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ORGINAL RESEARCH ARTICLE

Performance Analysis of Complex Series Parallel Computer Network with Transparent Bridge Using Copula Distribution

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Abstract

A network bridge is a computer networking device that creates a single aggregate network from multiple communication network or network segments. One of the type of network bridge is transparent bridge saddle with responsibility of checking incoming network traffic to identify media access control addresses. In this present research work on series parallel computer network performance, availability and cost analysis of complex computer network was considered to focus on a network that has four subsystems A, B, C and D and all the subsystems are arranged in series-parallel, subsystem A and B are working on 1-out-of-2: G and 2-out-of-3: F policy respectively, C subsystem behaved as a bridge with one unit and D subsystem has five units and are working in 3-out-of-5: G scheme. The system has two types of failure, degraded (partial failure) or complete failed states. The system was analyzed using supplementary variables techniques and Laplace transform, general distribution and copula family were employed to restore the partial failure and complete failure states. Computed results have been highlighted by the means of tables and graphs to investigate the performance of computer network. The result has shown that computer network with transparent bridge will be more reliable to filter incoming frame and forward it to media access control (MAC).

Keywords: Availability, Reliability, Sensitivity, Mean time to failure (MTTF), Gumbel-Hougaard family, Cost Analysis.

AMS Subject Classification: 90B25

1. Introduction

Many experts and researchers are constantly attempting to investigate the best approach of performance and effectiveness of series-parallel computer network due to its numerous applications in various areas such as industrials, health institutions, management, and financial institutions are always setting aside a special consideration to manage risk and explore a better performance computer network. The benefits of series-parallel complex systems in various disciplines, as well as their requirements in determining availability and reliability, are now becoming an important issue in system performance. System availability is the percentage of time that the system is available to operate. Failure or degradation of the system results in cost, danger, decreased production and profit, and even the loss of lives. To improve the system reliability and availability of a series-parallel computer network, redundant components must be implemented, with some units working while others remaining reserves for immediate action; this operational system style is knownas the k-out-of-n: G/F scheme. In this approach, k units must work from the system's domain n in order for it to function; failure of more than k units results in the system's complete failure. Among the vast amount of literature on reliability theory model that exists. Chauhan and Malik [1] investigated the reliability of series-parallel systems for arbitrary parameter values. Fadi and Sibai [2] investigated series-parallel photovoltaic module modeling and output power estimation. Hu *et al.* [3] analyzed availability analysis and design optimization in a repairable series-parallel system with failure

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dependencies. Khatab *et al.* [4] studied selective maintenance optimization for series-parallel systems with alternating missions and scheduled breaks of stochastic duration. Mohammad *et al.* [5] investigated the impacts of a method for distributing reliability in series-parallel systems while accounting for common cause failure. Mustafa [6] concentrated on improving the reliability of a series-parallel system using a modified Weibull distribution. Peng *et al.* [7] published their research work on reliability analysis and optimal structure of series-parallel phased mission systems subject to fault level coverage. Xie *et al.* [8] focused on the reliability and barrier assessment of series-parallel systems subject to system failure. Muhammad *et al.* [9] focused mainly on the cost benefit analysis of three different series parallel dynamo configurations, whereas Zhang [10] examined computer network reliability analysis using an intelligent cloud computing method. Potapov *et al.* [11] investigated reliability in the context of an information system with a client-server architecture. Kovalev *et al.* [12] investigated the reliability of distributed computer systems with client-server architecture. System performance is determined by system configuration and repair dynamics. Different authors have adopted various types of failure and repair; many of them have considered general repair, while many have adopted copula [13] which is accepted as an efficient approach for better performance results because it operates with more than one repair system compared to general repair. Abubakar and Singh [14] focused on assessment and performance of industrial systems using the Gumbel Hougaard copula approach. Kabiru *et al.* [15] have focused on reliability assessment of complex system with two subsystems using joint distribution. Ibrahim *et al.* [16] have analyzed the performance analysis of multi-computer system with three subsystems in series. Singh and Monika [17] examined on reliability analysis of n client's system under star topology. Pratap et al [18] have examined on the assessment of complex system with two subsystems and multi types failure and repair. Muhammad *et al.* [19] published their work on sensitivity analysis of three different series parallel dynamo configurations. Kabiru *et al.* [20] published their research work on availability and cost analysis of complex tree topology of computer network with multi-server using gumbel hougaard family copula approach, Druhv *et al.* [21] worked on reliability prediction of distributed system with homogeneity in software and serverusing joint probability distribution viacopula approach. From the previous research of computer network, little or no attention is paid on the reliability and performance analysis of computer network with transparent bridge, in this research work performance analysis of complex series parallel computer network with transparent bridge using copula distribution is studied.

