\alpha_4 \overline{P}_0(s) \tag{55}$$

solving equations (30) to (42) with the help of boundary conditions (43) – (55) and the shifting property of Laplace transformation.

$$\overline{P}_1(S) = \overline{P}_1(0,s)\left\{\frac{1 - \overline{S}_\varphi(S + \alpha_1 + 3\alpha_3)}{S + \alpha_1 + 3\alpha_3}\right\} \tag{56}$$

$$\overline{P}_2(S) = \overline{P}_2(0,s)\left\{\frac{1 - \overline{S}_\phi(S)}{S}\right\} \tag{57}$$

$$\overline{P}_3(S) = \overline{P}_3(0,s)\left\{\frac{1 - \overline{S}_\phi(S)}{S}\right\} \tag{58}$$

$$\overline{P}_4(S) = \overline{P}_4(0,s)\left\{\frac{1 - \overline{S}_{\beta_3}(S + 2\alpha_1 + 2\alpha_3 + 2\alpha_4)}{S + 2\alpha_1 + 2\alpha_3 + 2\alpha_4}\right\} \tag{59}$$

$$\overline{P}_5(S) = \overline{P}_5(0,s)\left\{\frac{1 - \overline{S}_{\beta_3}(S + \alpha_3)}{S + \alpha_3}\right\} \tag{60}$$

$$\overline{P}_6(S) = \overline{P}_6(0,s)\left\{\frac{1 - \overline{S}_\phi(S)}{S}\right\} \tag{61}$$

$$\overline{P}_7(S) = \overline{P}_7(0,s)\left\{\frac{1 - \overline{S}_{\beta_4}(S + 3\alpha_3 + \alpha_4 + 2\alpha_1)}{S + 3\alpha_3 + \alpha_4 + 2\alpha_1}\right\} \tag{62}$$

$$\overline{P}_8(S) = \overline{P}_8(0,s)\left\{\frac{1 - \overline{S}_\phi(S)}{S}\right\} \tag{63}$$

$$\overline{P}_9(S) = \overline{P}_9(0,s)\left\{\frac{1 - \overline{S}_\phi(S + 2\alpha_3 + \beta_4)}{S + 2\alpha_3 + \beta_4}\right\} \tag{64}$$

$$\overline{P}_{10}(S) = \overline{P}_{10}(0,s)\left\{\frac{1 - \overline{S}_{\beta_3}(S + \alpha_3)}{S + \alpha_3}\right\} \tag{65}$$

$$\overline{P}_{11}(S) = \overline{P}_{11}(0,s)\left\{\frac{1 - \overline{S}_{\beta_4}(S + 2\alpha_3 + \beta_3)}{S + 2\alpha_3 + \beta_3}\right\} \tag{66}$$

$$\overline{P}_{12}(S) = \overline{P}_{12}(0,s)\left\{\frac{1 - \overline{S}_{\beta_3}(S + \alpha_3)}{S + \alpha_3}\right\} \tag{67}$$

$$\overline{P}_{13}(S) = \overline{P}_{13}(0,s)\left\{\frac{1 - \overline{S}_{2\beta_1}(S)}{S}\right\} \tag{68}$$

Using the second shifting property of the Laplace transform equation (28) will reduce to:

Shifting property of Laplace transformation.

$$\int_0^\infty \left[e^{-sx} \cdot e^{-\int_0^x f(x)dx} \right] dx = L\left\{ \frac{1-\overline{S}_f(x)}{S} \right\} = \frac{1-\overline{S}_f(x)}{S}$$

$$\int_0^\infty \left[e^{-sx} \cdot f(x) e^{-\int_0^x f(x)dx} \right] dx = L\left\{ \overline{S}_f(x) \right\} = \overline{S}_f(x)$$

$$\begin{aligned} &[S+2\alpha_1+\alpha_2+3\alpha_3+2\alpha_4]\overline{P}_0(S)=1 \\ &+\overline{P}_1(0,s)\overline{S}_\varphi(S+\alpha_1+3\alpha_3) \\ &+\overline{P}_4(0,s)\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4) \\ &+\overline{P}_7(0,s)\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1) \\ &+(\overline{P}_3(0,s)+\overline{P}_2(0,s)+\overline{P}_7(0,s)+\overline{P}_8(0,s) \\ &+\overline{P}_6(0,s))\overline{S}_\varphi(S) \end{aligned} \tag{69}$$

