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Cost Benefit Analysis of a k-out-of-n: G Type Warm Standby Series System Under Catastrophic Failure Using Copula Linguistics

P. K. Poonia[1*] and A. Sirohi[2]

1. Department of General Requirements, Sur College of Applied Sciences, Oman
2. Department of Mathematics, Bharat Institute of Technology, Meerut, India

Abstract

This paper deals with the study of reliability measures of a complex engineering system consisting three subsystems namely L, M, and N in series configuration. The subsystem-L has three units working under 1-out-of-3: G; policy, the subsystem-M has two units working under 1-out-of-2: G policy and the subsystem-N has one unit working under 1-out-of-1: G; policy. Moreover, the system may face catastrophic failure at any time t. The failure rates of units of all subsystems are constant and assumed to follow the exponential distribution however, their repair supports two types of distribution namely general distribution and Gumbel-Hougaard family copula distribution. The system is analyzed by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to derive the differential equations and to obtain important reliability characteristics such as availability of the system, reliability of the system, MTTF, and profit analysis. The numerical results for reliability, availability, MTTF, and profit function are obtained by taking particular values of various parameters and repair cost using maple. Tables and figures demonstrate the computed results and conclude that copula repair is more effective repair policy for better performance of repairable systems. It gives a new aspect to scientific community to adopt multi-dimension repair in form of copula. Furthermore, the results of the model are beneficial for system engineers and designers, reliability and maintenance managers.

Keyword: K-out-of-n, G system, Availability, MTTF, Catastrophic failure, Gumbel-Hougaard family copula distribution.

Nomenclature

s , t	Laplace transform / Time scale variable
$\lambda_1 / \phi_1(x)$	Failure rate / Repair rate of each unit in subsystem-L.
$\lambda_2 / \phi_2(x)$	Failure rate / Repair rate of each unit in subsystem-M.
λ_3	Failure rate of the unit in subsystem-N.
λ_E	Deliberate failure rate when two units in subsystem-L and one unit in subsystem-M failed.
λ_C	Failure rate related to catastrophic failure mode.
$P_0(t)$	The state transition probability that the system is in S_i state at an instant for $i=0$.
$\bar{P}(s)$	Laplace transformation of the state transition probability $P(t)$.
$P_i(x,t)$	The Probability that the system is in state S_i for $i=1$ to 9, E, C and the system is under repair with elapsed repair time is x,t . x is repaired variable and t is time variable.
$E_p(t)$	Expected profit in the interval $\left[0,t\right)$.
K_1, K_2	Revenue generated and service cost per unit time respectively.
$\mu_0(x)$	An expression of the joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula is given as $\mu_0(x)=C_\theta\{u_1(x),u_2(x)\}$ $=\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}$ where $u_1(x)=\phi(x)$ and $u_2(x)=e^x$. Here θ is the parameter $1<\theta<\infty$.

1. Corresponding Author Email: pkpmrt@email.com

Introduction

Determining accurate reliability and availability of an existing structure or product is a crucial task in the reliability engineering. In case of failure, money and time will be wasted and even disaster may occur. In order to achieve reliable system functioning, components are designed to be highly reliable in the sense that they rarely suffer from sudden failures. Nevertheless, components might degrade gradually with usage. Redundant strategy is often used by engineers to

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ensure the reliability and availability of the systems and/or to improve these characteristics of the systems. Thus, variety of standby systems have been designed and analyzed during the last few decades. The main objective of these studies is to develop methods and tools for evaluation and demonstration of the reliability, availability, and cost analysis. Redundant systems, which have been widely used in practice, such as space shuttles, communication satellites, a hybrid car, Nuclear reactors, or a fighter plane, are frequently discussed in research literature. Initially, redundant parts are designed to improve the reliability of the system, meaning that some additional paths are created or identical components connected in such a way that when one component fails the others will keep the system functioning. It is a technique commonly used to improve system reliability and availability. Redundancies can be categorized into the following: (i) Cold standby in which the standby unit is only called upon when the primary or operating unit fails. These inactive components have a zero failure rate and cannot fail while in standby state; (ii) Hot standby in which the standby unit has the same failure rate as when it is run with the operating unit; (iii) Warm standby in which the standby unit runs in the background of operating unit. It can fail in this state but its failure rate is less than that of the operating unit. Moreover, redundancy is highly cost effective in achieving a certain reliability level of the system. Therefore, in order to enhance reliability k-out-of-n system structure in which at least k components of n must be functioned. In order to improve the reliability of k-out-of-n systems, numerous researches have presented their works and contributions by constructing different types of complex repairable systems under the different types of failure and repair distributions. For instance, authors consider warm standby system by She and Pecht [1], generalized multi state system by Huang et al. [2], repairable consecutive systems with r repairman by Wu and Guan [3], two-stage weighted systems with components in common by Chen and Yang [4], main unit with helping unit by Kumar and Gupta [5], Markov repairable system with neglected or delayed failures by Bao and Cui [6], evaluated exact reliability formula for consecutive repairable systems by Liang et al. [7], general system with non-identical components considering shut-off rules using quasi-birth-death process by Moghaddass et al. [8], and generalized block replacement policy with respect to a threshold number of failed components and risk costs by Park and Pham [9].