Gumbel Hougaard family distribution is one of the types of copula family distribution that takes into account more than one repair. In this paper, system of first order linear partial differential difference equations were to obtained and solved using copula approach to analyze a complex series-parallel computer network system with a transparence bridge, which had four subsystems named A,B,C, and D. Subsystem A operates under the 1-out-of-2:G scheme, Subsystem B operates under the 2-out-of-3:F scheme, Subsystem C has a single unit that serves as a bridge, and failure of the bridge results in system failure, and Subsystem D operates under the 3-out-of-5:M scheme. The objective of this research work are three. First is to obtain the expressions of reliability, availability, MTTF, cost functions. Secondly, is perform sensitivity analysis using MTTF. Thirdly, is capture the impact of time and other system parameters on reliability, availability, MTTF and cost function. The system works in both series and parallel modes, and the Gumbel Hougaard family copula distribution is used for computation and illustration. Finally, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10} represents the states of operation in degraded/partial failure while S_{11}, S_{12}, S_{13}, and S_{14} are completely failed states and S_0 is at perfect operational state. Degraded points have been repaired using general repair, and completely failed states have been repaired using the Gumbel Hougaard family copula. Supplementary variables and the laplace transformation were used to analyze the system, and tables and graphs are used to represent reliability measurements such as availability, reliability, and MTTF, as well as cost analysis.

2. STATE DESCRIPTION, ASSUMPTION AND NOTATIONS

State	State Description
S_0	The state S_0 represents a perfect state in which all the subsystems are in good working condition.
S_1	S_1, represents a degraded state/minor partial failure in the subsystem D, as the result of the failure of first unit of the subsystem.
S_2	State S_2 amount to another degraded state/minor partial failure in the subsystem D, due to the failure of the second units of the subsystem D.

State	State Description
S_3	S_3 represents a degraded state/minor partial failure in the subsystem B, when the failure of the first unit of the subsystem B occurred.
S_4	S_4, shows the degraded state or minor partial failure in the subsystem A, due to the failure of first unit of the subsystem.
S_5	S_5, represents a degraded state/ minor failure, due to the failure of one unit each in subsystems A and D.
S_6	S_6, represents a degraded state/ minor failure, as the result of failure of the one unit each in subsystems A and B.
S_7	This state accounts for a degraded state / minor partial failure, due to the failure of one unit each in subsystems B and D.
S_8	S_8, represents a degraded state with minor failure, due to the failure of the one unit in subsystem D and bridge (subsystem C).
S_9	This state accounts for a degraded state/ minor partial failure, due to the failure of one unit in each of the subsystems A and B.
S_{10}	S_{10} ,reveals a degraded state/ minor partial failure, due to the failure of one unit of the subsystem D.
S_{11}	S_{11}, shows the complete failure of the system, due to failure of more than two units in subsystem D and the system is under repair using copula
S_{12}	S_{12}, indicates total failure of the system, as the result of the failure of more than one unit in subsystem B. And the system is under repair using copula
S_{13}	The state S_{13} revealed another complete failed state of the system, when two units failed in subsystem A and goes to repair using copula.
S_{14}	The state S_{14} represents a complete failed state of the entire system as the result of failed stage of bridge (subsystem C) and considered to be under repair using copula distribution.

The state description highlights that, S_0 is a state where the system is in a perfect state where all the subsystems are in good working condition. S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8 S_9 and S_{10} are the states where the system is in degraded/ minor partial failure but operational mode and repair is being invited named as general distribution. The states S_{11}, S_{12}, S_{13}, and S_{14} are allin complete failed mode state, but repair is deployed to the complete failed state as Gumbel-Hougaard family copula distribution (Fig. 1).

Assumptions

The following assumptions are taken throughout the discussion of the model:

(i) Initially, S_0 is the state where all units in the systems are in its perfect good state.

(ii) The subsystems A and B are attached in working condition, with two and three units respectively, and failure of one unit in subsystem A tends the system to a partial failure (degraded) state, and at least two units most work in subsystem B otherwise degraded state, and both follows general distribution for repair, hence the complete failed state system is restore using copula.

(iii) The subsystem C served as a bridge between subsystems D and attached A and B, failure of the subsystem C amount the entire system to complete failed state and restore by copula

(iv) At least three units most work in the subsystem D, if more than three failed, the system leads to complete failure state, all complete failure stage are repair using copula distribution.

(v) It is assumed that a repaired system works like a new and no damage appears during repair.

(vi) As soon as the failed unit gets repaired, it performs its task normally.

(vii) All failure rates are constants and follow a negative exponential distribution.