Substituting the Laplace transformation boundary condition in (43) to (55) into (56) to (69) we obtain the solution of the partial differential equations from (1) to (14)

$$\overline{P}_1(S) = 2\alpha_1 \left\{ \frac{1-\overline{S}_\phi(S+\alpha_1+3\alpha_3)}{S+\alpha_1+3\alpha_3} \right\} \overline{P}_0(s) \tag{70}$$

$$\overline{P}_2(S) = 2\alpha_1^2 \left\{ \frac{1-\overline{S}_\phi(S)}{S} \right\} \overline{P}_0(s) \tag{71}$$

$$\overline{P}_3(S) = \alpha_2 \left\{ \frac{1-\overline{S}_\phi(S)}{S} \right\} \overline{P}_0(s) \tag{72}$$

$$\overline{P}_4(S) = 3\alpha_3 \left\{ \frac{1-\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4)}{S+2\alpha_1+2\alpha_3+2\alpha_4} \right\} \overline{P}_0(s) \tag{73}$$

$$\overline{P}_5(S) = 6\alpha_3^2 \left\{ \frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3} \right\} \overline{P}_0(s) \tag{74}$$

$$\overline{P}_6(S) = 6\alpha_3^2 \left\{ \frac{1-\overline{S}_\varphi(S)}{S} \right\} \overline{P}_0(s) \tag{75}$$

$$\overline{P}_7(S) = 2\alpha_4 \left\{ \frac{1-\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1)}{S+3\alpha_3+\alpha_4+2\alpha_1} \right\} \overline{P}_0(s) \tag{76}$$

$$\overline{P}_8(S) = 2\alpha_4^2 \left\{ \frac{1-\overline{S}_\phi(S)}{S} \right\} \overline{P}_0(s) \tag{77}$$

$$\overline{P}_9(S) = 12\alpha_1\alpha_3 \left\{ \frac{1-\overline{S}_\phi(S+2\alpha_3+\beta_4)}{S+2\alpha_3+\beta_4} \right\} \overline{P}_0(s) \tag{78}$$

$$\overline{P}_{10}(S) = 24\alpha_1\alpha_3^2 \left\{ \frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3} \right\} \overline{P}_0(s) \tag{79}$$

$$\overline{P}_{11}(S) = 12\alpha_3\alpha_4 \left\{ \frac{1-\overline{S}_{\beta_4}(S+2\alpha_3+\beta_3)}{S+2\alpha_3+\beta_3} \right\} \overline{P}_0(s) \tag{80}$$

$$\overline{P}_{12}(S) = 24\alpha_3^2\alpha_4 \left\{ \frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3} \right\} \overline{P}_0(s) \tag{81}$$

$$\overline{P}_{13}(S) = 4\alpha_1\alpha_4 \left\{ \frac{1-\overline{S}_{2\beta_1}(S)}{S} \right\} \overline{P}_0(s) \tag{82}$$

$$\left(S+2\alpha_1+\alpha_2+3\alpha_3+2\alpha_4 - \left(\begin{array}{l} 2\alpha_1\overline{S}_\varphi(S+\alpha_1+3\alpha_3)+3\alpha_3\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4) \\ +2\alpha_4\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1)+(\alpha_2+2\alpha_1^2+2\alpha_4+2\alpha_4^2 \\ +6\alpha_3^2)\overline{S}_\varphi(S) \end{array} \right) \right) \overline{P}_0(s)=1 \tag{83}$$