The occurrence of failure in any complex repairable or non-repairable engineering system is a natural phenomenon, which arises due to the different working conditions. The k-out-of-n effective policy plays a crucial role in maintaining the reliability of repairable systems. The researchers have focused on evaluating reliability and availability of the redundant repairable systems like k-out-of-n in series configuration. In particular, Singh et al. [10] analyzed an engineering system, which consists of two subsystems, viz. subsystem-1 and subsystem-2 with controllers in series. Subsystem-1 works under the k-out-of- n: good policy. Subsystem-2 consists of three identical units in parallel configuration. In this case, controllers control the working of both subsystems. Authors evaluated reliability characteristics using supplementary variable technique. Ram et al. [11] investigated the reliability of a standby system incorporating waiting time to repair. In this case, system consists of two units' namely main unit and standby unit. Whenever the main unit fails, the whole load is transferred to the standby unit instantaneously by a switching-over device. As regards to the repairing of the main unit, it has to wait for repair whenever it fails due to unavailability of repair facility. Munjal and Singh [12] analyzed a complex repairable system composed of two 2-out-of-3: G subsystems connected in parallel. Jia et al. [13] studied repairable multistate two-unit series systems when repair time can be neglected. Goyal et al. [14] studied the sensitivity analysis of a three-unit series system under k-out-of-n redundancy. Singh et al. [15] developed a model of a complex repairable system having two subsystems in series configuration. Both subsystems includes two units in parallel, and it is assumed to work till at least one unit of both the subsystems are in good operative condition. Gahlot et al. [16] assessed a repairable system in series configuration under different types of failure and repair policies using copula linguistics. Singh and Poonia [17] assessed 1-out-of-2: G system with correlated lifetimes under inspection.

Some specific papers related to this paper are as follows. Lado and Singh [18] analyzed an engineering system, which consists of two subsystems in series configuration operated by a human operator. Both the subsystems have two units in parallel. In this papers authors proved that copula repair is more reliable than general repair. Pundir and Patawa [19] studied repairable two dissimilar units' cold standby system waiting for repair facility after failure of system units. They stimulated exponential failures, arbitrary waiting and arbitrary repair rate. Singh et al. [20] studied two subsystems in series configuration with imperfect switch connected with both subsystems. Recently, Zhao et al. [21] and Singh et al. [22] studied some real system problems related to our study.

System Description

Researchers around the world have presented their research works on reliability analysis of complex repairable system however they have not focused on the study of the system consisting of three subsystems connected in series configuration with catastrophic failure. Catastrophic failure is a complete, sudden, often unexpected breakdown in the entire system. Such a break down may occur due to animal related disruption or change in environment related conditions like Corona virus nowadays. Sometimes a single component in a critical location fails; resulting in downtime for the entire system also comes under catastrophic failure. The term catastrophic failure is most commonly used for organizational failures, but has often been extended to many other disciplines in which total and irrecoverable loss occurs. Treating the above realities in the present study, the model consisting three subsystems in series configuration considering catastrophic failure. The subsystem-L has three identical units, subsystem-M has two identical units and subsystem-N has one unit only. The subsystem-L is working under 1-out-of-3: G; scheme, the subsystem-M is working under 1-out-of-2: G; scheme, however, the subsystem-N works under 1-out-of-1: G; scheme. The catastrophic failure is treated as a complete failing state. During operation, the system will be in any of the three states: perfect operation, partial failure, and complete failure. The failure rates of units of subsystems are constant and assumed to follow the exponential distribution, but their repair supports two types of distribution namely general distribution and Gumbel-Hougaard family copula distribution. Then, based on the behavior of the whole system, all the system states can also be classified into three subsets as follows.

Classification I: The system operates perfectly; in this situation, all the components in both subsystems are in the perfect functioning state.

Classification II: The system is partially working; in this situation, at least one component in one or both subsystems is in the failure state, and the remainder is perfectly functioning.

Classification III: The system is completely failed; in this situation, either subsystem L, M or N is in the complete failure state. Further, system may be completely failed due to catastrophic failure.

Therefore, the system remains working until one of the subsystems is completely failed. Based on the above-mentioned assumptions, the system could be modeled by a continuous-time stochastic process. The present study accomplished two objectives using supplementary variable technique. First the expressions for the reliability of the system, availability of the system, mean time to failure and profit function are obtained. Second numerical simulation with respect to profit function is performed. Explicit expressions for reliability, availability, MTTF, and cost analysis functions are obtained with help of MAPLE (software). Tables and graphs present a comparative analysis of results. The system configuration and transition state diagram of the designed model are shown in fig. 1(a) and 1(b) respectively.

Assumptions

The following assumptions are made through this paper:

1. Initially the system is in state S_0, and all the units of subsystem-L, M, and N are in proper working conditions.
2. The subsystem-L works successfully if minimum one unit is in proper working condition i.e. 1-out-of-3: G policy, the subsystem-M works successfully if minimum one unit is in proper working condition i.e. 1-out-of-2: G policy, and the subsystem-N works successfully if the lonely unit is in proper working condition i.e. 1-out-of-1: G policy.
3. As soon as repair of a unit in all of the three subsystems completed, it again becomes operational (as good as new). No damage reported due to repair of the system.
4. Whenever there is a failure in two units of subsystem-L and one unit in subsystem-M, the system goes to perilous state where system has to stop functioning deliberately to avoid further failures with emergency failure rate λ_E.
5. There may be unpredictable catastrophic failure to the system at any time (t).
6. One repairperson is available full time with the system and may be called as soon as the system reaches to partially or completely failed state.
7. All failure rates are constant and follows the exponential distribution.
8. The failure rate and repair rate in all the three subsystems is same unit wise, while different subsystem wise.
9. The complete failed system needs repair immediately. For this Gumbel-Hougaard, family of copula can be employed to restore the system.