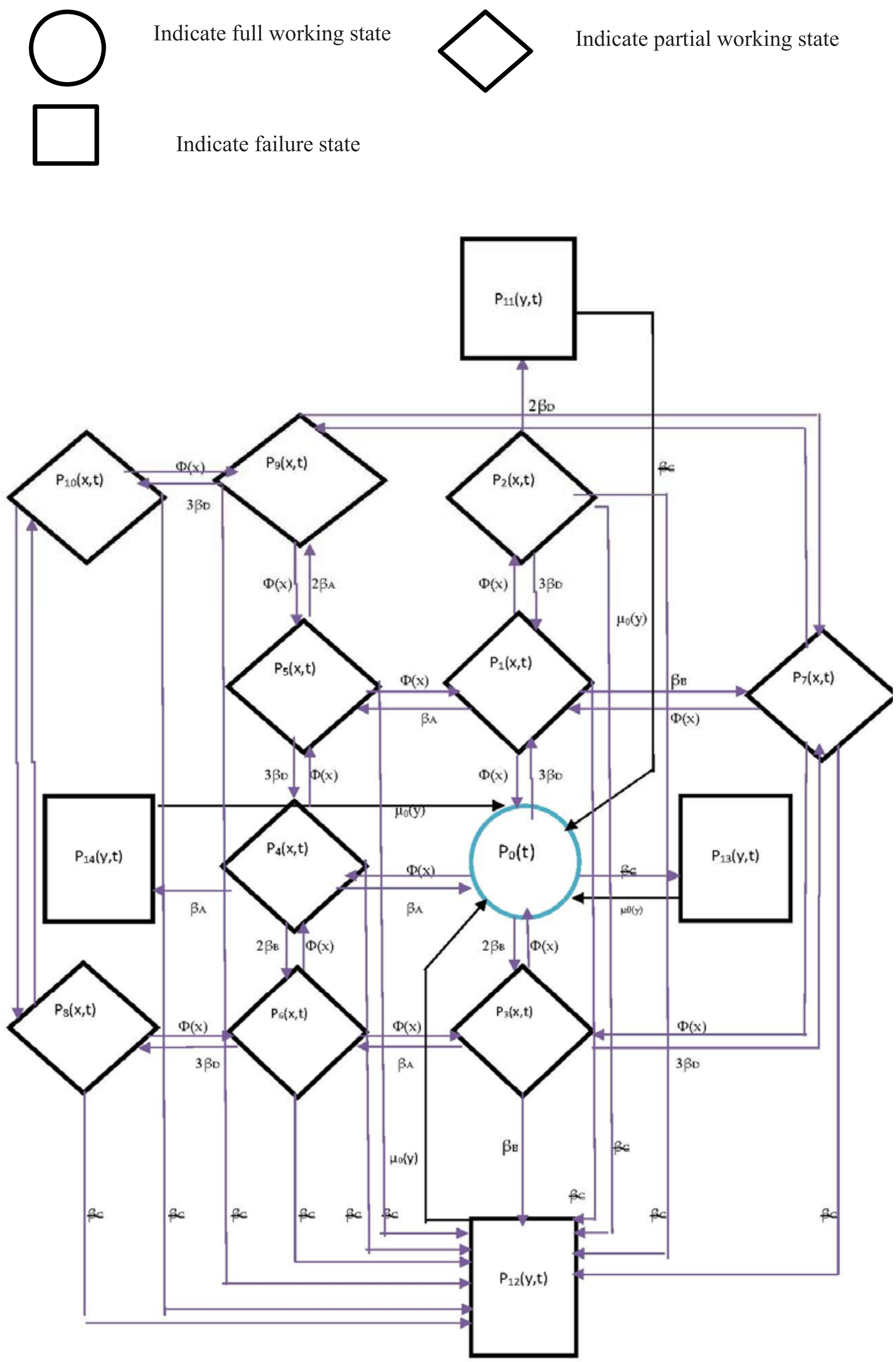


Figure 1. Transition Diagram of the System

Notations

t:	Time variable on time scale.
s:	A variable for Laplace transform for all expressions.
β_A / β_B/β_C/β_D:	Failure rates of units of subsystems A, B, C and D
$\boldsymbol{\varphi(x)}$	Repair rates for all unit of subsystems i.e. A, B, C and D
$\mu_0(x)$, $\mu_0(y)$:	Repair rates for complete failed states.
$\boldsymbol{P_i(x,t)}$:	The probability that the system is in S_i state at instant's' for i =0 to 12.
$\boldsymbol{\bar{P}_i(s)}$:	Laplace transformation of state transition probability P (t).
$\boldsymbol{E_p(t)}$	Expected profit during the time interval [0, t).
K_1, K_2:	Revenue and service cost per unit time in the interval [0, t) respectively.
$\boldsymbol{S_\varphi(x)}$	Standard repair distribution function $\boldsymbol{S_\varphi(x) = \varphi(x) e^{-\int_0^\infty \varphi(x)}}$
$\boldsymbol{L[S_\varphi(x)]}$:	$\boldsymbol{\bar{s}_\varphi(x) = \int_0^\infty e^{-sx}\, \varphi(x) e^{-\int_0^\infty \varphi(x)} dx}$ is the Laplace transform of $\boldsymbol{S_\varphi(x)}$
$\mu_0(x) = C_\theta(u_1(x), u_2(x))$	The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula is given as $C_\theta(u_1(x),u_2(x)) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where θas a parameter, $1 \le \theta \le \infty$.

3.1 Formulation of Mathematical Model

The following steps of the differential difference equation are derived for the probability of considerations.