But;

$$D(S) \times P(S) = 1 \tag{84}$$

It is clear that;

$$D(S) = \left(S+2\alpha_1+\alpha_2+3\alpha_3+2\alpha_4 - \left(\begin{array}{l} 2\alpha_1\overline{S}_\varphi(S+\alpha_1+3\alpha_3)+3\alpha_3\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4) \\ +2\alpha_4\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1)+(\alpha_2+2\alpha_1^2+2\alpha_4+2\alpha_4^2 \\ +6\alpha_3^2)\overline{S}_\varphi(S) \end{array} \right) \right) \tag{85}$$

$\overline{P}_{up}(S)$ Which is the sum of all operational states of the system is therefore

$$\overline{P}_{up}(S) = \overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_4(s)+\overline{P}_5(s)+\overline{P}_7(s)+\overline{P}_9(s)+\overline{P}_{10}(s)+\overline{P}_{11}(s)+\overline{P}_{12}(s) \tag{86}$$

$$\overline{P}_{up}(S)=\frac{1}{D(S)}\begin{bmatrix} 1+2\alpha_1\left\{\frac{1-\overline{S}_{\varphi}(S+\alpha_1+3\alpha_3)}{S+\alpha_1+3\alpha_3}\right\}+3\alpha_3\left\{\frac{1-\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4)}{S+2\alpha_1+2\alpha_3+2\alpha_4}\right\}+ \\ 6\alpha_3^2\left\{\frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\}+2\alpha_4\left\{\frac{1-\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1)}{S+3\alpha_3+\alpha_4+2\alpha_1}\right\}+ \\ 12\alpha_1\alpha_3\left\{\frac{1-\overline{S}_{\varphi}(S+2\alpha_3+\beta_4)}{S+2\alpha_3+\beta_4}\right\}+24\alpha_1\alpha_3^2\left\{\frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\}+ \\ 12\alpha_3\alpha_4\left\{\frac{1-\overline{S}_{\beta_4}(S+2\alpha_3+\beta_3)}{S+2\alpha_3+\beta_3}\right\}+24\alpha_3^2\alpha_4\left\{\frac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\} \end{bmatrix} \quad (87)$$

4. Formulation and Analysis of System Availability

Taking $S_{\alpha_0}(s)=\overline{S}_{\exp[x^{\theta}+\{\log\varphi(x)\}^{\theta}]^{1/\theta}}(s)=\frac{\exp[x^{\theta}+\{\log\varphi(x)\}^{\theta}]^{1/\theta}}{s+\exp[x^{\theta}+\{\log\varphi(x)\}^{\theta}]^{1/\theta}}$, $\bar{P}_{\phi}(s)=\frac{\phi}{s+\phi}\, but\phi=1$ and

$\alpha_1=0.0001, \alpha_2=0.0002, \alpha_3=0.0003, \alpha_4=0.0004\alpha_5=0.0005$

And all the repair rates are set to be equal to 1.

$$\phi(x)=\phi(y)=\phi(z)=\phi(k)=1 \quad (88)$$

And applying the inverse Laplace transform to (62), the expression for system availability is

$$\overline{P}_{up}(S)=\begin{bmatrix} 0.79136714\,13e^{=2.780051818t}+0.332643017e^{=1.062941614t}-0.008960516948e^{-1.038755477t}- \\ 37.84783592e^{-0.02655109014t}+0.0004194559848e^{-2.030000000t} \end{bmatrix} \quad (89)$$

Taking t = 0, 10,…,100, the availability of the system is obtained and presented in Table 2 and figure 3 below.