Copula

A d-dimensional copula is a distribution function on $[0, 1]^d$ with standard uniform marginal distributions. Let $C(u) = C(u_1, ..., u_d)$ be the distribution functions which are copulas. Hence C is a mapping of the form $C: [0, 1]^d \rightarrow [0, 1]$, i.e. a mapping of the unit hypercube into the unit interval. The following three properties must hold:

(i) $C(u_1, ..., u_d)$ is increasing in each component u_i.
(ii) $C(1, ...1, u_i, 1, ..., 1) = u_i$ for all i {1, ...d}, $u_i \in [0, 1]$.
(iii) For all $(a_1,..., a_d), (b_1,..., b_d) \in [0, 1]$ with $a_i \le b_i$ we have: Where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, ..., d\}$.

The copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval [0, 1]. The copula (joint probability distribution) approach is very natural when a complex system repaired in a couple of ways. For θ=1 the Gumbel- Hougaard copula $C_\theta(u_1, u_2) = \exp(-((-\log u_1)^\theta + (-\log u_2)^\theta)^{1/\theta}$, $1 \le \theta \le \infty$, θ=1 the Gumbel- Hougaard copula models become independence, and for θ→∞ it converges monotonically. Although the different copulas have employed by various researchers due to simplicity conventional purpose Gumbel- Hougaard family copula have employed to assessing analytical cases of the paper.

System Configuration and Transition Diagram

System configuration shown in Fig 1 (a) while transition diagram in Fig 1 (b). HereS_0 is perfect state, S_1, S_2, S_3, S_4 and S_5 partial failed/degraded and S_6, S_7, S_8, S_9, S_E and S_C are complete failed states. Due to failure of unit (s) in the subsystem-L, M or/and N, the transitions approaches to partially failed states S_1, S_2, S_3 S_4 and S_5. The state S_6, S_7, S_8 and S_9 are complete failed states due to failure of units in all the subsystems, while S_E is completely failed state due to deliberate failure. The states S_C is complete failed state due to catastrophic failure.

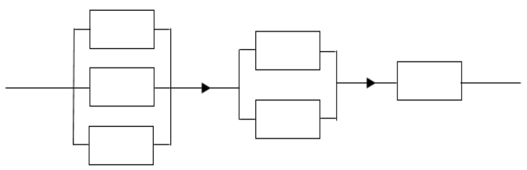

Fig. 1(a) System configuration

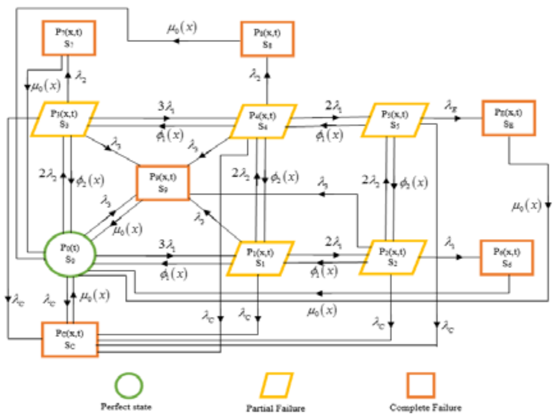

Fig. 1(b) State transition diagram of the model

In the transition diagram above S_0 is a state where all the subsystems are in good working condition. S_1, S_2, S_3, S_4 and S_5 are the states where the system is in partially failure mode/ degraded, and the general repair is employed, states S_6, S_7, S_8, S_9, S_E and S_C are the states where the system is in the totally failure mode. Repair is being applied using Gumbel-Hougaard family copula distribution.

Table 1. State Description

State	Description
S_0	This is a perfect state and all units of subsystem-L, M and N are in proper working condition.
S_1	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-L. All units of subsystem-M and N are in the proper operational state. The system is under general repair.
S_2	The indicated state is degraded but is in operational mode after the failure of two units in subsystem-L. All units of subsystem-M and N are in the proper operational state. The system is under general repair.
S_3	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-M. All units of subsystem-L and N are in the proper operational state. The system is under general repair.
S_4	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-L and one unit in subsystem-M. All units of subsystem-N are in the proper operational state. The system is under general repair.
S_5	The indicated state is degraded but is in operational mode after the failure of two units in subsystem-L and one unit in subsystem-M. All units of subsystem-N are in the proper operational state. The system is under general repair.
S_6, S_7 S_8, S_9 S_E, S_C	The states represent that the system is in completely failure mode and the system is under repair using Gumbel-Hougaard family copula distribution.

Formulation of mathematical model

By probability of considerations and continuity arguments, we can obtain the following set of difference-differential equations associated with the present mathematical model (see Appendix-1):

$$\left[\frac{\partial}{\partial t}+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right]P_0(t)=\int_0^\infty \phi_1(x)P_1(x,t)dx + \int_0^\infty \phi_2(x)P_3(x,t)dx+\sum_k\int_0^\infty \mu_0(x)P_k(x,t)dx \qquad (1)$$

where $k = 6,7,8,9,E,C$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]P_1(x,t)=0 \qquad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]P_2(x,t)=0 \qquad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2(x)\right]P_3(x,t)=0 \qquad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_1(x)+\phi_2(x)\right]P_4(x,t)=0 \qquad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_E+\lambda_C+\phi_1(x)+\phi_2(x)\right]P_5(x,t)=0 \qquad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\right]P_k(x,t)=0 \qquad (7)$$