$$\left(\frac{\delta}{\delta t} + \beta_A + 2\beta_B + \beta_C + 3\beta_D\right) P_0(t) = \int_0^\infty \phi(x) p_1(x,t)dx + \int_0^\infty \phi(x) p_3(x,t)dx + \int_0^\infty \phi(x) p_4(x,t)dx + \int_0^\infty \mu_0(y) p_{11}(y,t)dy + \int_0^\infty \mu_0(y) p_{12}(y,t)dy + \int_0^\infty \mu_0(y) p_{13}(y,t)dy + \int_0^\infty \mu_0(y) p_{14}(y,t)dy \tag{1}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_A + 2\beta_B + \beta_C + 3\beta_D + \phi(x)\right) P_1(x,t) = 0 \tag{2}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_C + 2\beta_D + \phi(x)\right) P_2(x,t) = 0 \tag{3}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_A + \beta_B + \beta_C + 3\beta_D + \phi(x)\right) P_3(x,t) = 0 \tag{4}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_A + 2\beta_B + \beta_C + 3\beta_D + \phi(x)\right) P_4(x,t) = 0 \tag{5}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + 2\beta_B + \beta_C + 2\phi(x)\right) P_5(x,t) = 0 \tag{6}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_C + 3\beta_D + 2\phi(x)\right) P_6(x,t) = 0 \tag{7}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_A + \beta_C + 2\phi(x)\right) P_7(x,t) = 0 \tag{8}$$

$$\left(\frac{\delta}{\delta t} + \frac{\delta}{\delta x} + \beta_C + 3\beta_D + \phi(x)\right) P_8(x,t) = 0 \tag{9}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\beta_C+3\beta_D+2\phi(x)\right)P_9(x,t)=0 \quad (10)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\beta_C+2\phi(x)\right)P_{10}(x,t)=0 \quad (11)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_{11}(y,t)=0 \quad (12)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_{12}(y,t)=0 \quad (13)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_{13}(y,t)=0 \quad (14)$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_{14}(y,t)=0 \quad (15)$$

Boundary Conditions

$$P_1(0,t)=3\beta_D P_0(t) \quad (16)$$

$$P_2(0,t)=9\beta_D^2 P_0(t) \quad (17)$$

$$P_3(0,t)=2\beta_B P_0(t) \quad (18)$$

$$P_4(0,t)=\beta_A P_0(t) \quad (19)$$

$$P_5(0,t)=6\beta_A\beta_D P_0(t) \quad (20)$$

$$P_6(0,t)=4\beta_A\beta_B P_0(t) \quad (21)$$

$$P_7(0,t)=12\beta_B\beta_D P_0(t) \quad (22)$$

$$P_8(0,t)=12\beta_A\beta_B\beta_D P_0(t) \quad (23)$$

$$P_9(0,t)=24\beta_A\beta_B\beta_D P_0(t) \quad (24)$$

$$P_{10}(0,t)=\left(72\beta_A\beta_B\beta_D^2+36\beta_A\beta_B\beta_D\right)P_0(\mathrm{t}) \quad (25)$$

$$P_{11}(0,t)=18\beta_D^2 P_0(\mathrm{t}) \quad (26)$$

$$P_{12}(0,t)=2\beta_B^2 P_0(\mathrm{t}) \quad (27)$$

$$P_{13}(0,t)=\beta_A^2 P_0(\mathrm{t}) \quad (28)$$

$$P_{14}(0,t)=\begin{pmatrix}\beta_C+\beta_C\beta_D+9\beta_C\beta_D^2+2\beta_B\beta_C+\beta_A\beta_C+6\beta_A\beta_C\beta_D+\\ 4\beta_A\beta_B\beta_C+12\beta_B\beta_C\beta_D+12\beta_A\beta_B\beta_C\beta_D+24\beta_A\beta_B\beta_C\beta_D\\ +\beta_C(72\beta_A\beta_B\beta_D^2+36\beta_A\beta_B\beta_D)\end{pmatrix}P_0(\mathrm{t}) \quad (29)$$

3.2 Solution of the Model

By taking the Laplace transformation of equations (1) to (29) we obtain the following results

$$\left(s+\beta_A+2\beta_B+\beta_C+3\beta_D\right)\overline{P_0}(s)=1+\int_0^\infty\phi(x)\overline{p}_1(x,s)dx+\int_0^\infty\phi(x)\overline{p}_3(x,s)dx+\int_0^\infty\phi(x)\overline{p}_4(x,s)dx+$$
$$\int_0^\infty\mu_0(y)\overline{p}_{11}(y,s)dy+\int_0^\infty\mu_0(y)\overline{p}_{12}(y,s)dy+\int_0^\infty\mu_0(y)\overline{p}_{13}(y,s)dy+\int_0^\infty\mu_0(y)\overline{p}_{14}(y,s)dy \quad (30)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_A+2\beta_B+\beta_C+3\beta_D+\phi(x)\right)\bar{P}_1(x,s)=0 \quad (31)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_C+2\beta_D+\phi(x)\right)\bar{P}_2(x,s)=0 \quad (32)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_A+\beta_B+\beta_C+3\beta_D+\phi(x)\right)\bar{P}_3(x,s)=0 \quad (33)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_A+2\beta_B+\beta_C+3\beta_D+\phi(x)\right)\bar{P}_4(x,s)=0 \quad (34)$$

$$\left(s+\frac{\delta}{\delta x}+2\beta_B+\beta_C+2\phi(x)\right)\bar{P}_5(x,s)=0 \quad (35)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_C+3\beta_D+2\phi(x)\right)\bar{P}_6(x,s)=0 \quad (36)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_A+\beta_C+2\phi(x)\right)\bar{P}_7(x,s)=0 \quad (37)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_C+3\beta_D+\phi(x)\right)\bar{P}_8(x,s)=0 \quad (38)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_C+3\beta_D+2\phi(x)\right)\bar{P}_9(x,s)=0 \quad (39)$$

$$\left(s+\frac{\delta}{\delta x}+\beta_C+2\phi(x)\right)\bar{P}_{10}(x,s)=0 \quad (40)$$