Table 2. System’s Availability against time

Time	Availability
0	0.9999
10	0.7706
20	0.5909
30	0.4531
40	0.3474
50	0.2664
60	0.2043
70	0.1567
80	0.1201
90	0.0921
100	0.0706

$$\overline{P}_{up}(S)=\left[\begin{array}{l} 1+2\alpha_1\left\{\dfrac{1-\overline{S}_{\varphi}(S+\alpha_1+3\alpha_3)}{S+\alpha_1+3\alpha_3}\right\}+3\alpha_3\left\{\dfrac{1-\overline{S}_{\beta_3}(S+2\alpha_1+2\alpha_3+2\alpha_4)}{S+2\alpha_1+2\alpha_3+2\alpha_4}\right\}+ \\ 6\alpha_3^2\left\{\dfrac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\}+2\alpha_4\left\{\dfrac{1-\overline{S}_{\beta_4}(S+3\alpha_3+\alpha_4+2\alpha_1)}{S+3\alpha_3+\alpha_4+2\alpha_1}\right\}+ \\ 12\alpha_1\alpha_3\left\{\dfrac{1-\overline{S}_{\varphi}(S+2\alpha_3+\beta_4)}{S+2\alpha_3+\beta_4}\right\}+24\alpha_1\alpha_3^2\left\{\dfrac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\}+ \\ 12\alpha_3\alpha_4\left\{\dfrac{1-\overline{S}_{\beta_4}(S+2\alpha_3+\beta_3)}{S+2\alpha_3+\beta_3}\right\}+24\alpha_3^2\alpha_4\left\{\dfrac{1-\overline{S}_{\beta_3}(S+\alpha_3)}{S+\alpha_3}\right\} \end{array}\right]\overline{P}_0(s) \tag{90}$$

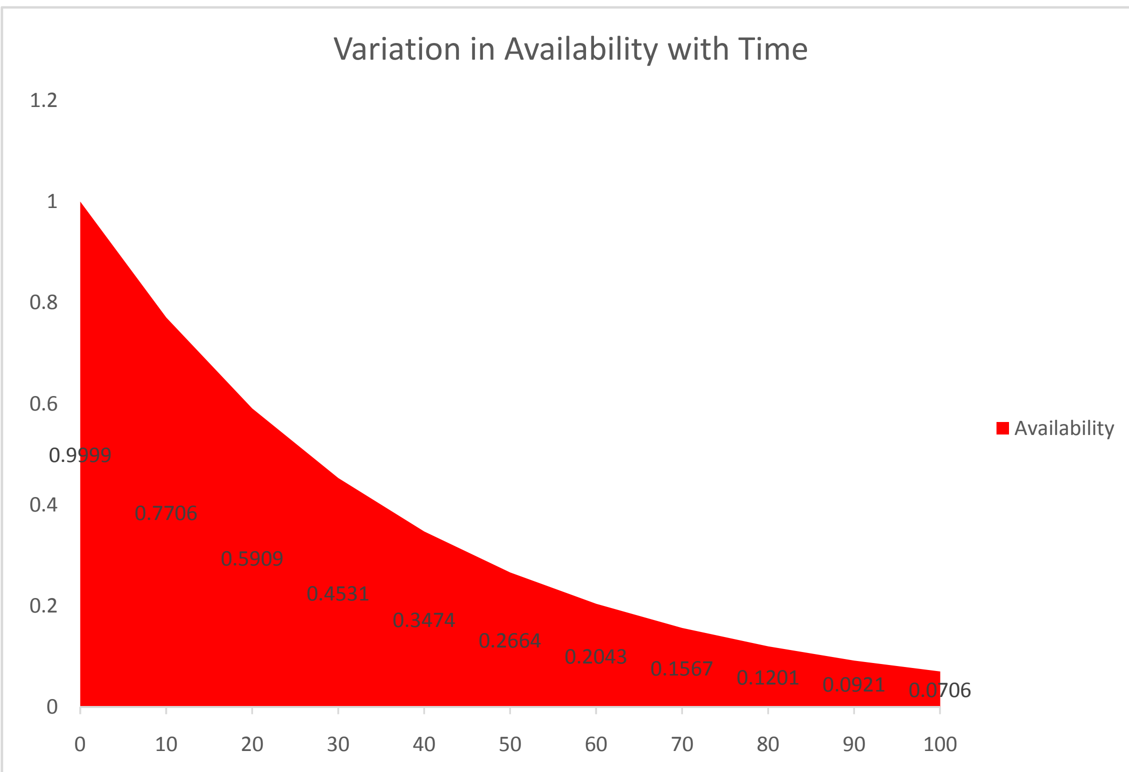

Figure 3. Variation of Availability with time

5. Formulation and Analysis of Reliability

Letting all repair rates, $\phi(x) = \phi(y) = \phi(z) = \phi(k) = 0$ in equation (88), Taking the failure rate values and applying the inverse Laplace transformation, the expression is reliability relation.