Where $k = 6,7,8,9,E,C$

Boundary Conditions,

$$P_1(0,t)=3\lambda_1 P_0(t) \quad (8)$$

$$P_2(0,t)=2\lambda_1 P_1(0,t)=6\lambda_1^2 P_0(t) \quad (9)$$

$$P_3(0,t)=2\lambda_2 P_0(t) \quad (10)$$

$$P_4(0,t)=3\lambda_1 P_3(0,t)+2\lambda_2 P_1(0,t)=12\lambda_1\lambda_2 P_0(t) \quad (11)$$

$$P_5(0,t)=2\lambda_2 P_2(0,t)+2\lambda_1 P_4(0,t)=36\lambda_1^2\lambda_2 P_0(t) \quad (12)$$

$$P_6(0,t)=\lambda_1 P_2(0,t)=6\lambda_1^3 P_0(t) \quad (13)$$

$$P_7(0,t)=\lambda_2 P_3(0,t)=2\lambda_2^2 P_0(t) \quad (14)$$

$$P_8(0,t)=\lambda_2 P_4(0,t)=12\lambda_1\lambda_2^2 P_0(t) \quad (15)$$

$$P_9(0,t)=\lambda_3 P_0(t)+\lambda_3 P_1(0,t)+\lambda_3 P_2(0,t)+\lambda_3 P_3(0,t)+\lambda_3 P_4(0,t) \quad (16)$$

$$=\lambda_3\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2\right)P_0(t)$$

$$P_E(0,t)=\lambda_E P_3(0,t)=2\lambda_2\lambda_E P_0(t) \quad (17)$$

$$P_C(0,t)=\lambda_C P_0(t)+\lambda_C P_1(0,t)+\lambda_C P_2(0,t)+\lambda_C P_3(0,t)+\lambda_C P_4(0,t)+\lambda_C P_5(0,t)$$
$$=\lambda_C\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2+36\lambda_1^2\lambda_2\right)P_0(t) \quad (18)$$

Initials conditions

$P_0(0)=1$, and other state probabilities are zero at $t=0$ (19)

Solution of the model

Taking Laplace transformation of equations (1) to (18) and using equation (19), we obtain

$$\left[s+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right]\bar{P}_0(s)=1+\int_0^\infty \phi_1(x)\bar{P}_1(x,s)dx+\int_0^\infty \phi_2(x)\bar{P}_3(x,s)dx+\sum_k\int_0^\infty \mu_0(x)\bar{P}_k(x,s)dx \quad (20)$$

Where $k=6,7,8,9,E,C$ and $\bar{P}_i(x,s)=\int_0^\infty e^{-st}P_i(x,t)dt$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]\bar{P}_1(x,s)=0 \quad (21)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]\bar{P}_2(x,s)=0 \quad (22)$$

$$\left[s+\frac{\partial}{\partial x}+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2(x)\right]\bar{P}_3(x,s)=0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_1(x)+\phi_2(x)\right]\bar{P}_4(x,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_E+\lambda_C+\phi_1(x)+\phi_2(x)\right]\bar{P}_5(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}\right]\bar{P}_k(x,s)=0 \quad (26)$$

Where $k=6,7,8,9,E,C$

Boundary Conditions,

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s) \quad (27)$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)=6\lambda_1^2\bar{P}_0(s) \quad (28)$$

$$\bar{P}_3(0,s)=2\lambda_2\bar{P}_0(s) \quad (29)$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(0,s)+2\lambda_2\bar{P}_1(0,s)=12\lambda_1\lambda_2\bar{P}_0(s) \quad (30)$$

$$\bar{P}_5(0,s)=2\lambda_2\bar{P}_2(0,s)+2\lambda_1\bar{P}_4(0,s)=36\lambda_1^2\lambda_2\bar{P}_0(s) \quad (31)$$

$$\bar{P}_6(0,s)=\lambda_1\bar{P}_2(0,s)=6\lambda_1^3\bar{P}_0(s) \quad (32)$$

$$\bar{P}_7(0,s)=\lambda_2\bar{P}_3(0,s)=2\lambda_2^2\bar{P}_0(s) \quad (33)$$

$$\bar{P}_8(0,s)=\lambda_2\bar{P}_4(0,s)=12\lambda_1\lambda_2^2\bar{P}_0(s) \quad (34)$$

$$\bar{P}_9(0,s)=\lambda_3\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2\right)\bar{P}_0(s) \quad (35)$$

$$\bar{P}_E(0,s)=\lambda_E\bar{P}_3(0,s)=2\lambda_2\lambda_E\bar{P}_0(s) \quad (36)$$

$$\bar{P}_C(0,s)=\lambda_C\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2+36\lambda_1^2\lambda_2\right)\bar{P}_0(s) \quad (37)$$

Change in Laplace transformation of boundary conditions after repair, if any

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s)+\int_0^\infty\phi_1(x)\bar{P}_2(x,s)dx \quad (38)$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)+\int_0^\infty\phi_2(x)\bar{P}_5(x,s)dx \quad (39)$$

$$\bar{P}_3(0,s)=2\lambda_2\bar{P}_0(s)+\int_0^\infty\phi_1(x)\bar{P}_4(x,s)dx \quad (40)$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(0,s)+2\lambda_2\bar{P}_1(0,s)+\int_0^\infty\phi_1(x)\bar{P}_5(x,s)dx \quad (41)$$