$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_{11}(y,s)=0 \quad (41)$$

$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_{12}(y,s)=0 \quad (42)$$

$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_{13}(y,s)=0 \quad (43)$$

$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_{14}(y,s)=0 \quad (44)$$

$$\bar{P}_1(0,s)=3\beta_D\bar{P}_0(s) \quad (45)$$

$$\bar{P}_2(0,s)=9\beta_D^2\bar{P}_0(s) \quad (46)$$

$$\bar{P}_3(0,s)=2\beta_B\bar{P}_0(s) \quad (47)$$

$$\bar{P}_4(0,s)=\beta_A\bar{P}_0(s) \quad (48)$$

$$\bar{P}_5(0,s)=6\beta_A\beta_D\bar{P}_0(s) \quad (49)$$

$$\bar{P}_6(0,s)=4\beta_A\beta_B\bar{P}_0(s) \quad (50)$$

$$\bar{P}_7(0,s)=12\beta_B\beta_D\bar{P}_0(s) \quad (51)$$

$$\bar{P}_8(0,s)=12\beta_A\beta_B\beta_D\bar{P}_0(s) \quad (52)$$

$$\bar{P}_9(0,s)=24\beta_A\beta_B\beta_D\bar{P}_0(s) \quad (53)$$

$$\bar{P}_{10}(0,s)=\left(72\beta_A\beta_B\beta_D^2+36\beta_A\beta_B\beta_D\right)\bar{P}_0(s) \quad (54)$$

$$\bar{P}_{11}(0,s) = 18\beta_D^2 \bar{P}_0(s) \tag{55}$$

$$\bar{P}_{12}(0,s) = 2\beta_B^2 \bar{P}_0(s) \tag{56}$$

$$\bar{P}_{13}(0,s) = \beta_A^2 \bar{P}_0(s) \tag{57}$$

$$\bar{P}_{14}(0,s) = \begin{pmatrix} \beta_C + \beta_C\beta_D + 9\beta_C\beta_D^2 + 2\beta_C\beta_B + \beta_C\beta_A + 6\beta_A\beta_C\beta_D + 4\beta_A\beta_B\beta_C + 12\beta_B\beta_C\beta_D + \\ 12\beta_A\beta_B\beta_C\beta_D + 24\beta_A\beta_B\beta_C\beta_D + \beta_C(72\beta_A\beta_B\beta_D^2 + 36\beta_A\beta_B\beta_D) \end{pmatrix} \bar{P}_0(s) \tag{58}$$

$$\bar{P}_0(s) = \frac{1}{D(s)}$$

$$\bar{P}_2(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_\phi(s + \beta_C + 2\beta_D)}{s + \beta_C + 2\beta_D} \right\} 9\beta_D^2 \tag{59}$$

$$\bar{P}_2(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_\phi(s + \beta_C + 2\beta_D)}{s + \beta_C + 2\beta_D} \right\} 9\beta_D^2 \tag{60}$$

$$\bar{P}_3(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_Q(s + \beta_A + \beta_B + \beta_C + 3\beta_D)}{s + \beta_A + \beta_B + \beta_C + 3\beta_D} \right\} 2\beta_B \tag{61}$$

$$\bar{P}_4(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_\phi(s + \beta_A + 2\beta_B + \beta_C + 3\beta_D)}{s + \beta_A + 2\beta_B + \beta_C + 3\beta_D} \right\} \beta_A \tag{62}$$

$$\bar{P}_5(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{2\phi}(s + \beta_A + 2\beta_B + \beta_C)}{s + \beta_A + 2\beta_B + \beta_C} \right\} 6\beta_A\beta_D \tag{63}$$

$$\bar{P}_6(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{2\phi}(s + \beta_C + 3\beta_D)}{s + \beta_C + 3\beta_D} \right\} 4\beta_A\beta_B \tag{64}$$

$$\bar{P}_7(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{2\phi}(s + \beta_A + \beta_C)}{s + +\beta_A + \beta_C} \right\} 12\beta_B\beta_D \tag{65}$$

$$\bar{P}_8(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_\phi(s + \beta_C + 3\beta_D)}{s + \beta_C + 3\beta_D} \right\} 12\beta_A\beta_B\beta_D \tag{66}$$

$$\bar{P}_9(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_\phi(s + \beta_C + 3\beta_D)}{s + \beta_C + 3\beta_D} \right\} 24\beta_A\beta_B\beta_D \tag{67}$$

$$\bar{P}_{10}(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{2\phi}(s + \beta_C)}{s + \beta_C} \right\} \left(72\beta_A\beta_B\beta_D^2 + 36\beta_A\beta_B\beta_D\right) \tag{68}$$

$$\bar{P}_{11}(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{\mu_0}(s)}{s} \right\} 18\beta_D^2 \tag{69}$$

$$\bar{P}_{12}(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{\mu_0}(s)}{s} \right\} 2\beta_B^2 \tag{70}$$

$$\bar{P}_{13}(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{\mu_0}(s)}{s} \right\} \beta_A^2 \tag{71}$$