$$R(t)=\left[0.3333333333\ e^{-0.04000000000t}+0.318000000\ e^{-0.3000000000t}\right. \\ \left.+0.348666666e^{-0.1300000000t}\right] \tag{91}$$

Taking t = 0, 10…100, units of time in equation (88), reliability is computed and presented in Table 3 and figure 3 below:

Table 3. Variation of reliability with time

Time	Reliability
0	1.0000
10	0.8034
20	0.5899
30	0.4210
40	0.2990
50	0.2133
60	0.1533
70	0.1110
80	0.0808
90	0.0592
100	0.0434

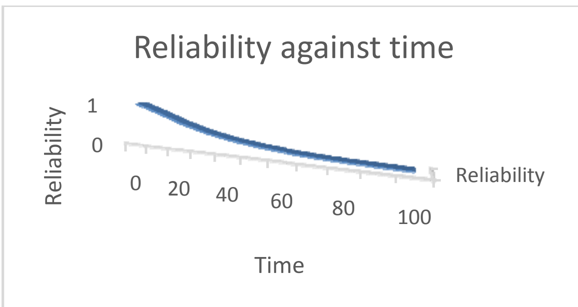

Figure 4. Variation of reliability with time

5.1 Cost Analysis

If the service facility is always available, then the expected profit during the interval [0, t) of the system can be obtained by the formula in [17].

$$E_p(t) = K_1 \int_0^1 P_{up}(t)dt - K_2 t \quad (92)$$

For the same set of parameters of (73) and (77). Therefore the subsequence equation follows

$$E_P = K_1[-5.73547937285 \times 10^{-7} e^{-2.718305013t} + 0.023814303064867 e^{-1.005991030t} -2.527015670 \times 10^{5} e^{-0.000003957272520t} +0.67014010019983768 e^{-2.001000000t} - 0.0017353437633 67 e^{-1.003100000t}] - K_2 t \quad (93)$$

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2, and\ 0.1$ respectively and varying $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$. Units of time, the results for profit can be obtained as shown in Table 4 below:

Table 4. Expected profit as a function of time

Time	$E_p(t)$				
0	0	0	0	0	0
10	4.7332	5.7332	6.7332	7.7331	8.7331
20	9.2806	11.2806	13.2807	15.2806	17.2806
30	13.6503	16.6503	19.6503	22.6503	25.6503
40	17.8453	21.8453	25.8453	29.8453	33.8453
50	21.8691	26.8691	31.8691	36.8691	41.8691
60	25.7247	31.7247	37.7247	43.7247	49.7247
70	29.4153	36.4154	43.4154	50.4154	57.4154
80	32.9441	40.9442	48.9442	56.9442	64.9442
90	36.3140	45.3140	54.3140	63.3140	72.3140
100	39.5279	49.5279	59.5280	69.5280	79.5280