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s)=\frac{1}{D(s)} \quad (42)$$

$$\bar{P}_1(s)=\frac{3\lambda_1}{D(s)}\frac{1-P}{s+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C} \quad (43)$$

$$\bar{P}_2(s)=\frac{6\lambda_1^2}{D(s)}\frac{1-Q}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_C} \quad (44)$$

$$\bar{P}_3(s)=\frac{2\lambda_2}{D(s)}\frac{1-R}{s+3\lambda_1+\lambda_2+\lambda_3+\lambda_C} \quad (45)$$

$$\bar{P}_4(s)=\frac{12\lambda_1\lambda_2}{D(s)}\frac{1-S}{s+2\lambda_1+\lambda_2+\lambda_3+\lambda_C} \quad (46)$$

$$\bar{P}_5(s)=\frac{36\lambda_1^2\lambda_2}{D(s)}\frac{1-T}{s+\lambda_E+\lambda_C} \quad (47)$$

$$\bar{P}_6(s)=\frac{6\lambda_1^3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{6\lambda_1^3}{D(s)}\frac{1-U}{s} \quad (48)$$

$$\bar{P}_7(s)=\frac{2\lambda_2^2}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{2\lambda_2^2}{D(s)}\frac{1-U}{s} \quad (49)$$

$$\bar{P}_8(s)=\frac{12\lambda_1\lambda_2^2}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{12\lambda_1\lambda_2^2}{D(s)}\frac{1-U}{s} \quad (50)$$

$$\bar{P}_9(s)=\frac{\lambda_3 V}{D(s)}\frac{1-U}{s} \quad (51)$$

$$\bar{P}_E(s)=\frac{2\lambda_2\lambda_E}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{2\lambda_2\lambda_E}{D(s)}\frac{1-U}{s} \quad (52)$$

$$\bar{P}_C(s)=\frac{\lambda_C\left(V+36\lambda_1^2\lambda_2\right)}{D(s)}\frac{1-U}{s} \quad (53)$$

Where,

$$D(s)=s+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C-3\lambda_1 P-2\lambda_2 R-UW$$

$$P=\frac{\phi_1}{s+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1}$$

$$Q=\frac{\phi_1}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1}$$

$$R=\frac{\phi_2}{s+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2}$$

$$S=\frac{\phi_3}{s+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_3}$$

$$T=\bar{S}_{\phi_3}\left(s+\lambda_E+\lambda_C\right)=\frac{\phi_3}{s+\lambda_E+\lambda_C+\phi_3},$$

$$U=\bar{S}_{\mu_0}(s)=\frac{\mu_0}{s+\mu_0},$$

$$V=1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2$$

and

$$W=6\lambda_1^3+2\lambda_2^2+12\lambda_1\lambda_2^2+2\lambda_2\lambda_E+V\left(\lambda_3+\lambda_C\right)+36\lambda_1^2\lambda_2\lambda_C$$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s) \quad (54)$$

$$\bar{P}_{down}(s)=1-\bar{P}_{up}(s) \quad (55)$$

Analytical Study

Availability Analysis

When repair follows general and Gumbel-Hougaard family copula distribution, we have

$$\bar{S}_{\mu_0}(s)=\frac{\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}}{s+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}}$$

setting $\bar{S}_{\alpha_i}(s)=\frac{\alpha_i}{s+\alpha_i}, i=1,2,3$. Following cases have been considered:

(a) Taking the values of different parameters as $\lambda_1=0.020,\ \lambda_2=0.025,\ \lambda_3=0.030,\ \lambda_E=0.040,$ $\lambda_C=0.015, \phi_i=1, x=1(i=1,2,3)$ in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$P_{up}(t)=-0.000324e^{-1.1150t}-0.001009e^{-1.1100t}-0.000149e^{-1.0550t}+0.020391e^{-2.7672t}-0.018120e^{-1.2268t}-0.000026e^{-1.1322t}+0.999239e^{-0.0037t} \quad (56)$$

(b) Taking the values of different parameters as $\lambda=0.025$, $\phi_i=1, x=1(i=1,2,3)$ in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$P_{up}(t)=-0.000253e^{-1.0500t}-0.001858e^{-1.1250t}+0.022656e^{-2.7740t}-0.022560e^{-1.2549t}+1.002016e^{-0.00599t} \quad (57)$$

(c) Repair follows general distribution by taking $\mu_0(x)=\phi_i(x)$ and same values of failure rates as in case (a) in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$P_{up}(t)=-0.000351e^{-1.1150t}+0.005406e^{-1.2561t}+8.8603\ 10^{-7}e^{-1.1322t}+0.027113e^{-1.0279t}+0.969217e^{-0.0036t}-0.000268e^{-1.0550t}-0.001118e^{-1.1100t} \quad (58)$$

For different values of time variable $t=0,10,20,30,40,50,\ 60,70,80,90$ and 100 units of time, one may get different values of $P_{up}(t)$ with the help of (56-58) as shown in table 2 and the figure 2.

Reliability Analysis

Taking all repair rates equal to zero and obtain inverse Laplace transform, we get an expression for the reliability of the system after taking the failure rates as $\lambda_1=0.020,\ \lambda_2=0.025,\ \lambda_3=0.030,\ \lambda_E=0.040$ and $\lambda_C=0.015$:

$$R_l(t)=3.000000e^{-0.1350t}+0.060000e^{-0.1150t}+2.000000e^{-0.1300t}+0.133333e^{-0.110000t}-4.196933e^{-0.1550t}+0.003600e^{-0.0550t} \quad (59)$$

For different values of time variable $t=0,10,20,30,40,50,60,70,80,90$ and 100 units of time, one may get different values of reliability $R_l(t)$ with the help of (59) as shown in table-3 and the corresponding figure 3.