$$\bar{P}_{14}(s) = \frac{1}{D(s)} \left\{ \frac{1 - \bar{S}_{\mu_0}(s)}{s} \right\} \begin{pmatrix} \beta_C + \beta_C\beta_D + 9\beta_C\beta_D^2 + 2\beta_C\beta_B + \beta_C\beta_A + 6\beta_A\beta_C\beta_D + 4\beta_A\beta_B\beta_C + 12\beta_B\beta_C\beta_D + \\ 12\beta_A\beta_B\beta_C\beta_D + 24\beta_A\beta_B\beta_C\beta_D + \beta_C(72\beta_A\beta_B\beta_D^2 + 36\beta_A\beta_B\beta_D) \end{pmatrix} \tag{72}$$

where: D(s) = $\left(s+\beta_A+2\beta_B+\beta_C+3\beta_D\right)-$

$$\left(\begin{array}{l} 3\beta_D\left\{\overline{S}_Q\left(s+\beta_A+2\beta_B+\beta_C+3\beta_D\right)\right\}+2\beta_B\left\{\overline{S}_Q\left(s+\beta_A+\beta_B+\beta_C+3\beta_D\right)\right\} \\ +\beta_A\left\{\overline{S}_Q\left(s+\beta_A+2\beta_B+\beta_C+3\beta_D\right)\right\}+\left(18\beta_D^2\right)\left\{\overline{S}_{\mu_0}(s)\right\}+\left(2\beta_B^2\right)\left\{\overline{S}_{\mu_0}(s)\right\} \\ +\beta_A^2\left\{\overline{S}_{\mu_0}(s)\right\}+\left[\begin{array}{l}\beta_C+\beta_C\beta_D+9\beta_C\beta_D^2+2\beta_B\beta_C+\beta_A\beta_C+6\beta_A\beta_C\beta_D \\ +4\beta_A\beta_B\beta_C+12\beta_B\beta_C\beta_D+12\beta_A\beta_B\beta_C\beta_D+ \\ 24\beta_A\beta_B\beta_C\beta_D+\beta_C\left(72\beta_A\beta_B\beta_D^2+36\beta_A\beta_B\beta_D\right)\end{array}\right]\left\{\overline{S}_{\mu_0}(s)\right\} \end{array}\right) \quad (73)$$

$$\overline{P}_{UP}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_3(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_9(s)+\overline{P}_{10}(s) \quad (74)$$

4. Analytical Analysis of the model for particular cases

This section deals with the reliability models formulation. Models such reliability, availability, MTTF, sensitivity and cost analysis were obtained.

4.1 Availability analysis:

By taking the inverse Laplace transform of equation (*) together with the values of failure rate, β_A=0.01, β_B=0.02, β_C = 0.03, β_D = 0.04, at $\Phi(x)$ = θ = x = 1, expression of availability with respect to time was derived.

$$S_{\mu_0}(s)=\overline{S}_{\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}}(s)$$

Setting: $=\dfrac{\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}}{s+\exp[x^{\theta}+\{\log\phi(x)\}^{\theta}]^{1/\theta}}$ and

$$\overline{S}_{\varphi_S}(s)=\frac{\phi_S}{s+\phi_S},$$

$$\overline{P_{up}}(t)=\left\{\begin{array}{l}-0.02240372415e^{-2.785124690t}-0.1229939365e^{-1.242493559t}+0.01953570936e^{-1.191148338t} \\ +1.097502394e^{-0.07953341223t}-0.0003842036733e^{-1.150000000t}-0.0002344172743e^{-1.030000000t} \\ -0.007861667140e^{-1.110000000t}-0.007967692680e^{-1.070000000t}\end{array}\right\}...(*)$$

Through variation of time t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9..., we obtained different values of $P_{up}(t)$ with the help of expression (*) as shown in Table 1 and figure 2.

Table 1. Variation of Availability with respect to time (t)

Time(t)	Availability
0	1
1	0.979
2	0.925
3	0.861
4	0.797
5	0.737
6	0.68
7	0.628
8	0.58
9	0.536

Figure 2. Variation of Availability with respect to time (t)

4.2 Reliability Analysis:

All repair rates are assumed to be zero i.e. $\phi(x)$ and $\mu_0(x)$ in equation (**), for the same values of failure rates as β_A=0.01, β_B=0.02, β_C = 0.03, β_D = 0.04, φ =1, θ = 1, x = 1, and then taking inverse Laplace transform, the expression reliability of the system is obtained as represented in equation (**) given as:

$$R(t)=\begin{Bmatrix} 0.001829647059e^{-0.03000000000t}+0.1600000000e^{-0.1100000000t}+2.e^{-0.1800000000t} \\ +0.09230769231e^{-0.07000000000t}+0.02176000000e^{-0.15000000000t}+7.23981900510^{-8}e^{-0.20000000000t} \\ (-1.762333210^{7}+1.79562510^{6}t) \end{Bmatrix} ...(**)$$

For, different values of time *t*= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 Units of time, different value of Reliability that shown in Table 2.and Figure 3.