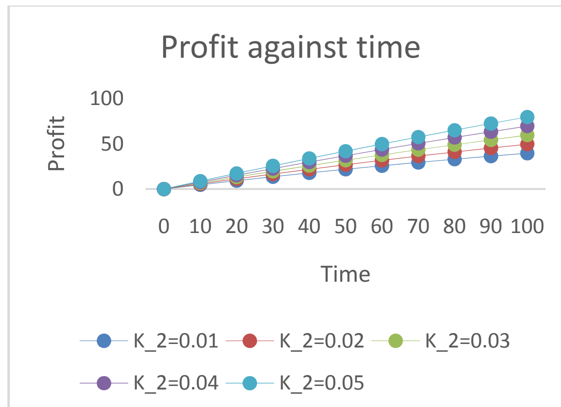

Figure 5. Box plot of Expected profit against

$$K_2 \in \{0.01, 0.02, 0.03, 0.04, 0.05\}$$

5.2 Mean time to failure (MTTF) Analysis

Taking all repairs to zero in equation (88) and the taking limit, as s, tends to zero, one can obtain the expression for MTTF as:

$$\text{MTTF} = \lim_{s \to 0} \bar{P}_{up}(S) \quad (94)$$

Setting , and varying the failure rates, one by one respectively as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and , varying one by one as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, in (94), one may obtain the variation of MTTF with respect to failure rates as shown in table 5 corresponding to figure 5.

Table 5. Variation of MTTF with failure rates α_k

Failure Rate	MTTF				
	Subsystem 1	Subsystem 2	Subsystem 3	Subsystem 4	Subsystem 5
0.001	71.6667	79.5455	83.3333	95.5556	99.3345
0.002	58.0000	71.6667	76.5152	86.0000	96.4597
0.003	50.7407	65.3846	71.6667	78.1818	87.9981
0.004	45.9524	60.2041	67.9487	71.6667	76.3219
0.005	42.5000	55.8333	64.9660	66.1538	71.6667
0.006	39.8765	52.0833	62.5000	61.4286	63.7781
0.007	37.8095	48.8235	60.4167	57.3333	59.0010
0.008	36.1364	45.9596	58.6275	53.7500	54.5569
0.009	34.7531	43.4211	57.0707	50.5882	52.1109

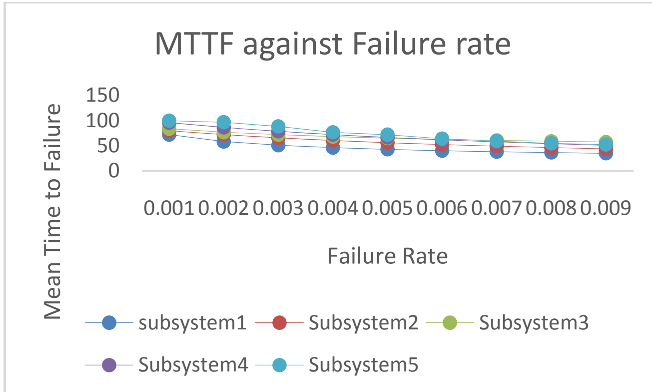

Figure 6. Variation of MTTF with failure rates

5.3 Sensitivity analysis corresponding to (MTTF)

The sensitivity of the system's MTTF can be studied by partial differentiation of MTTF with respect to the system's failure rates. Using the set of parameters as $\alpha_1 = 0.0001$, $\alpha_2 = 0.0002$, $\alpha_3 = 0.0003$, $\alpha_4 = 0.0004$, In partial differentiation of MTTF, the MTTF sensitivity may be calculated as indicated in table 6 and associated graphs in figure 7 below:

Table 6. Sensitivity as a function of time

Failure rate	$\frac{\partial(MTTF)}{\alpha_1}$	$\frac{\partial(MTTF)}{\alpha_2}$	$\frac{\partial(MTTF)}{\alpha_3}$	$\frac{\partial(MTTF)}{\alpha_4}$	$\frac{\partial(MTTF)}{\alpha_5}$
0.1	-837.499	-402.686	-239.333	-328.395	-218.345
0.2	-102.666	-284.722	-72.1074	-266.000	-122.012
0.3	-1.85185	-215.582	-6.94444	-219.834	-44.8341
0.4	19.89795	-170.699	21.1045	-184.722	0.235892
0.5	23.95833	-139.472	33.3819	-157.396	35.29612
0.6	23.18244	-116.637	38.3055	-135.714	52.98752
0.7	21.10204	-99.3079	39.6122	-118.222	71.99720
0.8	18.81887	-85.7718	39.1003	-103.906	83.85720
0.9	16.68381	-74.9538	37.6849	-92.0415	99.10386