Table 2. Variation in $P_{up}(t)$ with respect to time

Time	$P_{up}(t)$ (a)	$P_{up}(t)$	$P_{up}(t)$
0	1.00000	1.00000	1.00000
10	0.96280	0.94371	0.93491
20	0.92770	0.88880	0.90183
30	0.89387	0.83708	0.86991
40	0.86128	0.78837	0.83913
50	0.82988	0.74250	0.80943
60	0.79962	0.69930	0.78079

Time	$P_{up}(t)$ (a)	$P_{up}(t)$	$P_{up}(t)$
70	0.77047	0.65861	0.75315
80	0.74238	0.62028	0.72650
90	0.71531	0.58419	0.70079
100	0.68923	0.55020	0.67599

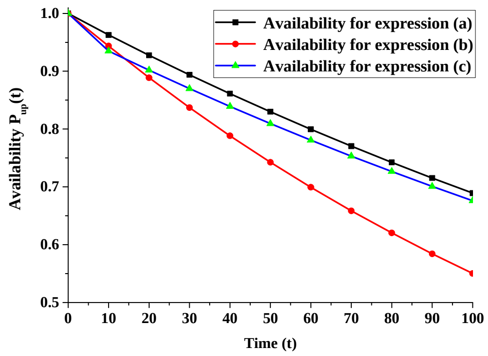

Fig.2. Availability as a function of time

Table 3. Variation in $R_i(t)$ with respect to time

Time	$R_i(t)$
0	1.00000
10	0.49745
20	0.18308
30	0.06013
40	0.01870
50	0.00567
60	0.00172
70	0.00053
80	0.00017
90	0.00006
100	0.00002

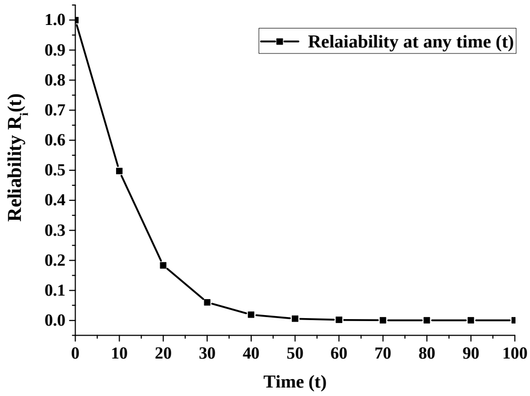

Fig. 3. Reliability as a function of time

Mean Time to Failure (MTTF)

If $R_i(t)$ is the reliability function obtained by taking inverse Laplace transformation of $\overline{P}_{up}(s)$ then average time to system failure for a continuous valued function is: MTTF $= \int_0^\infty R(t)dt = \lim_{s\to 0} R(s)$. Taking all repair rate to zero and the limit as s tends to zero in (54) for the exponential distribution; we can obtain the MTTF as:

$$MTTF = \frac{1}{\lambda}\left[1 + \frac{3\lambda_1}{2\lambda_1 + 2\lambda_2 + \lambda_3 + \lambda_C} + \frac{6\lambda_1^2}{\lambda_1 + 2\lambda_2 + \lambda_3 + \lambda_C} + \frac{2\lambda_2}{3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_C} + \frac{12\lambda_1\lambda_2}{2\lambda_1 + \lambda_2 + \lambda_3 + \lambda_C} + \frac{36\lambda_1^2\lambda_2}{\lambda_E + \lambda_C}\right] \text{ where } \lambda = 3\lambda_1 + 2\lambda_2 + \lambda_3 + \lambda_C \quad (60)$$

Now taking the values of different parameters as $\lambda_1 = 0.020, \lambda_2 = 0.025, \lambda_C = 0.030, \lambda_E = 0.040, \lambda_C = 0.015$ and varying $\lambda_1, \lambda_2, \lambda_3, \lambda_E$ and λ_C one by one respectively as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10$ in (60), the variation of MTTF, with respect to failure rates can be obtained as given in table 4 and figure 4.

Table 4. Computation of MTTF corresponding to the failure rates

Failure Rate	MTTF				
	λ_1	λ_2	λ_3	λ_E	λ_C
0.01	14.41	14.41	15.38	12.37	12.98
0.02	12.32	12.92	13.69	12.35	11.73
0.03	10.92	11.80	12.32	12.33	10.68
0.04	9.88	10.91	11.19	12.32	9.79
0.05	9.09	10.17	10.23	12.32	9.03
0.06	8.47	9.54	9.41	12.31	8.38
0.07	7.96	9.00	8.71	12.31	7.80
0.08	7.55	8.51	8.10	12.31	7.30
0.09	7.21	8.09	7.56	12.30	6.85
0.10	6.80	7.70	7.09	12.30	6.46

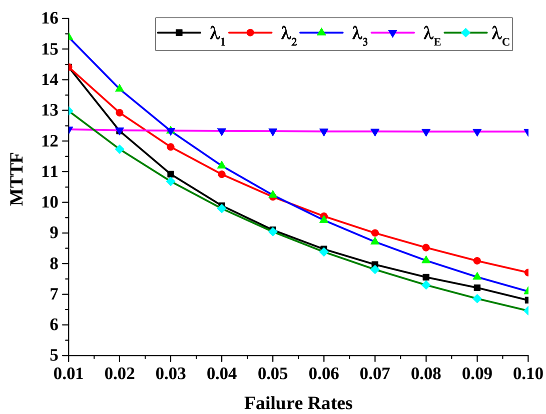

Fig. 4. MTTF as a function of failure rates

Cost Analysis

For the assumed failure and repair rates in section 6.1 and corresponding to the state transition diagram, we have computed the incurred profit for two cases when the system follows copula repair and general repair in (62 a) & (62 b). Let the service facility be always available, then expected profit during the interval $\left[0,t\right)$ is