Table 2. Reliability for different values of time (t*)*

Time(t)	Reliability
0	1
1	0.982
2	0.94
3	0.884
4	0.82
5	0.752
6	0.683
7	0.616
8	0.553
9	0.493

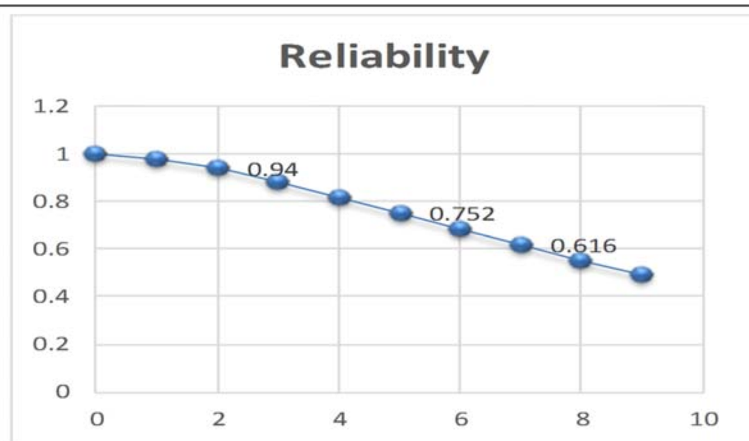


Figure 3. Reliability as a function of time (t)

4.3 Mean Time to Failure (MTTF):

Taking all repairs zero in equation (***), and the limit, as *s* tends to zero we obtain the expression for MTTF.

$$MTTF=\lim_{s\to 0}\overline{P}_{up}(s)=\frac{1}{\beta_A+2\beta_B+\beta_C+3\beta_D}\begin{Bmatrix} 1+\dfrac{3\beta_D}{\beta_A+2\beta_B+\beta_C+3\beta_D}+\dfrac{2\beta_B}{\beta_A+\beta_B+\beta_C+3\beta_D} \\ +\dfrac{\beta_A}{\beta_A+2\beta_B+\beta_C+3\beta_D} \end{Bmatrix}...(***)$$

by Setting β_A=0.01, β_B=0.02, β_C = 0.03, β_D = 0.04, and varying β_A, β_B, $\beta_C$and β_D one after another respectively as, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, in (***), the variation of M.T.T.F. with respect to failure rates is obtained as shown in adjacent Table3 and corresponding Figure 4.

Table 3. Variation of MTTF with respect to failure rates.

Failure Rate	MTTF β_1	MTTF β_2	MTTF β_3	MTTF β_4
0.1	10.96	11.65	13.4	17.38
0.2	10.71	10.96	12.04	14.38
0.3	10.48	10.44	10.96	12.39
0.4	10.27	10.02	10.07	10.96
0.5	10.07	10	9.307	9.876
0.6	9.893	9.36	8.651	9.026
0.7	9.726	9.095	8.078	8.34
0.8	9.57	8.86	7.572	7.775
0.9	9.425	8.65	7.124	7.302

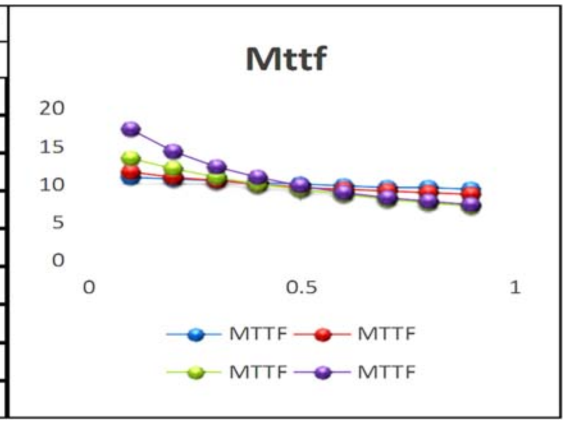


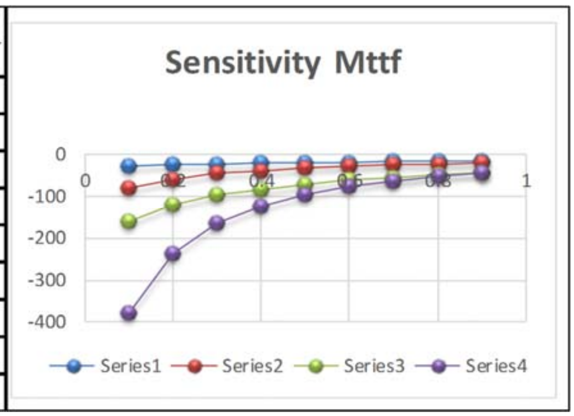
Figure 4. Variation of MTTF with respect to failure rates.

4.4 Sensitivity Analysis

The sensitivity in MTTF of the system is studied through the partial differentiation of MTTF concerning the failure rates of the system. By employing the set of parametric values of the failure rates after partial differentiation of MTTF with respect of failure rates and then varying β_A=0.01, β_B=0.02, β_C = 0.03, and β_D = 0.04, in resulting expression, one can calculate the MTTF sensitivity as shown in Table 4 and the corresponding graphs shown in Figure 5.