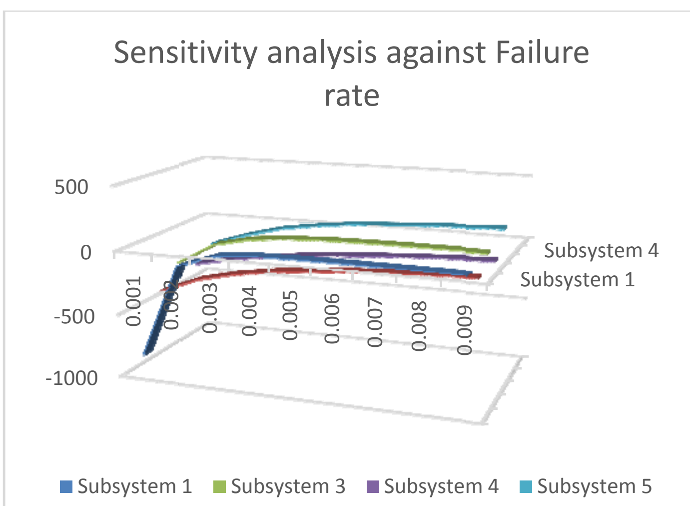

Figure 7. Sensitivity with respect to Failure rate

6. Discussion and conclusion

The simulation in Figure 3 shows that as time passes, availability decreases. When the time is less than 60 days, the chart clearly shows that the system's availability is higher. Figure 4 depicts the system's reliability over time in the same way. The graph shows that reliability decreases $t \in [0, 50]$ as time t goes from 0 to 100. On the other hand, the time interval has a higher level of trustworthiness. Table 2 and 3 and Figures 3 and 4 respectively show how more units are on standby, perfect repair in the event of an incomplete failure, replacing the affected subsystem with a new one in the event of a complete failure, regular inspection, and preventive maintenance, employing more repair machines. Other measures can improve the system's availability and reliability.

Table 5 and corresponding Figure 6 depict a simulation of mean time to failure vs. failure rate π_k. The graph shows that as π_k grows, the MTTF decreases. The MTTF decreases as π_k increases, resulting in a decrease in the system's longevity. To improve the system's MTTF and longevity, fault-tolerant components should be used. $K_2 \in \{0.01, 0.02, 0.03, 0.04, 0.05\}$ Figure 6 depicts the relationship between profit and time t. For any value of K2, the predicted profit decreases with increasing time, as shown in the graph. However, as the value decreases, the predicted profit rises. The expected profit can be increased by implementing the replacement mentioned above and redundancy suggestions. Table 5 and the corresponding figure 7 show the sensitivity analysis results in terms of failure rate.

6.1 Conclusion

Due to a lack of data on PV systems, the current study developed a reliability modeling technique to assess the PV system's overall strength, efficiency, and performance. The reliability, availability, MTTF, and profit function of this paper can all be evaluated. We present a novel solar system model with four subsystems: panel, inverter, battery bank, and control charger in this paper.

According to the paper's findings, reliability modeling can be used to assess a PV system's strength, efficiency, and performance. Once the PV system's strength, efficiency, and performance are determined, users can serve the cost of kerosene, gasoline, diesel, and other fuels that expose human hearths to air and land pollution for their household and commercial uses. As a result, the model's graphical representation demonstrates that for any given set of parametric parameters, the future behavior of a complex system can be confidently predicted at any time.

Reducing carbon dioxide emissions from conventional power generation can be achieved by adopting solar energy. Additionally, enterprises struggle with machine failure, slowing down technological growth globally due to power fluctuations.

7. Statements and Declarations

The authors did not receive support from any organization for the submitted work.

7.1 Competing interests

The authors have no competing interests to declare that are relevant to the content of this article.

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