$$E_p\left(t\right)=K_1\int_0^t P_{up}\left(t\right)dt-K_2t \qquad (61)$$

Where K_1 and K_2 are the revenue generation and service cost in unit time. For same set of parameters defined in (50), one can obtain expression for incurred profit as a function of time as:

$$\begin{aligned}E_p(t)=K_1\{&0.014770e^{-1.2268t}+0.000024e^{-1.1322t}\\&-0.007368e^{-2.7672t}+0.000909e^{-1.1100t}\\&+0.000291e^{-1.1150t}+0.000142e^{-1.0550t}\\&-269.039471e^{-0.0037t}+269.030703\}-K_2t\end{aligned} \qquad (62a)$$

$$\begin{aligned}E_p(t)=K_1\{&-7.8255\ 10^{-7}e^{-1.1322t}-0.0264e^{-1.0279t}\\&-0.001007e^{-1.1100t}-0.004304e^{-1.2562t}\\&+0.000315e^{-1.1150t}+0.000254e^{-1.0550t}\\&-269.001600e^{-0.0036t}+269.030703\}-K_2t\end{aligned} \qquad (62b)$$

Setting $K_1=1$ and $K_2=0.6,0.5,0.4,0.3,0.2$ and 0.1 respectively and varying $t=0,10,20,30,40,50,60,70,$ $80,90$ and 100 units of time, the results for expected profit can be obtain as per table 5 and 6 and figure 5 and 6.

Table 5. Profit computation for different vales of time for Copula repair

Time (t)	K_2					
	0.6	0.5	0.4	0.3	0.2	0.1
0	0.00	0.00	0.00	0.00	0.00	0.00
10	3.80	4.80	5.80	6.80	7.80	8.80
20	7.25	9.25	11.25	13.25	15.25	17.25
30	10.35	13.35	16.35	19.35	22.35	25.35
40	13.13	17.13	21.13	25.13	29.13	33.13
50	15.58	20.58	25.58	30.58	35.58	40.58
60	17.73	23.73	29.73	35.73	41.73	47.73
70	19.58	26.58	33.58	40.58	47.58	54.58
80	21.14	29.14	37.14	45.14	53.14	61.14
90	22.43	31.43	40.43	49.43	58.43	67.43
100	23.45	33.45	43.45	53.45	63.45	73.45

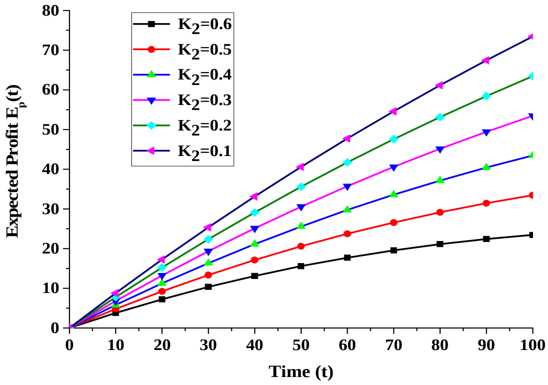

Fig.5 Expected profit as a function of time for Copula repair

Table 6. Profit computation for different values of time for general repair

Time (t)	K_2					
	0.6	0.5	0.4	0.3	0.2	0.1
0	0.00	0.00	0.00	0.00	0.00	0.00
10	3.54	4.54	5.54	6.54	7.54	8.54
20	6.73	8.73	10.73	12.73	14.73	16.73
30	9.58	12.58	15.58	18.58	21.58	24.58
40	12.13	16.13	20.13	24.13	28.13	32.13
50	14.37	19.37	24.37	29.37	34.37	39.37
60	16.32	22.32	28.32	34.32	40.32	46.32
70	17.99	24.99	31.99	38.99	45.99	52.99
80	19.39	27.39	35.39	43.39	51.39	59.39
90	20.52	29.52	38.52	47.52	56.52	65.52
100	21.41	31.41	41.41	51.41	61.41	71.41

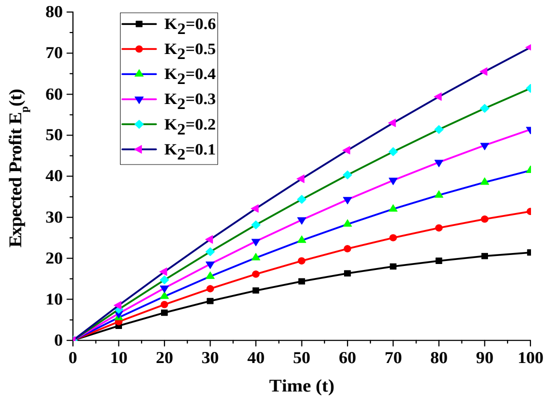

Fig.6. Expected profit as a function of time for general repair

Conclusion

This paper studies the reliability characteristics of a complex repairable standby system consisting of three subsystems in series configuration under catastrophic failure. First Subsystem-L is composed of three identical units in parallel configuration working under 1-out-of-3: G policy, second subsystem-M has two identical units working under 1-out-of-2: G: policy, while the third subsystem have one unit that working under a-out-of-a: G policy. Explicit expressions have been derived using supplementary variable technique. Warm standby redundancy has been used as an effective technique for improving the reliability of system design.

Table 2 and Figure 2 give the analysis of availability of the system in three different possibilities. One can clearly observe that availability of the system initially decreases with respect to time and later on it seems to be constant as the time increases. Table-3 and figure 3 give information for reliability of the system at different values of time. The graph showing a steep fall in reliability from the top to the lowermost in a very short period based on the failure rate of units. From table-2 and 3, one can observe that corresponding values of availability are greater than the values of reliability, which highlights the requirement of systematic repair for any complex systems for desirable performance. Additionally, availability is more in case (a) as compared to other cases that indicates that copula repair is far better than general repair.