Table 4. Sensitivity of MTTF as a function of failure rates

Failure Rate	$\partial(Mttf)/\beta_1$	$\partial(Mttf)/\beta_2$	$\partial(Mttf)/\beta_3$	$\partial(Mttf)/\beta_4$
0.1	-26.27	-80.14	-159.6	-379.2
0.2	-24	-58.93	-119.1	-238.3
0.3	-22.01	-43.54	-97.39	-166.3
0.4	-20.27	-38.35	-82.07	-123.3
0.5	-18.73	-32.55	-70.4	-95.3
0.6	-17.36	-28.22	-61.18	-75.93
0.7	-16.13	-24.86	-53.7	-61.95
0.8	-15.04	-22.19	-47.55	-51.51
0.9	-14.05	-20	-42.36	-43.51



Cost Analysis

If the service facility be always available, then the expected profit during the interval [0, t) canbe enumurate by the formula given as; $E_p(t) = K_1\int_0^t P_{up}(t)dt - K_2 t$,as notation explained in previous chapter. For the same set of the parameter of failure and repair rates in (**), the expression of cost analysis is obtained.

$$E_p(t) = \left\{ \begin{array}{l} -0.008044065040\,e^{-2.785124690t} + 0.09898959685e^{-1.242493559t} - 0.01640073594e^{-1.191148338t} - \\ -13.79926201e^{-0.07953341223t} + 0.0003340901e^{-1.150000000t} + 0.0002275895867e^{-1.030000000t} \\ +0.007082583009e^{-1.110000000t} - 0.007446357645e^{-1.070000000t} + 31.98263877 \end{array} \right\} \ldots(****)$$

By Setting K_1= 1 and K_2= 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively and varying t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, Units of time, the results for expected profit can be obtain as shown in Table 5 and graphical representation in Figure 6.

Table 5. Expected Profit in [0,t) t=0,1 ,2, 3, 4,…,9

Time (t)	EP(t): K2=0.6	EP(t): K2=0.5	EP(t): K2=0.4	EP(t): K2=0.3	EP(t): K2=0.2	EP(t): K2=0.1
0	0	0	0	0	0	0
1	0.39	0.49	0.59	0.69	0.79	0.89
2	0.75	0.95	1.15	1.35	1.55	1.75
3	1.04	1.34	1.64	1.94	2.24	2.54
4	1.27	1.67	2.07	2.47	2.87	3.27
5	1.44	1.94	2.44	2.94	3.44	3.94
6	1.55	2.15	2.75	3.35	3.95	4.55
7	1.6	2.3	3	3.7	4.4	5.1
8	1.61	2.41	3.21	4.01	4.81	5.61
9	1.56	2.46	3.36	4.26	5.16	6.06

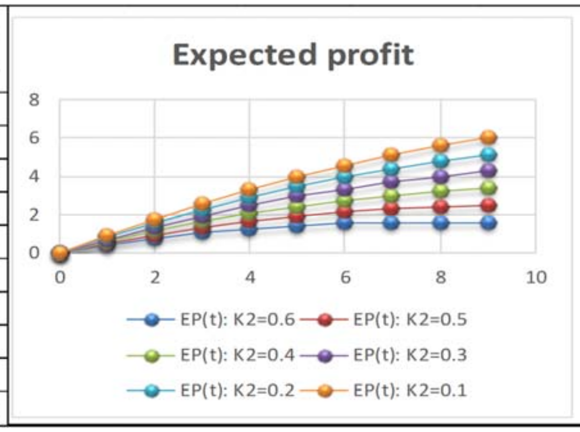


Figure 6. Profit with respect to failure rates.

5 Conclusions through result discussion

Table 1 and Table 2 provide a simple description of how the performance of the repairable series system changes with respect to time t as failure rates are set at different values. When failure rates are at lower values $\beta_A = 0.01$, $\beta_B = 0.02$, $\beta_C = 0.03$ and $\beta_D = 0.04$ availability and reliability of the system decreases gradually as the value of t increases, with the passage of time and ultimately become steady to the value zero after a long interval of time. For this reason, the future behavior of the repairable device can be accurately predicted at any point for any given set of parametric values. This is obvious from the graphical design of the platform. It is observed that when the repair is provided the system performance is far better than when the repair is not provided. The corresponding values of availability are greater than the values of reliability as evident from table 1 and table 2. This simulation suggests that regular repair should be invoked to improve system performance.

Furthermore, Table 3 and figure 3 yields the MTTF of the system with respect to variation in β_A, β_B, β_C, $and\ \beta_D$respectively when other parameters are kept constant.

Table 4 and the corresponding figure 4 displays the trends of cost function against the time t when the revenue cost per unit time K_1 is fixed at1, service cost $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$. It is evident in this table and figure that the cost increases with respect to time when the service cost K_2 decreases. The computed cost in table shows that $K_2 = 0.1$ is the maximum and $K_2 = 0.6$ is the minimum. Finally, it has been observed that as service cost decreases, the cost increases with variation of time. In general, for low service cost $(K_2 = 0.1)$ the cost function is high compare to high service cost$(K_2 = 0.6)$. This study will serve as a guide to engineers and computer system designers to design more critical systems to improve efficiency and reduce operational costs. As a result, existing work should integrate repairs and replacement for partial and full failure under the free renewal warranty. The future work will study the reliability and performance analysis of computer network using different type of transparent bridge.

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