Table 4 and figure 4 yield the MTTF of the system with respect to variation in failure rate $\lambda_1, \lambda_2, \lambda_3, \lambda_E$ and λ_C respectively, when other parameters were kept constant. MTTF of the system is decreasing concerning different failure rates. MTTF of the system is the highest for the failure rate of subsystem-3 and is the lowest concerning the catastrophic failure that indicates subsystem-3 is responsible for proper operation of the system. The MTTF in case of deliberate failure is almost the same for all λ_E . An acute examination from table-5 and 6 and figure-5 and 6 reveals that expected profit increases as service cost K_2 decreases, while the revenue cost per unit time is fixed at K_1=1 in case of both copula and general repair. The calculated expected profit is maximum for K_2= 0.1 and minimum for K_2=0.6. We observe that as service cost decreases, profit increase with variation of time. In general, for low service cost, the expected profit is high in comparison to high service cost. Conclusively, copula repair is more effective repair policy for better performance of repairable systems.

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Appendix 1

The state transition probability of the system are calculated under the presumption that the system is in state S_0, will remain in the state S_0 during the time $[t, t+\Delta t]$ and it will not move to any other state and if it is failed then after repair it will approach to state S_0. If the failure rate to move the state S_1, S_5, S_7 and S_8 during the time $[t, t+\Delta t]$ is $3\lambda_1\Delta t,\ 2\lambda_2\Delta t,\ \lambda_3\Delta t$ and $\lambda_C\Delta t$, then the rate that it will not move to the states will be $(1-3\lambda_1\Delta t),\ (1\text{-}2\lambda_2\Delta t),\ (1\text{-}\lambda_3\Delta t)$ and $(1\text{-}\lambda_C\Delta t)$. The state transition probability that the system is in state S_0 during t and $[t+\Delta t]$ is

$$P_0(t+\Delta t) = (1-3\lambda_1\Delta t)(1\text{-}2\lambda_2\Delta t)(1\text{-}\lambda_3\Delta t)(1\text{-}\lambda_C\Delta t)P_0(t)$$
$$+\left[\int_0^\infty \phi_1(x)P_1(x,t)\,dx\Delta t + \int_0^\infty \phi_2(x)P_3(x,t)\,dx\Delta t\right.$$
$$+\int_0^\infty \mu_0(x)P_6(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_7(x,t)\,dx\Delta t$$
$$+\int_0^\infty \mu_0(x)P_8(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_9(x,t)\,dx\Delta t$$
$$\left.+\int_0^\infty \mu_0(x)P_E(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_C(x,t)\,dx\Delta t\right]$$

$$P_0(t+\Delta t) = \left\{(1-3\lambda_1-2\lambda_2-\lambda_3-\lambda_C)\Delta t\right\}P_0(t)+$$
$$(\text{Product of two terms})(\Delta t)^2 + ..P_0(t) + \left[\int_0^\infty \phi_1(x)P_1(x,t)\,dx\Delta t\right.$$
$$+\int_0^\infty \phi_2(x)P_3(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_6(x,t)\,dx\Delta t$$
$$+\int_0^\infty \mu_0(x)P_7(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_8(x,t)\,dx\Delta t$$
$$+\int_0^\infty \mu_0(x)P_9(x,t)\,dx\Delta t + \int_0^\infty \mu_0(x)P_E(x,t)\,dx\Delta t$$
$$\left.+\int_0^\infty \mu_0(x)P_C(x,t)\,dx\Delta t\right]$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} + (3\lambda_1+2\lambda_2+\lambda_3+\lambda_C)P_0(t) =$$
$$\left[\int_0^\infty \phi_1(x)P_1(x,t)\,dx + \int_0^\infty \phi_2(x)P_3(x,t)\,dx\right.$$
$$+\int_0^\infty \mu_0(x)P_6(x,t)\,dx + \int_0^\infty \mu_0(x)P_7(x,t)\,dx$$
$$+\int_0^\infty \mu_0(x)P_8(x,t)\,dx + \int_0^\infty \mu_0(x)P_9(x,t)\,dx$$
$$\left.+\int_0^\infty \mu_0(x)P_E(x,t)\,dx + \int_0^\infty \mu_0(x)P_C(x,t)\,dx\right]$$

$$\left(\frac{\partial}{\partial t}+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right)P_0(t) = \left[\int_0^\infty \phi_1(x)P_1(x,t)\,dx\right.$$
$$+\int_0^\infty \phi_2(x)P_3(x,t)\,dx + \int_0^\infty \mu_0(x)P_6(x,t)\,dx$$
$$+\int_0^\infty \mu_0(x)P_7(x,t)\,dx + \int_0^\infty \mu_0(x)P_8(x,t)\,dx$$
$$+\int_0^\infty \mu_0(x)P_9(x,t)\,dx + \int_0^\infty \mu_0(x)P_E(x,t)\,dx$$
$$\left.+\int_0^\infty \mu_0(x)P_C(x,t)\,dx\right]$$

$$\left[\frac{\partial}{\partial t}+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right]P_0(t) = \int_0^\infty \phi_1(x)P_1(x,t)dx$$
$$+\int_0^\infty \phi_2(x)P_3(x,t)dx + \sum_k\int_0^\infty \mu_0(x)P_k(x,t)dx \qquad (1)$$

where $k = 6,7,8,9,E,C$